Applicability of Modal Dynamic Method in Seismic and Aircraft Crash Analyses of NPP Structures.

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Abstract: *This work is considered to overview the applicability of modal dynamic method in seismic and aircraft crash analysis of NPP structures. Soil-structure interaction is taken into account; problem of 20%-limiting of modal composite damping is researched; results of different methods (Direct integration method, modal method, and upgraded modal method, when projected damping matrix is fully populated) are compared on test model and on real structure model. Simple test was solved theoretically and in ABAQUS, global damping matrices, acquired by modal and direct integration methods are compared. It is worth noting that both methods are approved for use in dynamic analysis of NPP structures by ASCE 4-98, but in most cases direct integration method gives more conservative results.*

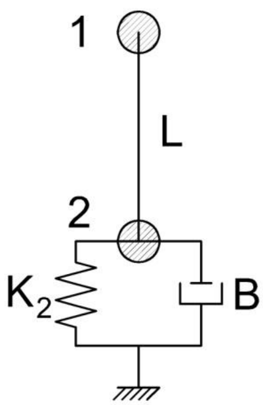
Keywords: *soil-structure interaction, modal composite damping, modal method, direct integration method.*

At present time in safety substantiation of NPP under special dynamic effects such as seismic and aircraft crash two methods of analysis are applied. These are modal method and direct integration method. However in "soil-structure" system, where radiation damping in soil essentially exceeds material damping in civil structures, modal method provides some inaccuracy in results obtained, besides the greater difference in damping in different parts of the system the greater inaccuracy is. It is related with inaccuracy in calculation of modal composite damping, which is determined as weighted value proportional to the stiffness of the element, and this inaccuracy increases in case of big radiation damping.

There is no such disadvantageous in direct integration method because of using "soil" dashpot instead of modal composite damping in dynamic analysis. At the same time there is important disadvantage in direct integration method – the Rayleigh damping in materials. The case is that Rayleigh damping supports very conservative results with respect to their normative values. Particularly in concrete under seismic loading of SSE level in accordance to ASCE 4-98 material damping should be equal to 7%, and Rayleigh damping on the working frequencies may droop to 2-3%. This leads to overestimating the dynamic response of the system. So in this paper, two above mentioned methods are compared with each other on the simple test and on the real structure test.

Let us consider simple beam structure shown on figure 1, which models vertical vibrations of the reactor building. In this test reactor building is modeled by the beam finite element, and it's mass corresponds to mass of the real building. Soil stiffness is modeled by spring, and radiation damping – by dashpot element. First frequency of vertical vibration in this model is equal to the same frequency of the real complex FE model of the building.

Initial data for the structure are presented below.



$E = 2.1 \cdot 10^7$ kN/m^2 – Young modulus of beam material,

L = 60 m – length,

F = 870 m^2 – section area,

$\rho = 3.72$ t/m^3 - density,

$K_2 = 5 \cdot 10^8$ kN/m – soil stiffness,

$B = 1.4 \cdot 10^7$ kNs/m – radiation damping in soil,

$\xi = 0.04$ - damping ratio in concrete.

Figure 1 – Soil – structure system model

Mass of the system is $M=\rho FL=194184t$, and stiffness of the beam – $K_1 = EF/L = 3 \cdot 10^8$ $\kappa N/m$. Let us $K = 10^8\, kN/m$, then $K_1 = 3K$, and $K_2 = 5K$.

So, consider two degree of freedom system, shown on figure 1.

To analyze the influence of the methods on the results, let us show obtaining of damping matrix in modal method and in direct integration method.

1. Stiffness and mass matrices have the following appearance:

$$[K] = \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 + K_2 \end{bmatrix} = \begin{bmatrix} 3K & -3K \\ -3K & 8K \end{bmatrix},$$

$$[M] = \begin{bmatrix} M/2 & 0 \\ 0 & M/2 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}.$$

Determining modes of vibration for the following system of equations:

$$[K - \lambda M]\phi = 0 \ , \tag{1}$$

where λ and ϕ - are eigenvalue and eigenvector of the system.

So demand for determinant of (1) equal to zero we write as following

$$\begin{bmatrix} 3k - \lambda m & -3k \\ -3k & 8k - \lambda m \end{bmatrix} = 0 \tag{2}$$

Or:

$(3k\text{-}\lambda m)(8k\text{-}\lambda m)\text{-}9k^2=0$), and

$\lambda^2\text{-}11\lambda\omega^2+15\omega^4=0$,

where $k/m=\omega^2=1030$.

