Fatigue life estimation of a non-linear system due to random vibration

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Abstract: *Random response analysis is a linear approach, while most real life random vibrations involve nonlinear components. It is challenge to analyze a nonlinear system subjected to random vibration. This paper presents an Abaqus FEA approach on the fatigue life calculation of an automobile assembly with rubber isolators subjected to random vibration. Random loading is categorized using Power Spectral Density (PSD). An equivalent dynamic analysis or a random response analysis was used to obtain the maximum stress level and location from random vibration. A MATLAB routine was used to post process Abaqus PSD response to calculate the stress cycles/peaks and the fatigue damages were estimated. Hyperelastic behavior of the rubber isolator was derived from Abaqus materials module and the corresponding tests. Random vibration test results with the same PSD input were used to tune and verify the FEA model.*

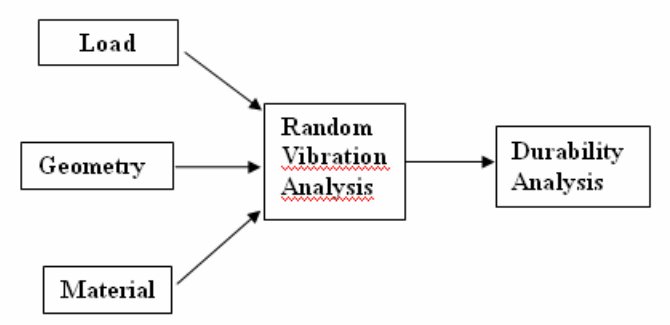
Keywords: FEA, Fatigue, Random vibration, PSD, Mises stresses, frequency, damage, rubber

Introduction

A metal bracket is used to hold ABS/ESP hydraulic actuator over a vehicle's lifetime. Rubber isolators are interfaced between the hydraulic unit, bracket and chassis frame to dampen the noise generated by the brake hydraulic component and reduce the road load transfer to hydraulic actuator. Although bracket is a simple part, it is a challenge to design a cost effective bracket that can fulfill the above functions during the entire vehicle life, especially when it is subject to random dynamic loading and interfaced with nonlinear rubber components.

This paper outlines a practical approach that has been used for estimating fatigue life or durability of such a system. Figure 1 shows the flow of the analysis. When a load is applied to a system, the load is transferred through the system from one component to another. The durability of a component is governed by the loading environment to which it is subject, the stress/strain arising from that load, and the response of the material that made up the components. System level random vibration analysis is performed where the random loading and response are categorized using Power spectral density (PSD) functions and the dynamic structure is modeled as a linear transfer function.

Figure 1 Random Vibration Fatigue Analysis Process of ABS/ESP bracket



Rubber Isolator Characterization

Rubber isolator is the major challenge for ABS/ESP bracket system modeling. Simulation of rubber is usually based on hyperelasticity and viscoelasticity. But for random vibration analysis of ABS/ESP actuation system, it is impractical to do computationally intensive transient dynamic analysis in the time domain, including actual isolator geometry and its hyperelasticity and viscoelasticity. Instead, an equivalent dynamic analysis or a frequency domain random vibration analysis is performed. The rubber isolator was modeled as spring and dashpot elements; alternatively the isolator was modeled as connector elements in which the friction and nonlinear stiffness can be specified. The stiffness and damping coefficients were estimated by separate tests and FEA models.

Two approaches were used to characterize the rubber isolator, both approaches were used to get ballpark rubber characteristics and the final values were tuned by a system level vibration test. The first approach is to measure the intrinsic properties of rubber, then build FEA model with actual isolator geometry to obtain the stiffness and damping coefficients. The advantages of this approach is that the intrinsic rubber hyperelasticity and viscoelasticity of rubber can be used for transient dynamic system modeling which includes actual component geometries and interfaces when enough computer power is available and such analysis become practical. The disadvantage is that the derived isolator stiffness and damping coefficient could be far away from actual value due to many affecting factors. The second approach is to measure the stiffness and damping characteristics of the actual isolator directly. This is a simpler and direct approach, but the obtained stiffness and damping characteristics can only be used for simplified system model; and difficulties in fixturing the isolator may result in large variation.

