Use of FEA and Radial Basis Functions for Reliability-based Design and Assessment of Tubular Connection Sealability

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Abstract: One of the significant challenges in oil wells is to maintain adequate structural and sealing capacities of casing and tubing connections. In particular, unconventional wells, such as thermal and HPHT wells, require elevated standards to evaluate structural integrity and sealability of tubular connections. This paper presents a Reliability Based Design & Assessment (RBDA) methodology that characterizes system safety using a quantitative estimate of reliability. In the RBDA approach, Finite Element Analysis (FEA) using Abaqus is employed to determine the connection sealing capacities as functions of several design parameters, such as connection geometry, material mechanical properties, and make-up and operational conditions. Based on the FEA results to characterize the effect of design parameters and variations, an implicit limit state function can be established using Radial Basis Functions (RBF). This paper also presents an FEA example of a generic premium connections to demonstrate the use of the proposed methodology.

Keywords: Casing; Connection; FEA; HPHT; Leakage Resistance; Radial-Based Function, RBDA; RBF, Reliability-Based Design Assessment; Sealability; Structural Integrity; Tubing.

1. Introduction

Oil wells are completed with casing and tubing strings consisting of many tubular joints (usually 12 to 14 m long) joined together by threaded connections. Tubular connections are one of the most critical components of an oil well in terms of risk of wellbore failure (Xie 2006). Payne and Schwind (1999) noted that, based on industry estimates, connection failures account for 85% to 95% of all oilfield tubular failures.

In recent decades, unconventional wells such as thermal and High Pressure and High Temperature (HPHT) wells pose more significant challenges to well designs. Thermal wells typically operate under cyclic high temperature in the range of 220°C to 350°C. For HPHT wells, high pressure (greater than 69 MPa) and high temperature (greater than 150°C) conditions are present. The tubular designs for unconventional well will require elevated standards to account for the plasticity load conditions resulting from high pressure, high temperature and formation movement.

Unconventional wells typically use premium connections, which generally show superior structural and sealing capacity over API round and buttress connections. Figure 1 shows a generic premium connection which uses a buttress-type thread form to meet the structural requirements of the full pipe body strength capacity, and incorporates a metal-to-metal radial seal for enhanced
sealability. Many premium connections also include a torque shoulder next to the seal region to control make-up torques and provide additional sealability.

**Figure 1. Schematic of a premium connection.**

To date, deterministic design approaches have been used to ensure the structural integrity and serviceability of tubular connections. In order to evaluate the sealability of connections, the deterministic approach applies a leakage resistance criterion to the Finite Element Analysis (FEA) results for the connection design with the worst geometry, minimum yield strength and worst combination of loads. However, the deterministic design approach appears to be overly conservative as it does not take into account the probabilities and uncertainties related to the variability in tubular connection geometry, material properties (e.g. yield and tensile strengths), and operational loads. To balance operational safety and cost, Reliability-Based Design and Assessment (RBDA) is recommended for use to optimize the connection design and minimize the risk of failure due to connection leakage (Xie et al. 2012, 2014).

In RBDA, the probability of failure, $P_f$, is predicted by explicitly addressing the uncertainty in capacity (e.g. resistance) and demand (e.g. load, deformation) through probability distributions. $P_f$ depends on the degree of overlap between these two distributions (Figure 2) (i.e. cases where demand exceeds capacity).

Connection sealability in unconventional wells, such as thermal, HPHT wells, is often associated with various uncertainties related to the variability in casing/connection geometry (e.g. thread and seal interferences and tapers), material properties (e.g. yield and tensile strength), cement completeness, formation properties, and operational loads (e.g. temperature and pressure). Due to the non-linear mechanical and material responses, it is virtually impossible to establish analytical limit state functions that represent the structural and serviceability requirements for tubular connections in HPHT and thermal wells. As such, Finite Element Analyses (FEA) are often used to assist the establishment of numerical limit state functions.
This paper presents a RBDA methodology that characterizes system safety using a quantitative estimate of reliability. In the RBDA approach, Finite Element Analysis (FEA) using Abaqus is employed to determine the connection sealing capacities as functions of several design parameters, such as connection geometry, material mechanical properties, and make-up and operational conditions. Based on the FEA results of design parameters and variations, an implicit limit state function can be established using Radial Basis Functions (RBF). This paper also presents an FEA example for a generic premium connection to demonstrate the use of the proposed methodology.

![Figure 2. Illustration of capacity and demand interference.](image)

### 2. FEA Model for Connection Sealability

FEA model of the connection should be developed to assess the following conditions:

- Make-up;
- Internal and external pressures;
- Axial tensile and compressive loads; and
- Curvature loading such as bending and shear.

