

A FINITE ELEMENT MODEL OF THE INTERVERTEBRAL DISC

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Abstract

Intervertebral Disc (IVD) Degeneration causes pain and movement restrictions in the majority (60%) of people during their lifetimes. Options for people suffering from this condition are limited. Total joint arthroplasty for the IVD is not currently available. The development of approaches for IVD arthroplasty is hindered by the insufficient understanding of the load requirements and the material properties of the IVD. Computational modeling is one approach that can be used to explore connections between IVD loading, properties, and disc degeneration. However, computational approaches utilize a finite element model (FE) that oversimplifies the construction of the IVD. The annulus fibrosis (AF) is often modeled as an isotropic material with consistent material properties throughout. Computer models of the IVD are typically validated against mechanical testing of isolated vertebral segments. Importantly, these current testing standards do not test spine segments in a manner that is physiologically relevant. Although the computer models can replicate the failure mode produced by in vitro mechanical testing, these models are limited in predicting in vivo degeneration and do not recreate the yielding and peripheral bulging of the IVD that occurs with progressive degeneration.

The aim of this study was to develop an FE model of the IVD that replicated the in vivo failure modes experienced clinically. The model created in this study utilized four distinct materials - the cartilaginous end plate (CEP), Nucleus Pulposus (NP), and two distinct layers of the Annulus Fibrosis (AF) - to represent the constituents of the IVD. The geometry of the model was based on average dimensions provided by computerized axial tomography. Material properties were assigned to each component based on published values from the literature. The model was validated by comparison to force/displacement data from mechanical testing.

Keywords: Intervertebral Disc, Physiologically Relevant Loading, Fatigue modeling, S-N Curves, Abaqus, Solidworks Simulation, Annulus Fibrosis, Nucleus Pulposus.

Problem Statement

To create a total joint arthroplasty option for the IVD, better understanding of the loading requirements and the material properties needed to support this load are required. Current computer models have had limited success in recreating *in vivo* failure modes. These models must be validated and accurate. Generally validating these models with mechanical testing alone has developed models that merely recreate testing load failure (Malandrino, Noailly et al. 2011). The more realistic a computer model is in portraying the anatomy of the IVD, the more difficult validation becomes (Lian, Liu et al. 2013). This difficulty comes from the variation in geometry and strength of material from person to person or specimen to specimen (Kelly and Bennett 2013). Mechanical properties for many biomaterials are given in a broad range of values, due to the anisotropic nature of the materials, and variability from specimen to specimen (Bass, Ashford et al. 2004). The aim of this study is to develop a model based on material properties provided by journal articles and mechanical testing data and with enough detail of the anatomy that the model re-creates the *in vivo* failure modes experienced clinically.

Hypothesis

The hypothesis of this study is that an FE model created with material properties found in published literature and correlated mechanical testing can be used to further the knowledge of the material requirements of the IVD. This model should be able to replicate mechanical testing and correlate to clinical deformation when subjected to physiologically relevant loading.

A. Significance

Total joint arthroplasty has been used to alleviate pain and maintain motion in the hip and knee joints for decades (Ratner, Hoffman et al. 2004). A more thorough understanding of the load requirements and the material properties of the IVD is required to develop a total joint arthroplasty option for the IVD (Ratner, Hoffman et al. 2004). Understanding how degeneration affects the mechanical stability of the IVD may aid in stopping or reversing disc degeneration (Adams and Roughley 2006). This project provides a better understanding of how degeneration affects mechanical loading over time. It contributes to a firm foundation for understanding the mechanics of the IVD. Full understanding is the first step to finding solutions to chronic back pain that stop or reverse degeneration or to provide a total joint arthroplasty option for trauma patients (Adams and Roughley 2006). An accurate FE model limits the need for expensive and timely mechanical testing. Developing an FE model based on testing to determine material properties is common (Ratner, Hoffman et al. 2004). Testing methods currently used do not reflect *in vivo* loading on the spine (Kelly and Bennett 2013). If the model reflects clinical expressions of wear and degeneration when loaded in a physiologically relevant manner, it may guide material development. Ultimately, this understanding will create more options for individuals with back injuries.

1. Methodology

This analysis will utilize results of mechanical testing conducted by Kalaf et al. in pure compression as shown in Figure 1 (Kalaf 2013). The natural curve of the spine would create a significantly more complex loading scenario than that applied in this test (Marieb and Hoehn 2010). The loading and test configuration utilized in this study follow the standard testing procedure of ASME 169 testing standards for mechanical compression testing (Heuer, Schmidt et al. 2008). This testing configuration is repeatable and accurate; however, it simplifies the complex loading of the spine.

