Simpack User Meeting 2007

Modeling and Simulation of a Hexapod Machine Tool for the Dynamic Stability Analysis of Milling Processes

C. Henninger, P. Eberhard

Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

project funded by the DFG
within the framework of the SPP 1099
‚Parallel Kinematic Machine Tools'

Outline

- motivation: PKM machine tools and dynamic stability of cutting processes
- modeling of the machine structure
- modeling of the milling process
- analysis of dynamic stability
- techniques for efficient computation of stability diagrams
- pose-dependent stability diagrams for the hexapod machine tool Paralix

PKM Machine Tool PARALIX


IfW, University of Stuttgart

spatial assembly of links
→ high static stiffness

no bending moments in links, only normal forces
→ light-weight design of struts

equal components in identical links
→ reduction of production costs

- 6-DOF hexapod machine tool
- struts with constant length
- sledges driven by linear direct drives
- 5-DOF high-speed milling of metal

nonlinear kinematics and dynamics
nonlinear position control necessary

pose-dependency of stiffness
force/velocity transmission and dynamic properties

kinematic singularities
loss of control of DOF,
reduction of workspace

Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

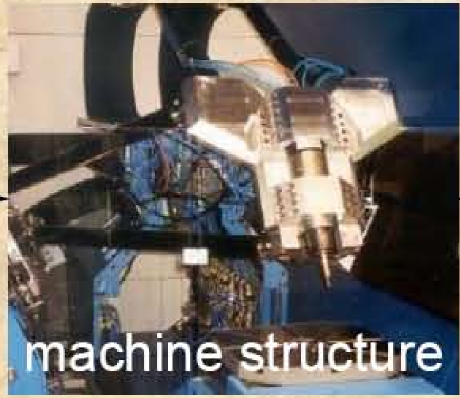
Dynamic Stability of Cutting Processes for PKM

characteristics of PKM:
- light-weight structures
- pose-dependent dynamics

?

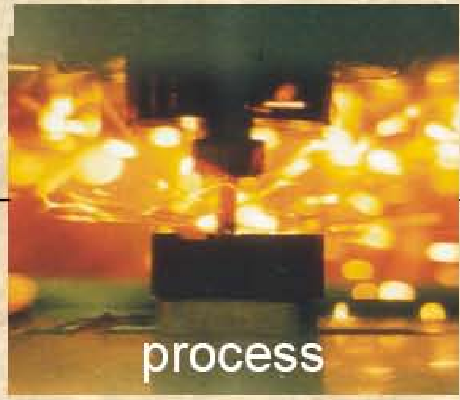
process reliability
dynamic stability
surface quality