The static stiffness was determined by three steps. The first step is to perform a set of ASME standard tests (uniaxial tension/compression, biaxial tension, pure shear, volumetric compression), the second step is to use Abaqus to derive intrinsic hyperelastic materials model based on the test data, the third step is to estimate the stiffness of the isolator from FEA model which use actual isolator geometry and the hyperelastic materials parameters derived from previous steps. Figure 2 and Table 1 show the hyperelastic materials models on one type of rubber based on uniaxial tests.

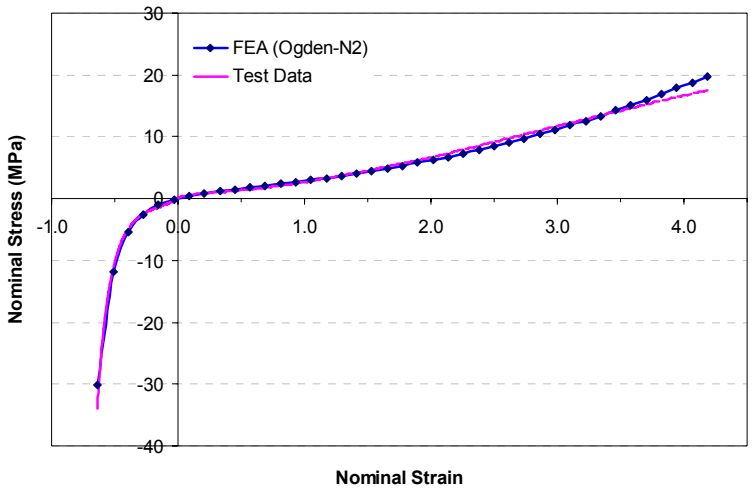


Figure 2 Materials evaluation – uniaxial compression and tension

	μ_1	α_1	μ_2	α_2	D_1	D_2
Ogden	0.63	3.376	1.143	-2.16	1.4123E-3	2.6139E-5

Table 1 Hyper-elastic material parameters

Figure 3 shows the displacement contour when an isolator is compressed. Figure 4 shows the compression force versus compressed displacement, the compression stiffness can be estimated by getting the slope of the linear fitted line. It can be seen that the stiffness increases as amount of compression (strain) increases.

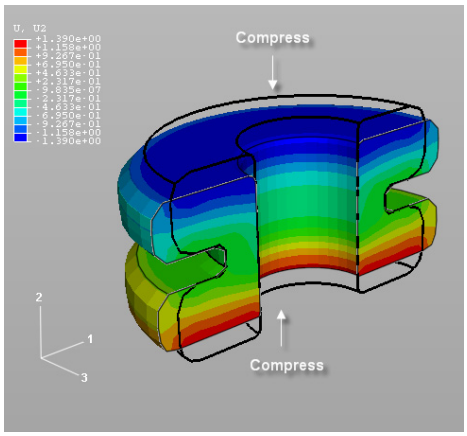


Figure 3 Displacement contour of an isolator

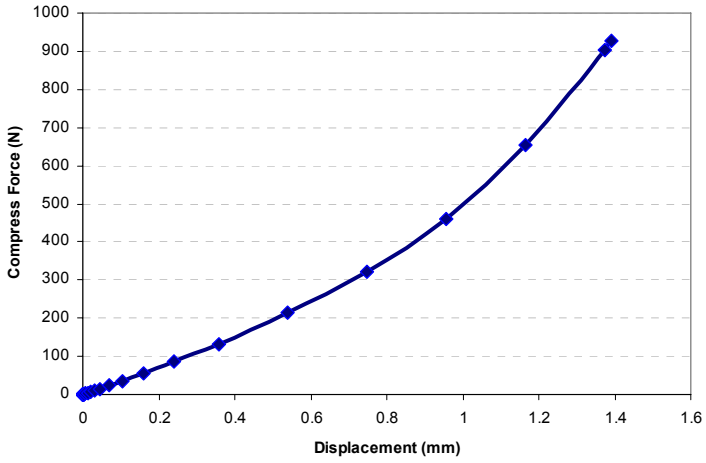


Figure 4 Isolator Reaction force versus Displacement

Dynamic stiffness is determined by performing DMA (Dynamic Mechanical Analysis). Figure 5 shows the stiffness versus frequency from the direct measurement on an isolator. Figure 6 & 7 show the modulus versus frequency from a materials test specimen at various temperatures.