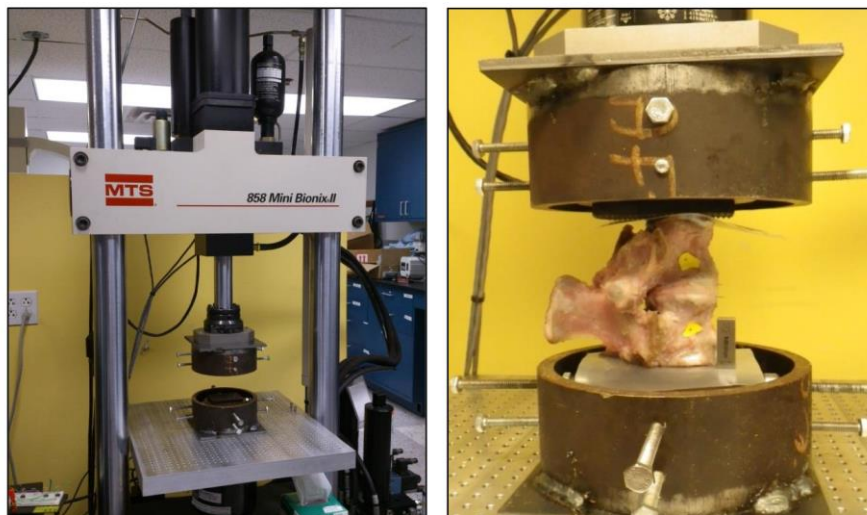


Figure 1 - Motion segment (right) in the MTS 858 (left) for dynamic testing used to establish material properties (Kalaf 2013).

The mechanical testing was conducted on complete functional spinal units taken from young bovine specimens (Kalaf 2013). The primary focus of this research was on the NP (Kalaf 2013). This work showed how enzymatic activity affects degeneration of the NP (Kalaf 2013). Load and corresponding displacement information are shown in [Table 1](#). The specimens were tested -50 N to -150 N cyclic loading at a frequency of 2 Hz to simulate “extremely mild loading conditions” (Kalaf 2013). Data collected from this work was utilized in calculating the material properties of the NP and verifying the force vs. displacement accuracy of the static simulations. In addition, the

cyclic loading information was used in development of the S-N curves required for a Finite Element Analysis (FEA) in Fatigue.

Using material properties provided in journal articles, the remaining properties were derived from the Hooke's Law formulas listed below in [Equation 1](#) (Sharpe Jr, Yuan et al. 1996). These equations are used for determining the material properties of linearly elastic materials; the IVD consists of different viscoelastic materials (Guilak, Ting-Beall et al. 1999). As simulations were run, the results were compared visually to results described by Heuer et al. and Buckland et al. The Elastic Modulus and the Poison's Ratio are provided in a range of possible values for the CEP, the NP, and the AF layers. Numerous static simulations were conducted in Solidworks until a result was achieved that was consistent with testing data (Kotwal, Mohan et al. 2003)). The material properties that were used in the final simulation are listed in the [Table 2](#). Solidworks simulation was used for this portion of model development based on the need for multiple iterations. Once the model yielded results that were consistent with clinical findings and matched the force deflection data from testing. The simulation was verified in ABAQUS.

$$E = 2G(1 + \nu) \quad F = \frac{EA_0\Delta L}{L_0}$$

Where:

G = Shear Modulus
E = Elastic or Young's Modulus
ν = Poison's Ratio
F = Applied Force
A₀ = Initial Cross Sectional Area
L₀ = Initial Length
ΔL = Change in Length

Equation 1 - Material Properties obtained from Hooke's Law Approximation(Dowling 1993).

Table 1 - Functional Spinal Unit Displacement Utilized in Development of FE - Model

Cycle Number	Displacement (mm)
5000	0.082
10000	1.048
15000	1.148
20000	1.335
25000	1.422
39272	1.517

In the mechanical testing conducted minimal information was provided as to initial static test or the first cycle ran (Kalaf 2013). This displacement was referred to as doubling in the first five thousand cycles. Therefore the initial displacement utilized to verify the model was 0.041 mm.

Table 2 - Material Properties Used in FE Model that exhibited the desired failure mode

Material	Poisons Ratio	Elastic Modulus	Shear Modulus
Nucleus Pulposes	0.495	1422480 Pa	2188430 Pa
Annulus Fibroses (1)	0.35	12.6 MPa	5.22 MPa
Annulus Fibroses (2)	0.3	40.6 MPa	14.1 MPa
Cartileneogous End Plate	0.27	9.0 Mpa	5.5 MPa

The results of the simulation under static loading are shown in Figures 3 and 4. A second simulation was conducted in Abaqus/CAE to validate the static results. Although these results are not identical to the Solidworks results they are consistent, these results are shown in Figure 4. The variances are attributed to differences in how the mesh was applied, and the size/geometry variation of the mesh.