This paper presents the connection model considerations taken into account when using the commercial finite element analysis software, Abaqus v6.14 (2014).

Figure 3 shows the FEA mesh for a generic premium connection and an adjoining section of pipe body. Note that due to the symmetry of deformation about the connection center under the imposed loading (except for in the case of shear deformation which is not discussed here), only half of the coupling is modeled.

At the coupling centre plane, symmetric boundary conditions are assumed. For axisymmetric loading analyses, such as make-up, axial forces and pressure loading, the connection is modeled using axisymmetric solid elements. However, for the curvature loading analysis, such as bending and shear, the connection is modeled using axisymmetric solid elements with non-linear, asymmetric deformation. As noted in the Abaqus documentation (2014), these elements are intended for the non-linear analysis of structures which are initially axisymmetric but may undergo non-linear, non-axisymmetric deformation, which is the case for the connection and load scenarios examined here. Contact between the pin and coupling elements within the connection is modeled using slidelines.
The tubular connection material response should be modeled using an elastic-plastic, non-linear kinematic hardening constitutive relationship. A comprehensive material model should incorporate both temperature-dependent and time-dependent characteristics of the casing material. These modeling considerations are needed to properly capture material behavior variation with temperature, stress relaxation and Bauschinger effects (i.e. a reduced yield stress upon load reversal after the plastic deformation that occurs during initial loading).

Tubular connections should be modeled with sufficient mesh density, especially in critical areas such as the thread root, seal and shoulder regions, to ensure accuracy and convergence of analysis results. Over the seal region, a mesh density of 0.1 mm/element is recommended along the seal direction, to ensure an accurate prediction of seal contact intensity. To accurately predict the structural behavior of the connection threads under the various loading conditions considered, a more refined mesh density (e.g. 0.04 mm/element) is suggested for the areas next to the thread roots where relatively high stress/strain concentrations are expected to be present.

Connection make-up is modeled through the interference fit in the thread, seal and shoulder. The interference amount in the threads and the seal is typically defined by the connection drawing. The shoulder interference is defined, in this paper, in correlation with the make-up torque values.

For modeling the load conditions, the axial tensile/compressive forces are modeled by applying global forces over the pin end section of the model; the external pressure is modeled by applying the pressure loading on the OD surface of the connection, on the pin and coupling surfaces over the thread region, and on the partial sealing region up to the anticipated maximum contact stress point on the seal; and the internal pressure is modeled by applying the pressure loading on the ID surface of the pin and coupling surface and on the torque-shoulder region.

Details of the FEA methodology for assessing tubular connections can be found in several publications (Xie 2007; Xie 2011; Xie et al. 2011).
3. Limit State for Connection Sealability

A limit state function represents the relationship between capacity and demand parameters, as well as differentiates between safe and failure domains:

\[ g(C, D) = \text{Capacity} - \text{Demand} \]  \hspace{1cm} (1)

The sealing capacity of a premium connection is determined by the magnitude and distribution of the metal-to-metal contact stress that exists over the seal region under each load condition. As a necessary requirement, the seal contact stress must be higher than the differential pressure across the inner and outer surfaces of the casing in order to maintain sealability. Figure 4 presents an illustration of seal contact stress in a premium connection. The following is the limit state for sealability of premium connections:

\[ g_{sealing} = S^C - S^D \]  \hspace{1cm} (2)

where \( S^C \) is the connection sealing capacity to resist leaking, and \( S^D \) is the differential pressure demand. \( S^C \) is typically expressed as a function of the contact stress profile (i.e. magnitude, length, and distribution) over the seal region, and \( S^D \) is often expressed in terms of differential pressure of gas/liquid and sealing compound.

Note that to be consistent with ISO 13679 requirements (ISO 2002), the torque shoulder region is not considered to be a part of the sealing mechanism for the leakage resistance assessment.

In a more generalized assessment approach, the sealing capacity of a premium connection can be represented by the “Seal Contact Intensity” (\( f_s \)), which is defined as the integration of the seal contact stress (\( \sigma_c \)) over the effective seal length (\( L_{ES} \)):

\[ f_s = \int_{L_{ES}} \sigma_c dx \]  \hspace{1cm} (3)

Based on Xie et al. (2012), a seal contact intensity value of equal to or greater than 250 N/mm is considered sufficient for thermal wells where the fluid pressure is typically low (e.g. less than 15 MPa).
For HPHT wells, Xie (2013) recommended using a weighted area of seal contact stress for evaluating connection sealability:

\[ W_a = \int_0^L P_c^n (l) \, dl \]  

(4)

where \( P_c(l) \) is the seal contact pressure, \( L \) is the seal length, and \( n \) is a correlation exponent. Based on test results, Murtagian et al. (2004) proposed values of 1.2 and 1.4 for the exponent \( n \), for connections with and without sealing compounds, respectively.