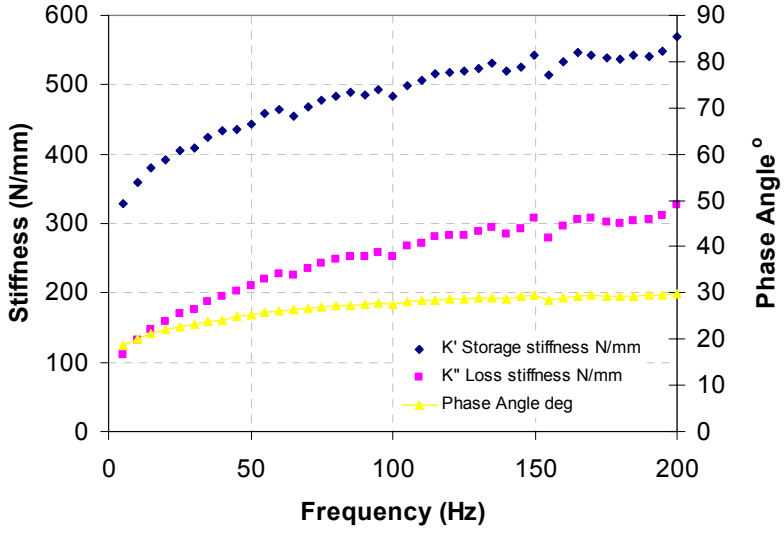


Figure 5 Isolator Stiffness as a Function of Frequency

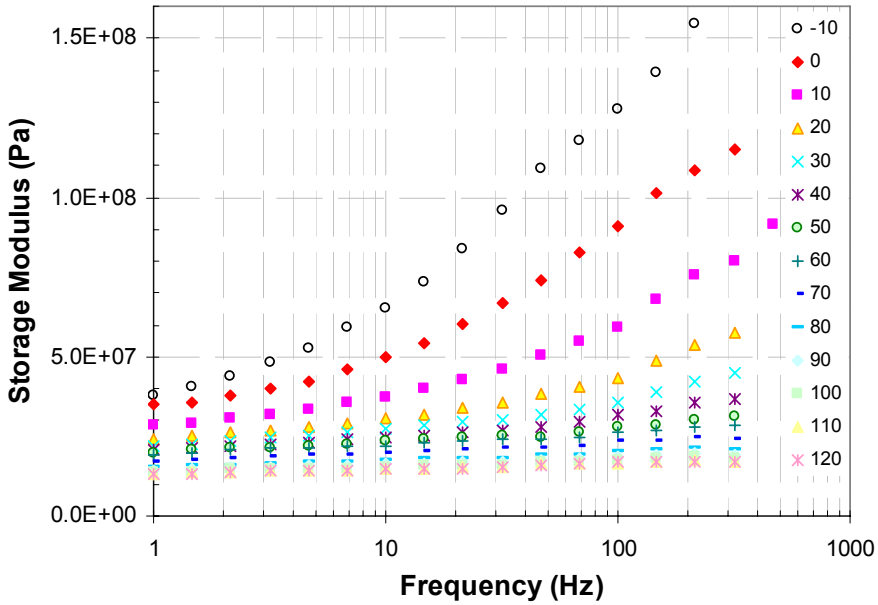


Figure 6 Storage Modulus as a Function of Frequency at 14 Temperatures

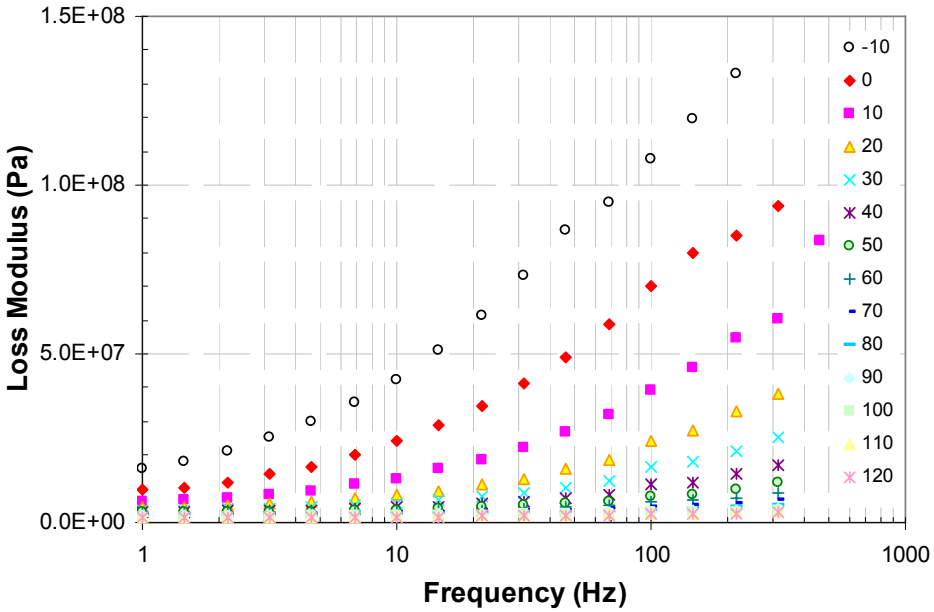


Figure 7 Loss Modulus as a Function of Frequency at 14 Temperatures

Random Vibration Analysis

Figure 8 shows a simplified model for a ABS/ESP bracket assembly. The FEA model includes the ABS/ESP bracket and a concentrated mass representing the ABS/ESP hydraulic unit. The bracket consists of welded top and lower parts, planar shell elements were used to model the bracket. Since the mid-surface is not available from CAD, the outer surface was used to generate the shell and the offset property was used to account for the shell thickness if two shells are in contact. The top and the lower bracket parts were connected at the welding region by tie constraints to constrain all degrees of freedoms. The isolators are modeled as 3 dimensional spring and dashpot elements, the isolator stiffness and damping coefficients were estimated as described above. The point mass with rotary inertial effect was connected to one end of the isolators with rigid coupling constraints; the other end of each isolator