Once the Elastic Modulus, Shear Modulus, Poisson's Ratio, Ultimate Tensile Strength, and Yield Strength were estimated, and a satisfactory resulting static simulation was achieved, a fatigue analysis was initiated. The basic requirement of a fatigue analysis is the development of an "S-N Curve" for all materials in the construct being analyzed (Dowling 1993). S-N curves are developed by testing a material (traditionally metals) under cyclically applied loads of varying amplitude to determine fatigue life (Dowling 1993). An S-N Curve plots alternating stress values versus the number of cycles required to cause a failure. Once an S-N Profile for a specified material is developed, the fatigue life may be estimated (Dowling 1993). To create this type of curve, a material must be tested under numerous stress loading profiles at a constant frequency (Dowling 1993). The various materials in the disc FE model created in this study used one loading scenario from testing conducted by Kalaf et al. The S-N Curves developed are estimations, made using numerous assumptions. These assumptions were validated by the simulation failure modes and how well they correlated with testing and journal data. The use of estimating an S-N Curves from a single set of testing data for the aim of creating a fatigue simulation is novel, and is not validated by another study. The results of simulation and the comparison to clinical descriptions of failure were the aim of this model.

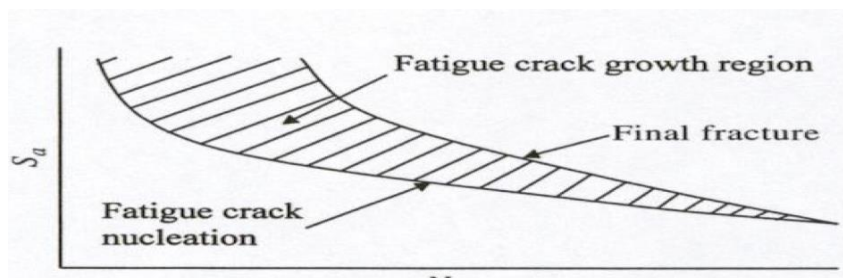


Figure 2- Typical S-N Curve where S_a is the Stress and N is the Number of Cycles

Using the S-N curves developed to run a Fatigue Analysis yielded the results depicted in Figures 5,7,8 and 9. The damage and total life portrayed are consistent with those described by Brickman et al (Brinckmann, Biggemann et al. 1988)]. The initial fatigue curve created a fatigue simulation that did not last 30,000 cycles at 2 Hz, it lasted until 7,231 cycles prior to "failure." Failure of the fatigue life is defined by the Solidworks simulation software, as when the Elastic Modulus of one of the components is no longer applicable and the component begins to plastically deform^{1,2}(Dowling 1993). As the S-N Curves utilized in the simulation were approximations made by numerous assumptions as previously stated. A sensitivity analysis was conducted to determine how these curves needed to be altered to ensure the simulation ran to 30,000 cycles at 2 Hz. It was found that increasing the Yield Strength by 2 orders of magnitude for the NP and inner portion of the AF enabled the simulation to run the stated number of cycles. The Percent Damage Plot that shows the percentage of the life of the structure consumed by the defined fatigue events and the Biaxiality Indicator which plots the ratio of the smaller alternating principal divided by the larger alternating principal stress showed the exact same failure modes, as the original SN-Curves. The only variation in the results was number of cycles run prior to failure. Once model material properties yielded FE results that mimicked the load and displacement data from testing, a loading scenario was applied to the model that was based on "Poor Posture(Claus, Hides et al. 2009)]. This loading scenario applied a -100N force normal to the Disc as well as a -50N load 28 mm anterior, and 10mm lateral from the midline. This load is consistent with that constant force of

¹ Elastic Modulus - the ratio of the force exerted upon a substance or body to the resultant deformation. Young's modulus, it is a mechanical property of linear elastic solid materials.

² Plastic Deformation - When a sufficient load is applied to a metal or other structural material, it will cause the material to change shape permanently Yu, H. Y. (2009). "Variation of elastic modulus during plastic deformation and its influence on springback." Materials Design 30(3): 846-850.

“Poor Posture,” creating a bending moment (Claus, Hides et al. 2009). The FE model static results are shown in Figure 6.