A premium connection might be considered to provide acceptable seepage resistance when the weighted area of seal contact stress \( W_a \) is greater than some critical value \( W_{ac} \). Based on experience, further review of the available data and incorporation of the ISO 13679 leakage rate limit, the following equation for calculating the critical value of \( W_{ac} \) was recommended for HPHT tubular connections (Xie 2013):

\[ W_{ac} = 0.01 \left( \frac{P_{gas}}{P_{atm}} \right)^{0.838} \]  

(5)

Surface roughness could also have an impact on the leakage rate for premium connections. The effect of the roughness properties on the leakage rate could be studied through analytical simulations and physical tests. Further studies are also required to develop suitable sealing criteria for sour gas wells.

4. RBDA of Connection Sealability

Reliability analysis is performed using Monte Carlo simulation on the limit state functions defined either explicitly or implicitly. Since FEA is used to determine the connection sealability, the limit state function may not be explicitly defined as a function of the design variables. In the absence of an explicit format, surrogate functions need to be developed to approximate the limit states using techniques such as the Response Surface Method (RSM) (Rajashekhar 1993; Haldar 2000) or Radial Basis Functions (RBF) (Buhmann 2003). The proposed RBDA approach for connection integrity includes the following three steps:

**FEA Sampling**

Due to the large number of relevant variables in connection design and load conditions, a significant number, possibly millions, of Monte Carlo simulations may be required to derive a prediction of probability of failures. It is then impossible to perform detailed FEA for each Monte Carlo simulation point. A more practical approach would be to perform the reliability analysis based on selective FEA on parameters with more significant influence. The FEA sampling cases can be defined through trial analyses.

**Surrogate Function**

A surrogate function can be developed based on the FEA data to facilitate the Monte Carlo simulations. The surrogate function can be developed by using a linear Radial Basis Function (RBF) interpolation and extrapolation from the FEA dataset. The RBF uses the Euclidian distance between points to perform the interpolation (Buhmann 2003). To improve the accuracy of the
RBF interpolation, all input data are normalized to unit length using the range of the FEA data. This method was configured to ensure that the surrogate function passed exactly through the FEA data points (i.e. the nodal points). Deng (2006) noted that unlike the response surface method, with RBF based approaches, it is not necessary to know the underlying relationship or to suppose a relationship between the input variables and the output. The RBF is a universal approximator and can be used to approximate linear or non-linear, implicit or explicit performance functions. The RBF based Monte Carlo simulation is especially useful for reliability problems with implicit and nonlinear performance functions where other reliability methods are not applicable.

According to Buhmann (2003), an RBF-based surrogate function is calculated from the weighted sum of radial basis functions that typically use the n-dimensional Euclidian distance from the interpolation point to each of the FEA input data points:

\[ y(x) = \sum_{i=1}^{N} w_i \phi(\|x - x_i\|) \]  

where

- \( r_i = \|x - x_i\| \) = the Euclidian distance to the coordinates of FEA data point \( x_i \)
- \( \phi(r_i) \) = the radial basis function for \( x_i \)
- \( w_i \) = weight for \( \phi(r_i) \)

The surrogate function weights \((w_i)\) can be calibrated using the least squares method, because the function is linear in \( w_i \) (or an iterative neural network learning method) (Buhmann 2003; Deng 2006).

Isight (2014) implemented a methodology to calibrate the shape function variable of a power spline RBF, for minimizing the errors resulting from data interpolations/extrapolations during cross-validation.

For the work presented in this paper, linear RBF was used for ease of implementation and flexibility in performing Monte Carlo Simulation for the RBDA.

**Monte Carlo Simulation**

Monte Carlo simulation is implemented to determine the reliability of connection integrity by establishing the portion of cases that would have the weighted area of seal contact stress less than the threshold value. This method provides a means to establish the effect that changes to the design (shifted mean, \( \mu \)) or improved consistency (reduced standard deviation, \( \sigma \)) of a particular design parameter would have on the reliability of the tubular connection.