was connected to the bracket by a rigid coupling constraint. The bracket was rigidly fixed at the mounting holes and the base motion was applied to the fixed boundaries. This model is constructed to simulate the vibration test that was carried out to check the functional influence during the lifetime of the road load vibration. A road load profile representing lifetime of vehicle road load in PSD form was used to generate input vibration signal, the accelerated vibration load is applied at each axis with total test time equivalent to vehicle lifetime.

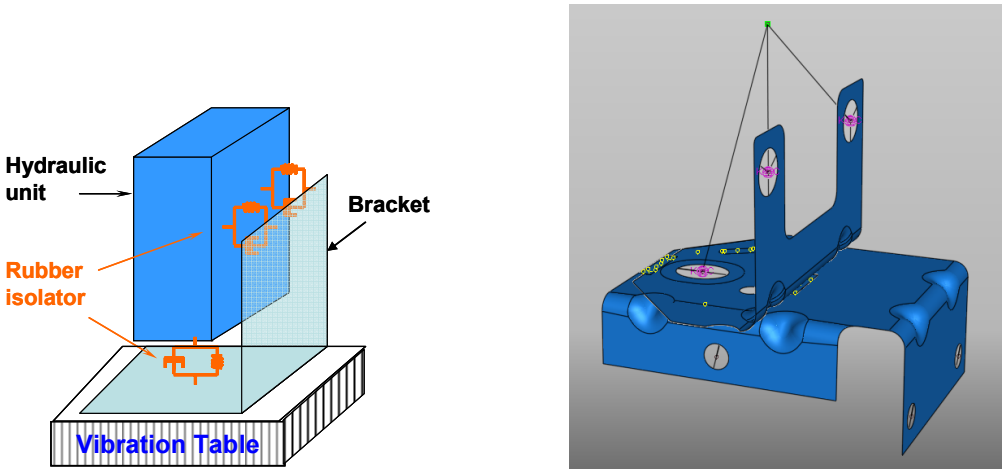


Figure 8 Model Representation

Random vibration is a vibration that can be described only in a statistical sense. The instantaneous magnitude is not known at any given time; rather, the magnitude is expressed in terms of its statistical properties (such as mean value, standard deviation, power spectrum density, and probability of exceeding a certain value). The random nature of the vehicle vibration makes it almost impossible to perform the direct transient nonlinear dynamic FEA analysis on ABS/ESP bracket system because it is very time consuming, cumbersome, and inconvenient. Two simplified methods were used to model the ABS/ESP random vibration:

The first method is an equivalent dynamic method. The bracket will go into resonance if the resonant frequency of the bracket occurs within the range of the frequencies provided by the input PSD. A bracket

will experience the greatest degree of fatigue damage when exposed to its resonant frequency. The resonance is powered by the level of energy provided at resonant frequency and amplified by the phenomenon of resonance, with a degree of magnification Q.

Equivalent Dynamic Force: $F = G_{out} * Weight$

Where

W = the weight of ABS/ESP unit

$G_{out} = \sqrt{\frac{\pi}{2} \times G_{in} \times F_n \times Q}$ - Acceleration at CG

F_n = Resonant Frequency (from FEA)

G_{in} = the level of energy (read from PSD at resonant frequency)

$Q = 2 \times \sqrt{F_n}$ - Transmissibility (amplification factor)

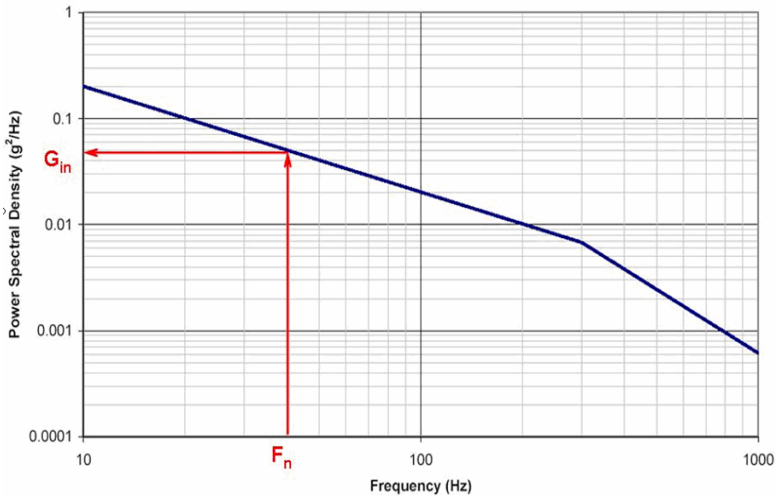


Figure 9 Example of Input vibration profile

The equivalent dynamic force F was applied at Center of Gravity (CG) in each direction and the nonlinear static analysis was performed; the static stiffness and damping coefficients of the rubber isolator were used for spring and dashpot elements. The resulting Mises stress distribution was obtained and was later used for durability analysis. The advantage of this method is its simplicity and low cost of calculation, we use this method for quickly sorting through design options and identifying the better designs in the early design stage. Our test results show that this approach is on the conservative side, which may be resulted from the calculation of transmissibility (amplification factor).

The second method is random response analysis in frequency domain, which reduced the system degrees of freedom to a set of modes. FEA performs random response analysis as a post step after frequency analysis.

The frequency analysis is used to generate the transfer function. The input PSD multiplies the transfer function to form a response PSD. The frequency dependent stiffness can be incorporated into analysis by dividing random vibration analysis into different frequency zones. The RMS Mises stress is calculated from RMS value of the stress components and the highest RMS Mises stress location is used for fatigue evaluation. More direct method of calculating RMS Mises stress of random vibration is through time integration of the stress invariants at each time step or through Segalman's method [2]. The simplified RMS calculation gave similar stress distribution to the equivalent dynamic analysis. A python script was used to post process Abaqus result file and perform such calculation automatically.

Durability Analysis

After obtaining random vibration analysis results, durability analysis base on an example bracket was performed as follows:

Step 1 - Determine the highest stress location/value from FEA calculation, either from equivalent dynamic analysis or random vibration analysis. Figure 10 shows the bracket's Mises stress distribution from equivalent dynamic analysis, Figure 11 shows the RMS Mises stress distribution from random vibration analysis. Both analysis show that the maximum stress for this bracket happens when the system was vibrated in X-axis direction, and the highest stress region is at the end of the back fillet weld on the base.