The same S-N Curves used in the testing simulation were used in the “Bad Posture” Simulation. The results of these simulations predicted an earlier failure due to increased stresses on the posterior portion of the spine.

A. Assumptions

The geometry of the solid models is based on average dimensions of the IVD. There is considerable variation between the geometry of the model and the specimens tested. In the contents of the published data reviewed there was no information on the geometric variations of the specimens tested (Kalaf 2013). There is a considerable difference between the size and loading of a bovine spine compared to that in a human spine (Spera, Genovese et al. 2011).

To focus on the IVD deflection and deformation the vertebral bodies were omitted from this simulation. The loading of the disc is distributed uniformly onto a thin disc with very stiff material properties to simulate the distributed loading provided by the vertebral body. The vertebral body has a complex geometry that obscures the posterior portion of the IVD from view (Buckland 2011).

The loading applied in the FE model is based on loading applied during compression testing of bovine spinal segment (Kalaf 2013). This FE Model is limited by the lack of specific material data. When possible human material properties were used, however when no human data was available porcine and bovine information was used. All material data provided by published articles was given in a range.

The Finite Element software used in this study assumes material failure modes similar to metal (Ciambella, Destrade et al. 2009). Once a material under the specified load reaches a stress value equal to the yield point and the elastic modulus is no longer applicable for that material the simulation stops (Kurowski 2013). All the materials in the IVD are viscoelastic, but they are modeled as elastic materials (Guilak, Ting-Beall et al. 1999).

2. Results

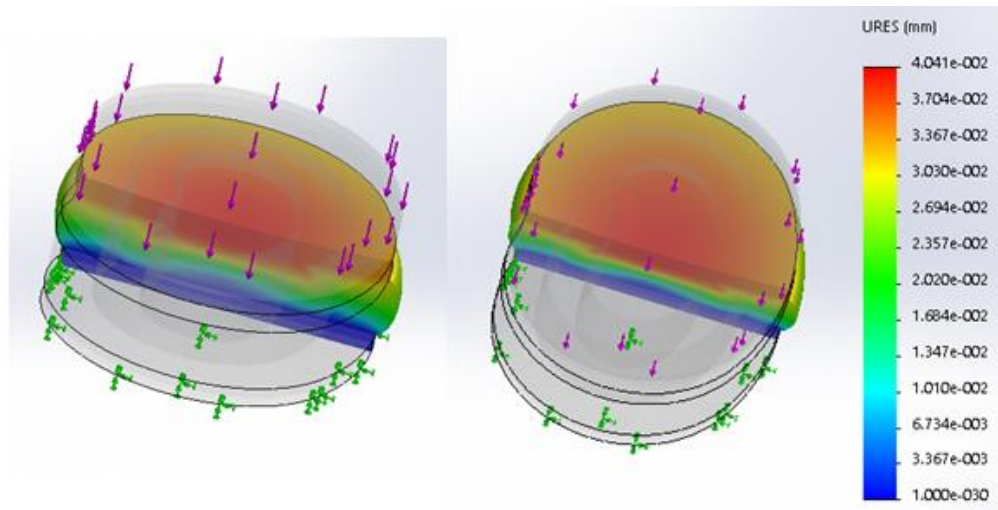
The FE Model created in this study yields results consistent with static mechanical testing (Kalaf 2013). A comparison of static testing data and the FE Model data is shown [Table 3](#). Due to the discrepancy between the defined failure modes in testing versus Solidworks Simulation the cyclic force deflection data does not correspond.

Table 3 - Testing to Model Deflection Discrepancy

Testing to Model Discrepancy	Deflection (mm)	error
Testing No Degeneration	0.0400	0
Solidworks Model No Degeneration	0.0410	-0.001
ABAQUS Model No Degeneration	0.0380	0.002

The resulting reaction forces and deformation produced in this model are consistent with those described in literature (Heuer, Schmidt et al. 2008). When a physiologically relevant load is applied the bulging of the disc is similar to that described by clinicians (VANHARANTA, GUYER et al. 1988). Clinicians cite a “bulging” in the posterior portion of the IVD, offset from midline (VANHARANTA, GUYER et al. 1988). To quantify the bulging, the displacement orthogonal to the top surface of the disc model when the testing scenario loading is applied is uniform. This same measurement varies in the physiologically relevant loading scenario the displacement in the posterior portion of the disc, slightly offset from midline shows 0.02644mm more of deflection. The total displacement is approximately equivalent, however there is concentration of displacement in the area indicated. This concentration produces an increase in strain by approximately 2% more than the test loading scenario.