From the last equation we obtained:

$\lambda_1=1.6\omega^2=1650;\ \lambda_2=9.4\omega^2=9682$

or $f_1 = 6{,}4\ Hz$ and $f_2 = 15{,}6\ Hz$. Then we determine eigenvectors as following

$\phi_2 = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$, and

$$(3k - 1.6k; -3k)\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = 0$$

From this equation we obtain $\phi_1 = U_1\begin{pmatrix} 1 \\ 0.47 \end{pmatrix}$

From mass normalization $\phi_1^T M \phi_1 = 1$: from which

$U_1^2(1, 0.47)\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} 1 \\ 0.47 \end{pmatrix} = 1$, where

$U_1 = \dfrac{1}{\sqrt{m} \cdot \sqrt{1 + 0.47^2}} = 2.9 \cdot 10^{-3}$ and

$$\phi_1 = \begin{pmatrix} 29 \\ 14 \end{pmatrix} \cdot 10^{-4}$$

Similarly, second eigenvector is easily determinable by analogy:

$$\phi_1 = \begin{pmatrix} 14 \\ -29 \end{pmatrix} \cdot 10^{-4}$$

It should be noted that this eigenmodes and eigenvectors are well matching with calculations by Abaqus code[2].

Now, define modal composite damping by using following formula:

$$\xi_i^M = \frac{\varphi_i^T [\widetilde{K}] \varphi_i}{\varphi_i^T [K] \varphi_i}, i = 1{,}2 \tag{3}$$

In (3) $\widetilde{K}$- modified stiffness matrix, each element of which is formed by multiplying value of stiffness on damping ratio in the element. Damping ratio in spring element is determined as:

$$\xi_2 = \frac{B}{2\sqrt{K_2 M}} 100\% = 70\%$$

As a result $\widetilde{K}$ can be obtained as:

$$[\widetilde{K}] = \begin{bmatrix} \xi_1 K_1 & -\xi_1 K_1 \\ -\xi_1 K_1 & \xi_1 K_1 + \xi_2 K_2 \end{bmatrix} = \begin{bmatrix} 0.04 \cdot 3 & -0.04 \cdot 3 \\ -0.04 \cdot 3 & 0.04 \cdot 3 + 0.7 \cdot 5 \end{bmatrix} \cdot 10^8 = \begin{bmatrix} 0.12 & -0.12 \\ -0.12 & 3.62 \end{bmatrix} \cdot 10^8$$

Then we calculated modal composite damping for different shapes as:

$$\xi_1^M = \frac{(29\,,14)}{(29\,,14)} \frac{\begin{pmatrix} 0.12 & -0.12 \\ -0.12 & 3.62 \end{pmatrix} \begin{pmatrix} 29 \\ 14 \end{pmatrix}}{\begin{pmatrix} 3 & -3 \\ -3 & 8 \end{pmatrix} \begin{pmatrix} 29 \\ 14 \end{pmatrix}} = 43\%$$

by analogy $\xi_2^M = 32\%$

Abaqus gives close results for modal composite damping also.

On this basis global damping matrix, calculated through modal and direct integration approach, shall be defined. It should be noted that in present analysis the direct integration approach is used as reference one, and, in spite of the fact that generally the approach gives more conservative results due to Rayleigh damping, however, in this particular case, it has been avoided. This is possible because in system with two degrees of freedom, unlike complex systems, Rayleigh damping exactly matches with the standard normative values.

Rayleigh damping for $\omega_1=2\pi f_1$=40.2 rad/s and $\omega_2=2\pi f_2$=97.97 rad/s and ξ=0.04 is defined below:

$$\alpha = 2\xi\omega_1\omega_2/(\,\omega_1 + \omega_1) = 2.28,$$

$$\alpha = 2\xi/(\,\omega_1 + \omega_1) = 0.00058,$$

$$C = \alpha M + \beta K = 2.28 \begin{pmatrix} 97092 & 0 \\ 0 & 97092 \end{pmatrix} + 0.00058 \begin{pmatrix} 3 & -3 \\ -3 & 8 \end{pmatrix} 10^8 = \begin{pmatrix} 3.9 & -1.74 \\ -1.74 & 6.85 \end{pmatrix} 10^5$$