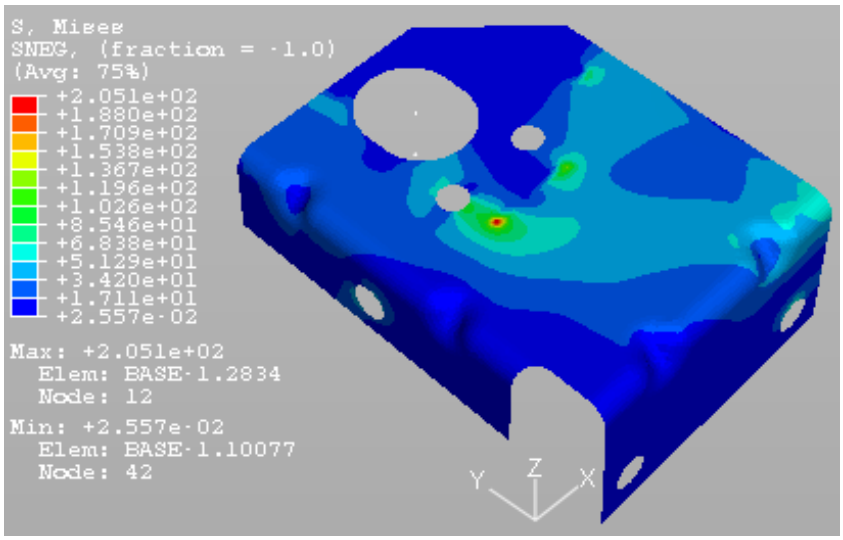


Figure 10 Maximum Mises stress distribution of the bracket (Equivalent dynamic analysis - excited at X axis)

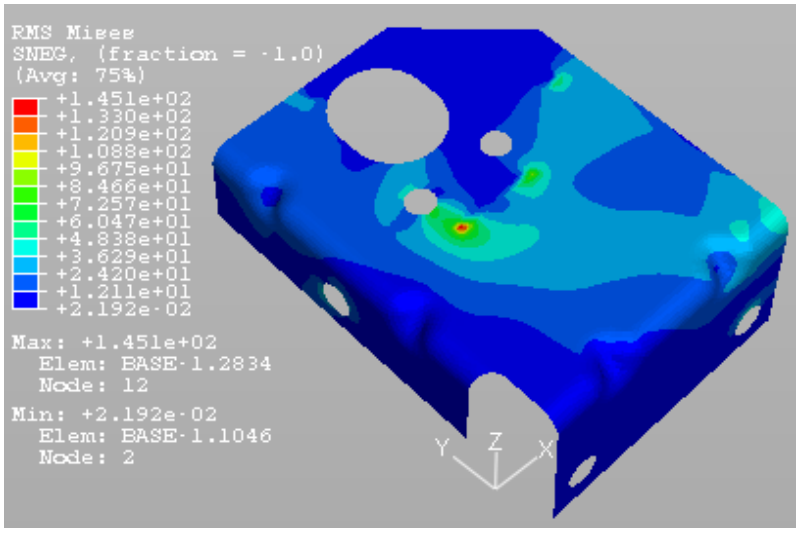


Figure 11 Maximum RMS Mises stress distribution of the bracket (Random response analysis - excited at X axis)

Step 2 - Determine local Life to Failure (cycles) of the highest stress location when apply the highest stress from step 1. The local Fatigue Strength (life to failure) is determined by the one sigma stress level, two sigma stress level and three sigma stress level from S-N curve of the bracket materials.

Step 3 - Convert PSD of the highest stress component to time based signal, calculate positive zero crossings. Matlab programs were developed to perform these calculations. Positive crossings of the stress component corresponding to the dominate mode were used for cycle counting. Figure 12 shows a dominate stress component in PSD and time based form.