Figure 3 - Static Simulation Mechanical Test Loading Displacement



There is variation in the results depicted in Abaqus/CAE when compared to the results shown in Solidworks Simulation FEA as seen in Figure 4. This is attributed in part, to a variation in the way the solid models were meshed and the display options available by the software packages (Khennane 2013). In Solidworks a curvature based mesh of max element size 0.00169759 mm, and min element size of 0.000565856 mm used. In Abaqus a Hexahedron mesh of average size 0.0000055mm was use(Khennane 2013).

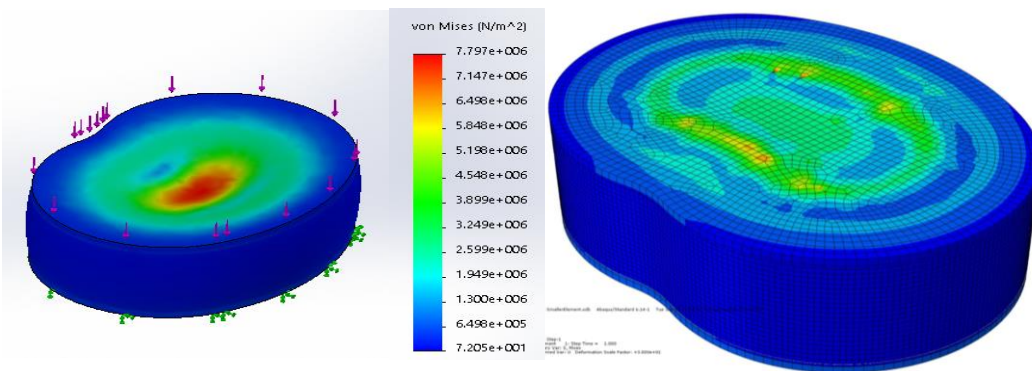


Figure 4 - A) Solidworks Simulation Von Mises B) Abaqus/CAE Von Mises

The fatigue simulations varied from the mechanical testing, the FE simulations stopped at 7,231 cycles when the stress on the NP exceeded the Yield Strength as defined by the material properties (Kalaf 2013). Simulations were run that reached 30,000 cycles by considerable alteration of the S-N Curves and the Yields Strength of the NP and AF materials. These altered simulations completed 30,000 cycles prior to failure, but the mode of the failure was the same as the previous simulations. These simulations reflected clinical and laboratory failure by the NP yielding prior to other materials under cyclic loading³(Urban, Roberts et al. 2000). When the NP yields the mechanical loading on the inner AF increase(VANHARANTA, GUYER et al. 1988)]. This is shown specifically by the Biaxiality plot, depicting how the compressive loading becomes hydrostatic as the cycles accumulate (Kurowski 2013). The Fa-

³ Damage Percentage This plot shows the percentage of the life of the structure consumed by the defined fatigue events.

tigue simulation is stopped when the stress on a component exceeds the Yield Stress of the material (Kurowski 2013).

The Yield Point is the location on a Stress-Strain Curve where the Young's Modulus is no longer applicable (Kurowski 2013)⁴. It would require more mechanical testing to determine if this stress corresponded with the initial yielding of the NP; Solidworks simulation stopped the fatigue analysis when the first material in the construct began to plastically deform (Kurowski 2013). The mechanical testing conducted was stopped at 29,274 cycle (Kalaf 2013). The motion segments were cut open to observe the structure of the discs after testing was complete (Kalaf 2013). It was documented that the NP expanded out of the plane of separation (Kalaf 2013). This indicates that cycles were run for a considerable amount of time after the structure of the NP plastically deformed and ultimately failed (Kalaf 2013).

The static simulations that were used to represent a physiologically relevant loading scenario showed stress concentrations similar to clinical findings. There was a region of high stress due to compression in the anterior portion of the disc offset medially where the load was applied. This loading scenario showed an area of increased stress in the posterior portion, on the opposite side, medial to the midline. These areas of high stress coincide with the location in the lumbar spine where 2.5mm of bulging has been known to cause severe pain.