Full damping is determined by adding lumped damping to Rayleigh global damping matrix, i.e.

$$C = \begin{pmatrix} 3.9 & -1.74 \\ -1.74 & 6.85 \end{pmatrix} 10^5 + \begin{pmatrix} 0 & 0 \\ 0 & 1.4 \cdot 10^7 \end{pmatrix} = \begin{pmatrix} 3.9 & -1.74 \\ -1.74 & 146.9 \end{pmatrix} 10^5 \qquad (4)$$

Global modal damping is defined by the following formula:

$$C = 2M\Phi\Omega Z(M\Phi)^T, \tag{5}$$

where

M - mass matrix,

Φ - a modes shaped matrix,

Ω – diagonal matrix composed of own natural frequencies (rad/s),

Z – diagonal matrix composed of modal composite damping values. Above matrices are follows:

$$\Omega = \begin{bmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{bmatrix} = \begin{bmatrix} 40.2 & 0 \\ 0 & 97.97 \end{bmatrix}$$

$$Z = \begin{bmatrix} \xi_1^M & 0 \\ 0 & \xi_2^M \end{bmatrix} = \begin{bmatrix} 0.43 & 0 \\ 0 & 0.32 \end{bmatrix}$$

$$M\Phi = (M\Phi)^T = \begin{bmatrix} 97092 & 0 \\ 0 & 97092 \end{bmatrix} \begin{bmatrix} 29 & 14 \\ 14 & -29 \end{bmatrix} 10^{-4} = \begin{bmatrix} 28157 & 13593 \\ 13593 & -28157 \end{bmatrix} 10^{-2}$$

Multiplying matrices using formula (5) we get:

$$C = \begin{bmatrix} 39 & -10.6 \\ -10.6 & 56 \end{bmatrix} 10^5 \tag{6}$$

From comparison of matrices calculated by direct integration approach (standard) and modal approach (see formula 4 & 6), it can be seen that diagonal member C(1,1) comes significantly less in former case than in latter and vice-versa, in case of member C(2,2), thus leading to contraction of results at Point No 1 calculated using modal approach, if compared to results obtained by direct integration approach. A carried out comparison of results at Point No 2 gives adverse effect.

Further, calculating accelerations at Point No 1 & No 2 using Abaqus at single amplitude harmonics with frequency equal to 6,4 Hz. To ensure the availability of resonance a total number of 50 cycle impacts were applied to the supporting point. Results of calculations are represented in Tab. 1. The given table shows results of calculations with no account of dampers and under progressive rise of wave damping up to 70% inclusively.

Tab. 1 shows maximum accelerations at Point No 1 & No 2 calculated by direct integration method, modal method and SIM architecture measures. Additionally, the table gives results of calculations found by modal approach with 20% limitation on modal composite damping, starting from the moment the value exceeded 20%.

Tab. 1- results of calculation for seismic testing
$$\left(\bar{B} = \frac{B}{2\sqrt{K_2 M}}\right)$$

$\bar{B}$	0	14%	28%	35%	42%	70%
B	0	$0{,}28\cdot10^7$	$0{,}56\cdot10^7$	$0{,}70\cdot10^7$	$0{,}84\cdot10^7$	$1{,}40\cdot10^7$
Direct integration approach	a^1_{max} =15 m/s^2	a^1_{max} =6.3 m/s^2	a^1_{max} =4.0 m/s^2	a^1_{max} =3.5 m/s^2	a^1_{max} =3.2 m/s^2	a^1_{max} =2.6m/s^2 a^2_{max} =1.2m/s^2
modal approach	a^1_{max} =15 m/s^2	a^1_{max} =6.3 m/s^2	a^1_{max} =4.0 m/s^2	a^1_{max} =2.9 m/s^2	a^1_{max} =2.5 m/s^2	a^1_{max} =1.7m/s^2 a^2_{max} =1.3m/s^2
Sim-architectutre	a^1_{max} =15 m/s^2	a^1_{max} =6.3 m/s^2	a^1_{max} =4.0 m/s^2	a^1_{max} =3.5 m/s^2	a^1_{max} =3.2 m/s^2	a^1_{max} =2.6m/s^2
modal approach with 20% limitation						a^1_{max} =3.1m/s^2 a^2_{max} =1.8m/s^2
modal composite damping	4%	9,7%	17,6%	21,6%	25,6%	41,5%