**RBDA Flow Chart**

Figure 5 presents the flow chart for RBDA which is composed of FEA sampling, establishment of surrogate function using RBF, and Monte Carlo Simulation.
5. Analysis Example

An analysis example is presented here to demonstrate the use of the proposed RBDA approach for casing connections in HPHT application. The analysis example used a 177.8 mm, 34.2 kg/m P110 generic premium casing connection. The connection model includes the basic features common to many of the premium connections currently used in HPHT well applications (e.g. buttress thread form, axial torque shoulder, and radial metal-to-metal seal) so that the analysis results would be reasonably representative for premium connections in the HPHT application.

5.1 Load Scenarios

Guidelines for qualification of tubing and casing connections have been established by ISO 13679, “Petroleum and natural gas industries — Procedures for testing casing and tubing connections” (ISO 2002). According to ISO 13679, tubular connections for HPHT applications should meet the requirements of a Connection Application Level (CAL) IV.

The RBDA of the analysis example considered the response of the generic premium connection subjected to combined load conditions (i.e. axial tensile/compressive forces, and internal/external pressures) as specified by the ISO 13679 CAL-IV Test Series A category.
Figure 6 presents the Test Series A load path for a 177.8 mm, 34.2 kg/m P110 generic premium connection, with a rated axial load capacity greater than or equal to the pipe-body in compression. The load path was calculated based on guidelines provided by ISO 13679 (2002). There are a total of 14 load points with various combinations of internal/external pressures, and tensile/compressive forces included in the analysis.

![Figure 6. Test Series A load path for a connection with an axial load rating greater than or equal to the pipe body in compression (ISO 2002).](image)

5.2 Base Case Results

Figure 7 presents the contour plots of radial stress for the base case at make-up, Load Point 4 (axial tension and internal pressure) and Load Point 13 (axial compression and external pressure). The figure shows that at make-up, the threads are quite uniformly engaged through the thread length, and the contact stress is distributed over the seal length. At Load Point 4, the internal pressure appears to enhance the contact stress at the seal. However, at Load Point 13, the seal contact stress is reduced significantly under external pressure.
Figure 7. Contour plot of radial stress (MPa) at make-up (top), Load Point 4 (middle) and Load Point 13 (bottom).

Figure 8 presents the weighted contact stress area $W_a$ and the critical value of $W_{ac}$, calculated using Equations 4 and 5. Since the internal/external pressures vary through the load points, as shown in Figure 6, the critical value of $W_{ac}$ also varies in Figure 8. As shown for this base case, the results of the weighted area of seal contact stress are higher than the critical values for all load points. However, the base case analysis suggests that the potential leakage scenarios could include:

- Load Points 2 through 4 for containing the internal pressure when the connection is subjected to nearly the maximum internal pressure and the maximum axial tensile force;
- Load Points 10 through 14 for containing the external pressure, when the connection is subjected to nearly the maximum external pressure and a high tensile force.

Among all the load points, Load Point 13 with a combination of axial compression force and external pressure appears to be the most critical in terms of connection leakage resistance. As such, Load Point 13 was chosen for the reliability assessment in this paper.
5.3 Parametric Analyses

The following parameters were considered for the sensitivity analyses to facilitate the reliability assessment of connection sealing capacity:

- thread and seal interferences,
- thread and seal tapers for pin and coupling,
- make-up torque, and
- material yield strength.

In defining the distribution of the varying geometric parameters, the tolerances specified by the connection drawing were considered to cover a range of four standard deviations centered on the mean value (i.e. $\mu - 2\sigma < x < \mu + 2\sigma$, where $\mu$ is the mean value and $\sigma$ is the deviation). Such defined range would cover up to 95.5% of all possible cases given a normally distributed variable.

Figure 9 presents a tornado plot of the effect of each variable on the weighted area of seal contact stress, $W_a$, based on the trial FEA. As shown in the figure, the thread and seal interference are identified as the top two primary variables. The other parameters are considered to be secondary variables.

For the two primary variables, a full factorial study was performed with five treatments for each variable with the following levels: $\mu - 3\sigma; \mu - 1.5\sigma; \mu; \mu + 1.5\sigma; \mu + 3\sigma$. This procedure required a total of 25 (i.e. $5^2$) simulations for the two top influential variables. Approximately 200 FEA simulations were performed to cover the sensitivities of all variables.