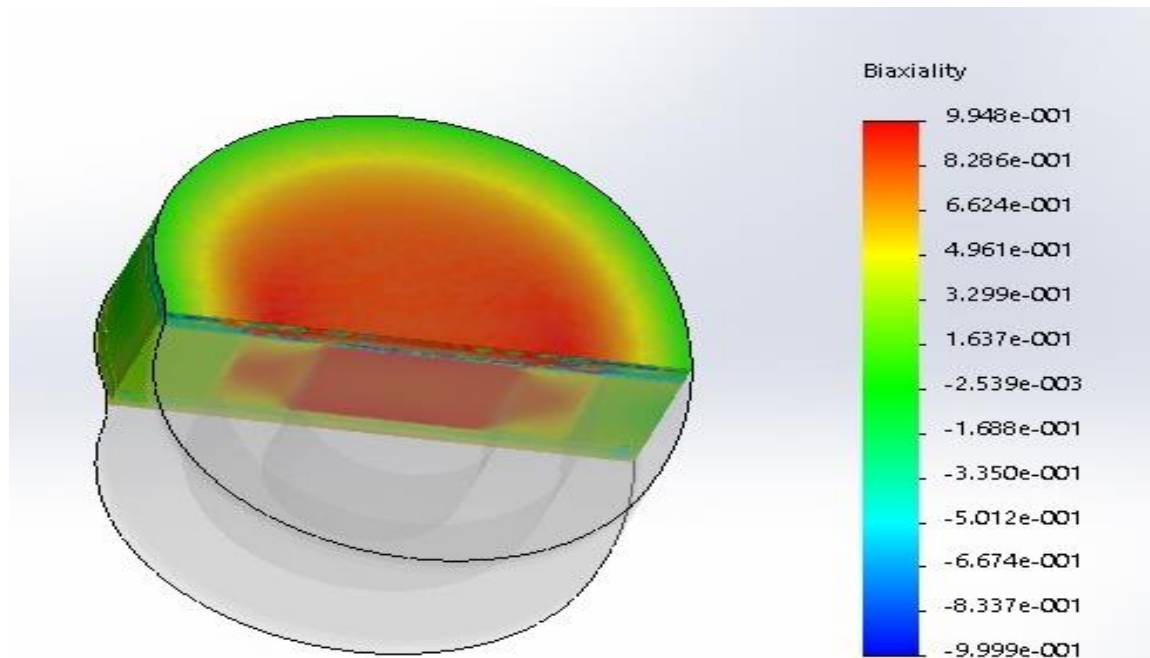


Figure 5 - Biaxiality Plot from Test Loading Scenario

⁴ Young's Modulus or Modulus of Elasticity - is a measure of stiffness of an elastic material. It is used to describe the elastic properties of objects when they are stretched or compressed. It applies during the linear portion of the Stress/Strain Curve. Dowling, N. E. (1993). Mechanical behavior of materials: engineering methods for deformation, fracture, and fatigue, Prentice hall.

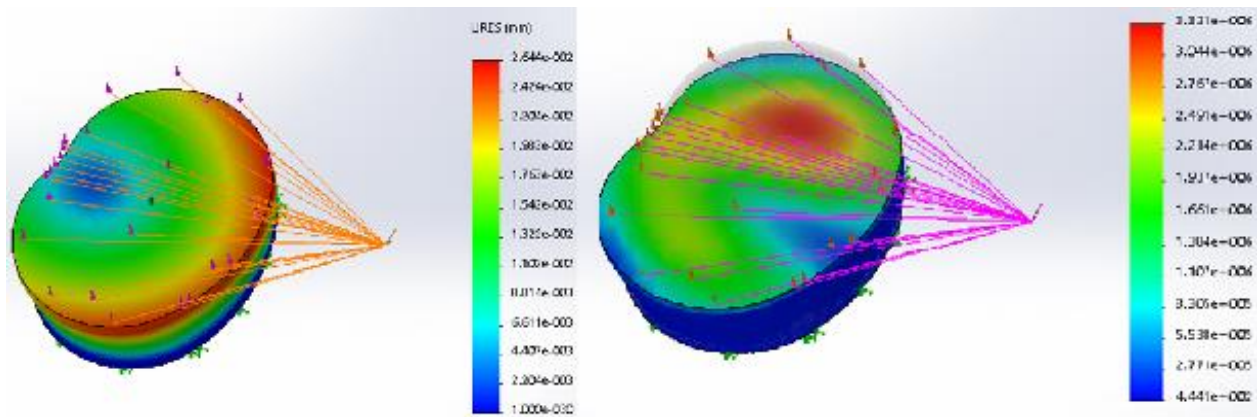


Figure 6 - Static Simulation Physiologically Relevant Loading A) Displacement B) Von Mises