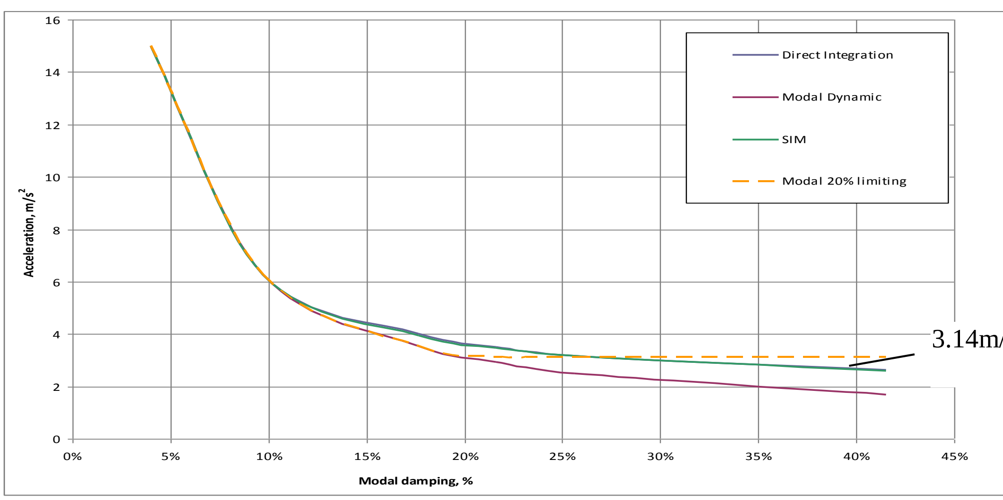


Fig. 2. correlation of maximum acceleration and modal composite damping

Tab. 2 shows maximum accelerations during crash of Boeing 747 at 200 m/s speed. Point No 1 corresponds to crash impact. Fig. 3 shows graph representing the impact.

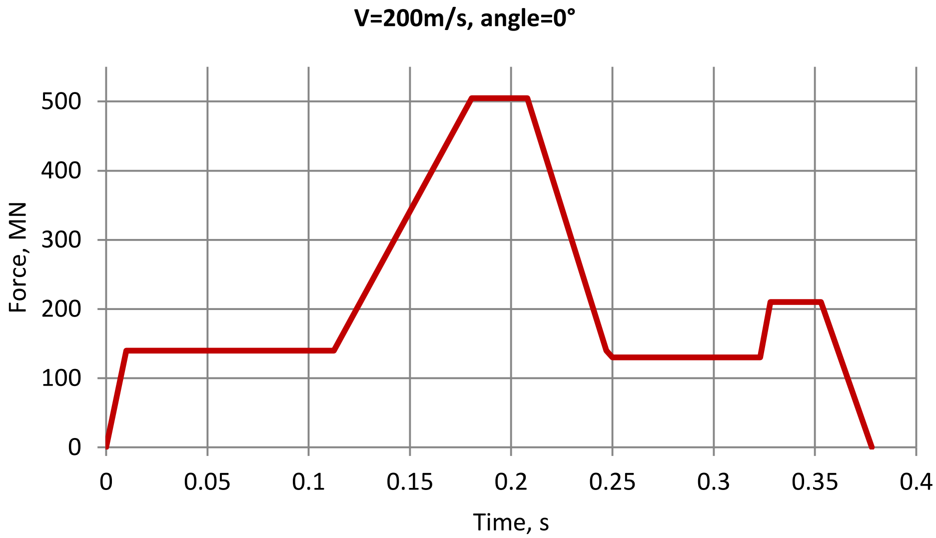


Fig. 3. Aircraft Boeing 747 crash action

Tab. 2 - results of calculation of aircraft crash impact ($\overline{B}$=70%)

	calculation approach	acceleration
a^1_{max}	direct approach	2,6 m/s^2
	modal approach	1,8 m/s^2
	modal approach with 20% limitation	2,8 m/s^2
a^2_{max}	direct approach	0,86 m/s^2
	modal approach	0,79 m/s^2
	modal approach with 20% limitation	1,40 m/s^2

A carried out comparative study shows that results found by direct integration approach almost perfectly match with Sim-architecture across the whole damping range. From the beginning the results found through direct integration and modal approach are again perfectly matching to each other (up to damping 12%). However, later on, at higher values of modal composite damping the modal approach tends lowering the results. At that, the higher is damping the higher is error, even with realistic damping of 70%. In this case a total difference comes to 57%. As mentioned above the reason for the difference lays in the inaccuracy of calculation of modal composite damping due to sharp surge in damping in different parts of the system. This shortcoming relates to modal approach only. Therefore, as seen from test results (not contradicting to guide document ASCE 4-98[3]), it is necessary to include 20% limitation, which gives more conservative result as against to results found through direct integration approach as for seismic impact and plane crash event.