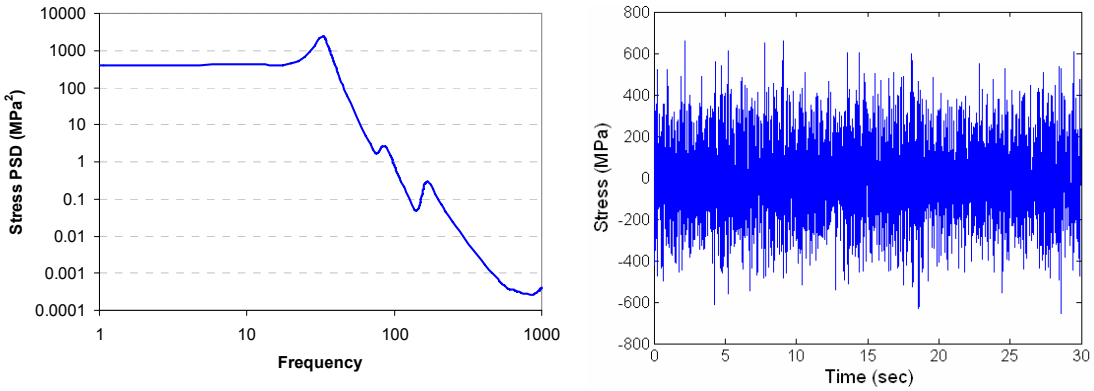


Figure 12 Stress component in PSD and time based form (positive zero crossing = 30)

Step 4 – Calculate number of operating cycles during the vibration process, conservatively assume stress cycles are zero crossings of the maximum dominate stress component. Assume vibration level varies as described by a normal distribution. Then the stress level of plus and minus one sigma will occur 68.3% of the time. A higher vibration level of plus and minus two sigma will occur 27.1% of the time, and even higher level of vibration (plus and minus three sigma) will occur 4.33% of the time. All three of these stress levels will be operating randomly during the vibration process.

Step 5 - Estimate fatigue damage and/or fatigue life.

By Miners rule, the fatigue damage is:

$$\text{Fatigue damage} = \sum_{i=1} \frac{ni}{Ni}$$ where n is the operating cycles, N is the life to failure

Miner's rule assumes that the part will fail when accumulated damage reaches 1, but significant scatter in the value of Damage was observed in some cases. Miner specified that the damage value of 1 is only an average. For decreasing level of loading sequence ($\sigma_1 > \sigma_2 > \ldots > \sigma_n$), there is in general failure for Damage<1, whereas failure occurs with Damage>1 when one applies the levels increasingly ($\sigma_1 < \sigma_2 < \ldots < \sigma_n$). For random vibration fatigue, damage rule can be best described by a statistical formulation:

$$D = \sum_{i=1} \frac{ni}{Ni} = \Delta$$

Where Δ, index of damage to failure, is a random variable of median value near 1, with a scatter characterized by the coefficient of variation V_Δ, the distribution being regarded as log-normal.

The failure takes place when $D > \Delta$, with a probability given by:

$Pf = P(D>\Delta)$

For example, one experiment (560 samples) shows that with a cumulative damage of $D < 0.3$, the probability of non-failure is 95%.

Concluding Remarks

In this paper we have introduced a practical approach for durability analysis of a non-linear system due to random vibration. A case study is discussed on assessing the durability of a ABS/ESP bracket.

Abaqus has good nonlinear materials capabilities for modeling hyper elastic and visco elastic behavior of the vibration isolator. But Abaqus does not have fatigue solver. RMS Mises stress calculation for random vibration does not exist, and the stress cycles need to be calculated separately by Matlab or other softwares. Also, random response analysis ignore local damping, estimated equivalent damping or post processing direct solutions is needed to account for localized damping.

With current software capabilities and computer power, it is challenging to evaluate random vibration fatigue of nonlinear system. But we are continuously enhance our modeling and test capabilities, with proper simplification on simulation and various test capabilities within Bosch CC/ECH-NA, we will be in a good position to successfully evaluate durability of ABS/ESP bracket assembly containing rubber isolator.

References

1. Christian Lalanne, “Random Vibration”, Mechanical Vibration and Shock, Vol. III., Hermes Penton Ltd, pp 33 – 135, 2002
2. Christian Lalanne, “Fatigue Damage”, Mechanical Vibration and Shock, Vol. IV. Hermes Penton Ltd, pp 69 – 190, 2002
3. Daniel J. Segalman et al, “An Efficient Method for calculating RMS von Mises Stress in a Random Vibration Environment”, Sandia National Lab. Report, 1988.