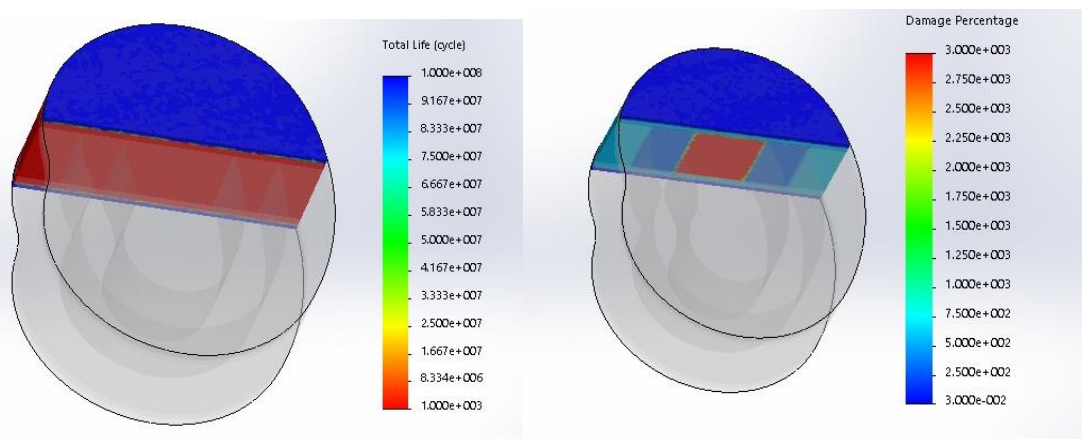


Figure 7 - Total Fatigue Life Plot - Test Scenario Figure 8 - Fatigue Damage Plot - Test Scenario

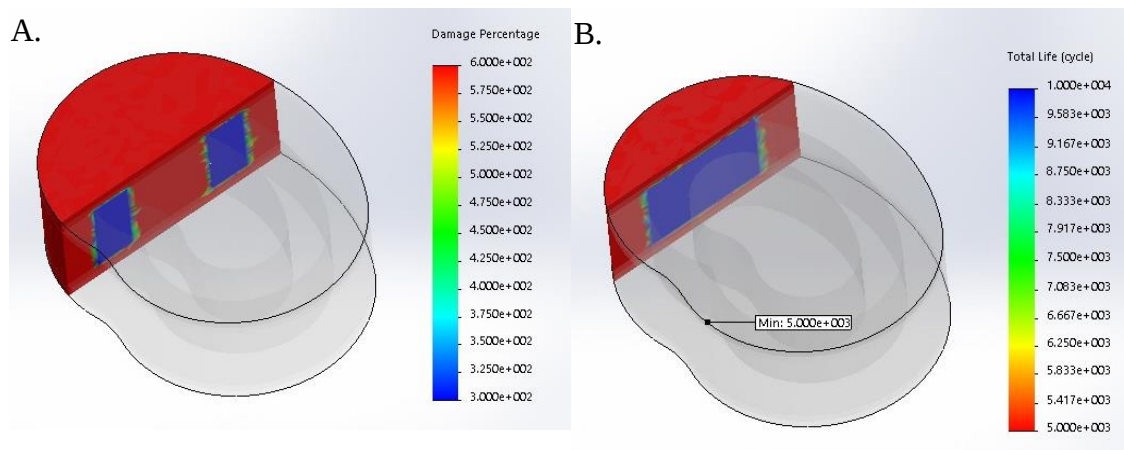


Figure 9 - A. Fatigue Damage Plot - Physiologically Relevant Scenario B. Fatigue Plot - Total Life - Physiologically Relevant Scenario

The fatigue simulations based on physiologically relevant loading stopped at 6,943 cycles. There were fewer cycles run in the physiologically relevant loading scenario, due to the higher stress concentrations in the posterior portion of the disc. The percent life and damage percentage plots are very similar to the plots for the test loading scenario. The location of the max and min points for these plots were different and reflected the offset loading. The Biaxiality plot is considerably different, showing a significantly more complex stress distribution to the test scenario plot.

3. Discussion

This investigation showed that it is possible to obtain clinically relevant results from an FE model created by published material data and uniaxial testing (Adams, McNally et al. 1996). The biaxial nature of the fatigue loading profiles shown in Figure 5 and 7 illustrate the compressive loading results in an axial load (Urban, Roberts et al. 2000). The results of FE models were consistent with the static testing conducted by Kalaf et al. however, it must be noted that this testing was used in constructing the model. Separate testing would be required to validate the model.

In the study by Buckland et al. it was stated that 0.9mm of bulging was not clinically relevant (Buckland 2011). A clinically relevant displacement was not achieved in the testing conducted by Kalaf et al. until after 20,000 cycles at this point the testing and FEA start to show “clinically relevant” displacement (Buckland 2011). It should be noted that in the posterior portion of the spine is where 2.5-3mm of bulging has been shown clinically caused severe pain (Ricci, Stewart et al. 2006). The desired failure mode of “bulging” coupled with a force/displacement behavior matching the test results is the validation mode of this model.