As for the second task, compare response spectra of realistic block-type building due to aircraft crash, see Fig. 4. The figure shows FE model indicating point of impact. An equivalent stiffness and damping in soil, being considered by use of parallelepiped soil, then gets shared across foundation bed of the building (stiffness is shared according to saddle law and damping is shared evenly). Following integrated values (corrected) of damping were used in the calculations: bx=45%, by=44%, bz=33%. Damping in concrete is found equaled to 7%. Rayleigh damping has been defined proceeding from the stipulation that upper and lower limit frequency (in question) are equaled to 4 Hz and 100Hz accordingly. In modal approach the duration of integration step comes to 0.001s and in direct integration approach the calculations have been fulfilled using variable integration step not exceeding 0.001s. The number of tones of natural vibration considered in the modal approach was taken up to 100 Hz.

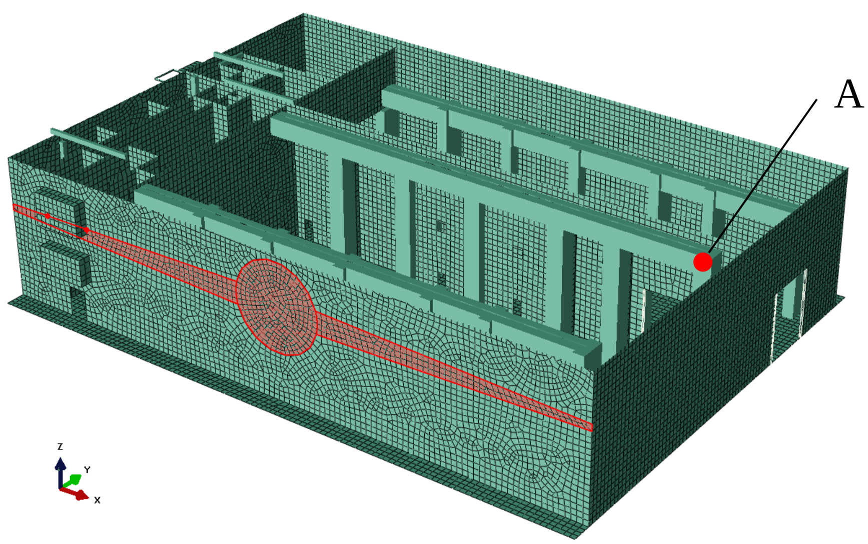
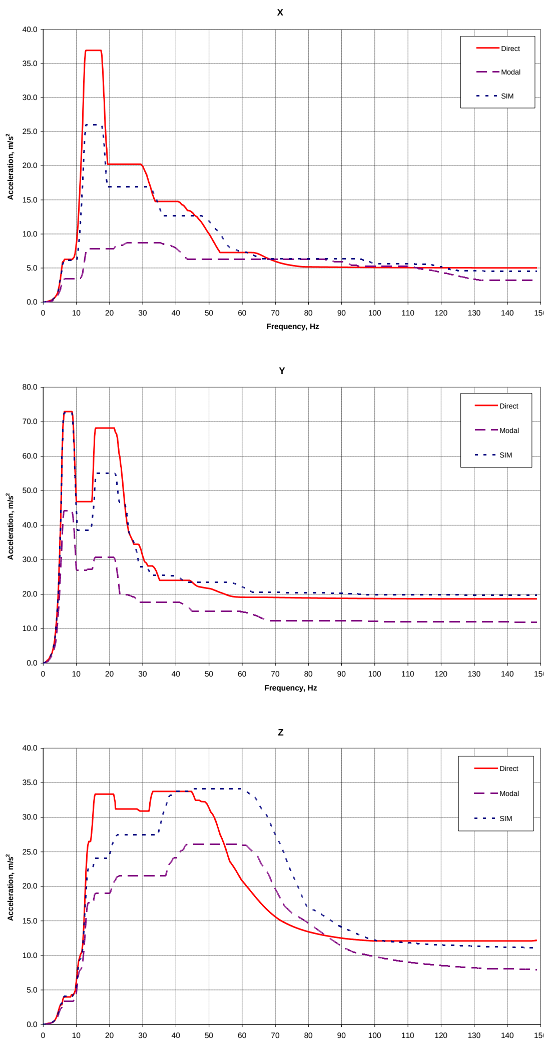


Fig. 4. Building FE model indicating point of impact.