FEA results of $W_a - W_{ac}$ values for the full factorial FEA conducted on the two most influential variables are presented as hollow circles in Figure 10. It shows that the weighted area of seal contact stress decreases with the thread interference and increases with the seal interference.
5.4 **Surrogate Function and Reliability Analysis**

Based on the FEA results discussed above, a surrogate function was established using a linear RBF interpolation. Figure 10 presents $W_a - W_{ac}$ values interpolated and extrapolated using RBF surrogate function for a range of thread and seal interference domain. The FEA dataset is shown as hollow circles in the figure. The figure demonstrates that the surrogate function generates a response that is smooth and produces a representative output in the extrapolation regions.

To evaluate the accuracy of the surrogate function, a “leave one out” cross-validation was performed. The procedure tested the surrogate function at each FEA data point by removing that point from the dataset and using that reduced dataset (i.e. dataset-1) to calibrate the surrogate function. Figure 11 shows the comparison of all the FEA data points to the response from the surrogate functions with reduced datasets. Interpolated/extrapolated data points represent generated points based on interpolation/extrapolation from the reduced FEA dataset, using the RBF method. The reasonably accurate correlation suggests that the proposed method for developing the surrogate function was adequately accurate in the region of the assessment limit ($W_a-W_{ac}=0$) as a result of the finer resolution of the dataset in this region. In all cases, the call between pass and failure is made correctly (i.e. above or below the assessment limit).

Note Isight (2014) offers calibration of power spline RBFs by minimizing the errors between the FEA data points left out of the dataset and the corresponding interpolated/extrapolated results.
Figure 10. $W_a - W_{ae}$ (mm·MPa$^{1.4}$) values using RBF surrogate function in thread interference and seal interference domain.

Figure 11. “Leave one out” cross-validation of surrogate function.
Monte Carlo simulation was performed to determine the reliability of the connection sealability. The results of the Monte Carlo simulation can establish the portion of connections that would have a weighted area of seal contact stress lower than the limit established by Equation 5 based on the probability functions of the design parameters.

The method presented in this paper provides a means to establish the effect that changes to the design (i.e. shifted mean, \( \mu \)) or the manufacturing consistency (i.e. reduced standard deviation, \( \sigma \)) of any given design parameter would have on the reliability of the connection sealability. For example, by simultaneously adjusting the mean values of the three most significant parameters (i.e. seal interference, thread interference and yield stress), the effect on the probability of failure due to connection sealability is shown Figure 12. The \( \alpha \) value in the figure is the shift factor over the standard deviation \( \sigma \). If the means for these three design parameters deteriorate from the mean value by one \( \sigma \), the probability of failure will increase dramatically from 0.0008\% (without shift) to 0.18\% (with the maximum shift), as shown in Figure 11.

![Figure 12. Effect on probability of failure due to mean value deterioration.](image)

6. Platform for RBDA

The key objective for the proposed approach is to assist the designers/manufacturers to optimize the connection design by adjusting design parameters, thereby minimizing the risk of failure due to connection leakage. A platform can be developed to facilitate the RBDA.

Figures 13 shows the interface of the platform for calculating capacity, demand and reliability. The data input for connection design in embedded in the capacity interface, which allows a user to specify statistical distribution (e.g. distribution type, mean and standard deviation) of connection design parameters (e.g. geometry, material property, and make-up torque). The distribution types for the data input include normal, uniform, log-normal, weibull, and triangular. The reliability calculation interface allows a user to perform calculations to determine the reliability and probability of failure for connection sealability under internal and external pressures.
The presented RBDA platform can be used for other connection designs, provided additional FEA of various parametric cases are performed and the additional FEA data are stored in the platform dataset. Designers/manufacturers can use this platform to adjust design parameters to reach a targeted reliability for acceptance of the product.

7. Summary

This paper presents a methodology for RBDA of tubular connection sealability for oil wells. The RBDA was based on FEA of tubular connections under various load conditions such as make-up, axial tension and compression, internal and external pressures, and curvature loading. A surrogate function is established using a linear RBF interpolation to facilitate the Monte Carlo simulations to determine the reliability of connection sealability. An example analysis was performed to demonstrate the use of the proposed methodology for RBDA of connection sealability for HPHT well application.
The presented FEA-based RBDA methodology can be used by manufacturers to optimize the connection design by adjusting the mean value and the distribution of key design parameter, to reach a minimal risk of connection failure due to leakage.

8. References


2015 SIMULIA Community Conference
www.3ds.com/simulia
9. Acknowledgements

The work presented in this paper was supported by C-FER Technologies (1999) Inc., Canada. The authors would like to sincerely acknowledge Dr. Maher Nessim, Chief Engineer at C-FER and FCAE, for his technical guidance in the development of the reliability analysis model; Dr. Nader Yoosf-Ghodsi, Senior Research Engineer at C-FER, for his technical review of this study.