Static testing results for the test loading configuration, showed that the Von Mises Stress was $6.336 \times 10^6 \text{ N/m}^2$ across more than 80% of the IVD construct. This value is considerably higher than that estimated by dividing the load applied by the cross sectional area of the model IVD which was $2.84 \times 10^6 \text{ N/m}^2$. Von Mises Stress. This indicates that the majority of the stress in the IVD construct is not in the plane of the compression as indicated by the simplified equation. Von Mises stress gives a better indication of the stress that each component is exposed to and how it varies throughout the part and assembly analyzed.

The physiological relevant loading scenario showed an average Von Mises Stress value of $3.321 \times 10^6 \text{ N/m}^2$, and less overall deflection (.02644mm) than the test loading configuration. It should be noted, that although there is the same load applied in the physiologically relevant construct, the stress is not evenly distributed. In the cyclic fatigue simulation this loading scenario ran 6,943. This is 238 fewer cycles than the test loading configuration. The discrepancy indicates that the loading scenario affects the fatigue life of the IVD. This indicates that the more physiological relevant loading scenario wears the disc at a faster rate.

The discrepancy between fatigue cycles run in testing and simulation is attributed to the discrepancy of the failure point definition in Solidworks and that used in the mechanical testing. Solidworks assumes the materials to be linearly elastic, and stops the fatigue simulation when the Yield Strength of the material is reached (Kurowski 2013). As previously stated the model S-N Curves could be altered and the Yield Stress could be increased to meet or exceed 30,000 Cycles, however this simulation would not be directly related to testing data.

The Elastic Modulus used to identify the stiffness of a material is only applicable in the Elastic Portion of the stress strain curve. If the rate at which a load is applied to a polymer changes rapidly the mechanical response may be drastically different (Dowling 1993). This is due to the complex modulus of polymers (Dowling 1993). The IVD constituents behave similarly to polymers when applied load changes rapidly (Malandrino, Noailly et al. 2011). Further testing would be required to determine if the NP truly begins to yield when predicted by the FE model, and if 2Hz allows adequate time for the viscoelastic material to react to the loading (Lin, Hedman et al. 2009). Water content of the IVD changes over time, the level of hydration drastically effects the mechanical properties of the IVD (Ellingson, Mehta et al. 2013). The frequency of the loading unloading cycles may need to be varied and multiple tests conducted to determine the optimal frequency for future testing.

This model used material properties from various studies, utilized previous mechanical testing, and did not create a cyclic loading scenario simulation that replicated the testing data. The model did deform in a way consistent with clinical findings when loaded with a physiologically relevant loading, re-created the bi-axial stress developed in the

disc when loaded in pure compression, and most importantly, reflects the modes of failure in fatigue. There are definite similarities between the failure modes experienced clinically, and those shown in the FE model developed in this study. This study indicates that further work may produce a practical computer representation of the IVD, eliminating the need for expensive and timely mechanical testing.

The scope of this project is to create a FE model of an intervertebral disc, which illustrates how degeneration of the NP affects the IVD response to loading both static and fatigue. The resulting failure modes are compared to that seen *in vivo*. This project is limited in that all computational models will be validated by testing conducted on bovine spines in a separate study (Kalaf 2013). The testing used to create the model is not capable of validating the model. This model is validated by how well it can replicate *in vivo* failure when a physiological loading scenario is applied. Further mechanical testing is required to validate the models ability to predict mechanical failure.

Hooke's Law approximations were used to calculate material properties. Use of Hooke's Law assumes linearly elastic material properties and the constituents of the IVD are viscoelastic (Guilak, Ting-Beall et al. 1999).

The FE model load displacement results were compared to descriptions of failure and wear, subjectively. This limits the results of this study to indicative not conclusive.

4. Conclusions and Future Work

The FE model created is capable of manifesting failure modes seen *in vivo*, as well as, exhibiting the correct force deflection data consistent with mechanical testing. This study created a model that is not capable of running a fatigue simulation that reflected the number of testing cycles completed.

Further mechanical testing is required to validate the model. This testing would need to isolate load and cycle when the NP permanently yields, as well as a frequency that allows the viscoelastic energy to fully recover from loading. Mechanical testing that utilized loading scenarios developed by Heuer et al. would create more accurate physiologically relevant loading scenarios (Heuer, Schmidt et al. 2008). Testing in a six degree of freedom force controlled robotic testing device like that developed by Kelley et al. would yield results that would validate this this model further (Kelly and Bennett 2013).

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