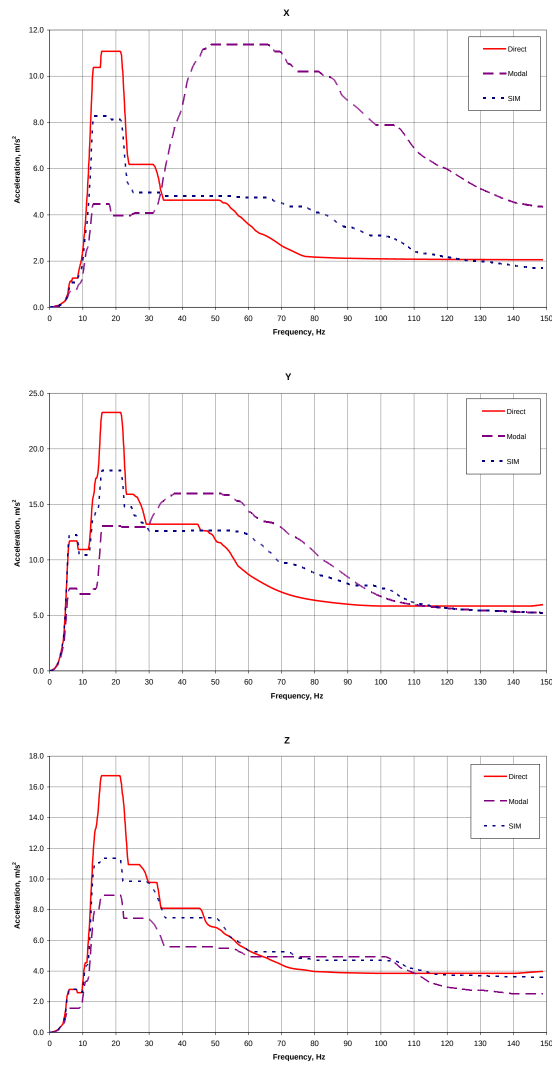
Response spectra is calculated at Point No A.

Fig. 5 shows response spectra arrived by using modal approach with 20% limitation of modal composite damping, by direct integration approach and by SIM-architecture in the upper top column and at foundation slab near to base of the column. From comparison of spectra in the upper point, it is seen that modal approach in general gives drastically lower results of spectral accelerations and the same conforms to task conclusions. This is due to conservatism available in the calculations by direct integration approach. Comparison of spectra in the lower point shows that direct integration approach at high frequencies gives lower values and vice-versa. The same may be explained in terms of effects by two aspects available at foundation when direct integration approach is applied, i.e. direct effect by soil damper leads to lower spectral accelerations and Rayleigh damping leads to higher spectral accelerations wrt modal approach.

In general, SIM–architecture gives averaged mediate results wrt modal and direct integration approach.



Comparison of response spectra (ξ=5%) at point A.



Comparison of response spectra (ξ=5%) at foundation slab at column base

Fig. 5. resulted response spectra

Conclusions:

To carry out linear dynamic calculations for NPP civil structures for aircraft crash event the modal approach is found suitable, provided consideration of number of frequencies up to 100 Hz and 20% limitation on modal composite damping. In major number of calculations carried out such approach gives more lower values than direct integration approach. Conclusions does not contradict the last safety report of IAEA [4].

Main definitions:

interaction system «soil-structure» - calculation model comprising building structure and soil foundation

«soil damper» - viscous damper characterizing damping value in soils (as per ASCE4-98)

SIM – architecture – new architecture by ABAQUS software comprising fully filled damping matrix for «structure-soil» system modal calculations. Earlier, such matrix was diagonally shaped.

Reference materials:

1. NP-031-01. Norms of development of seismic-resistant NPP. Moscow, 2002.

2 Abaqus V13.1, Dassault Systemes SIMULIA CORP.

3. ASCE4-98. ASCE STANDARD. Seismic analysis of safety related nuclear structures and commentary. Approval 1992.

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