external excitation
unbalance excitation
inertia forces



process forces

displacements

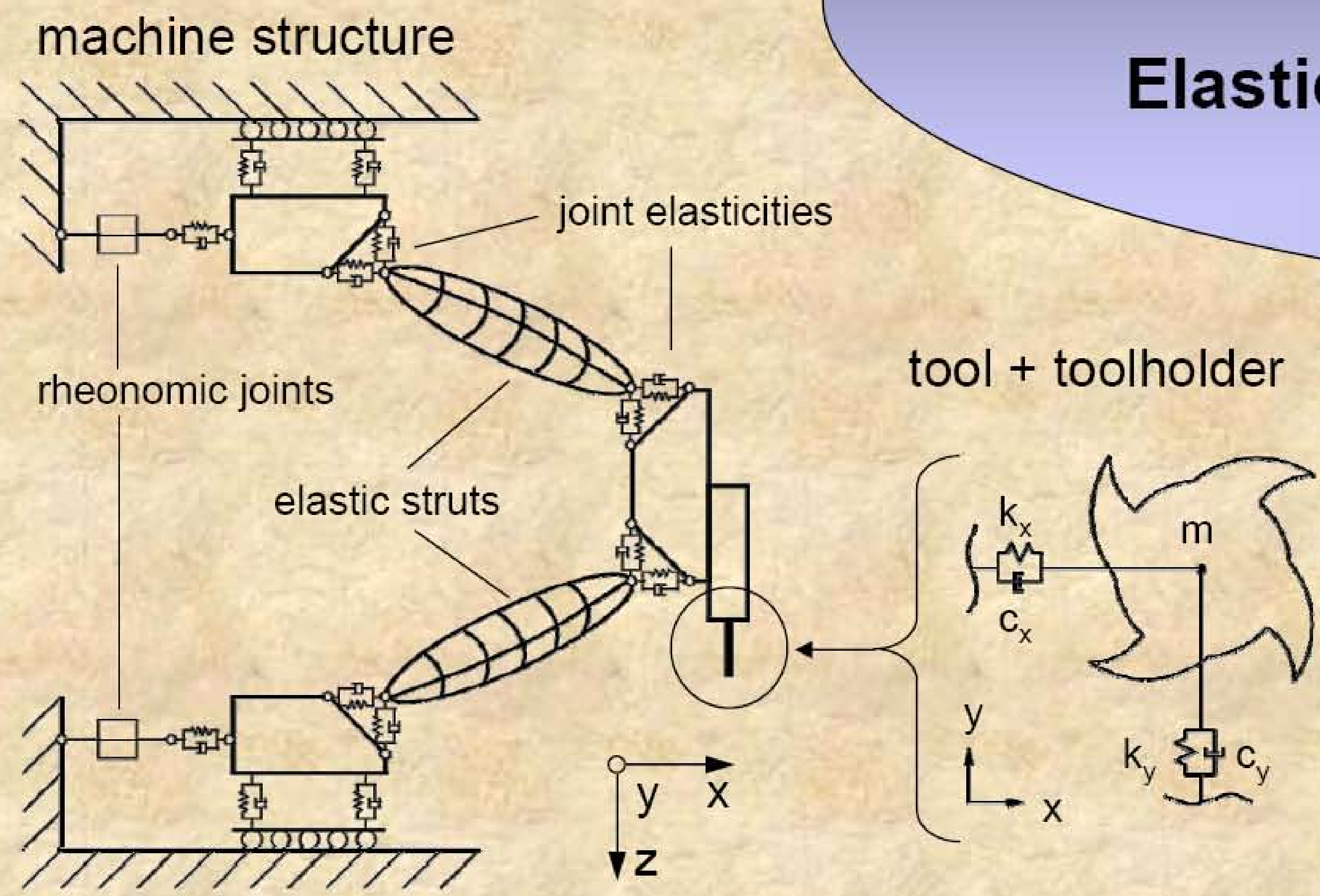


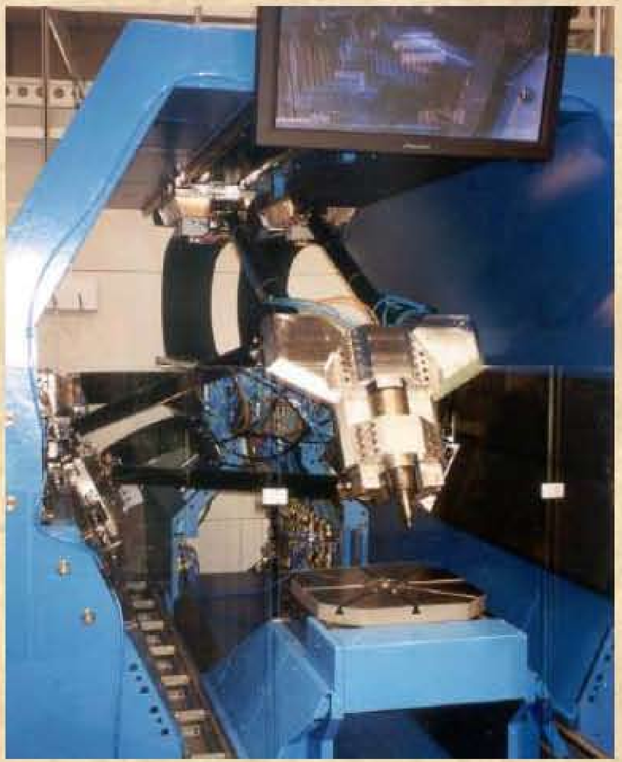
mechanisms of excitation
external and self-excited vibrations

process parameters
spindle speed,
feed,
depth of cut, ...

Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

Elastic Multibody System of PKM Paralix





sledges, platform as rigid bodies

finite element modeling of struts (beam elements)

elastic Cardan joints and linear joints

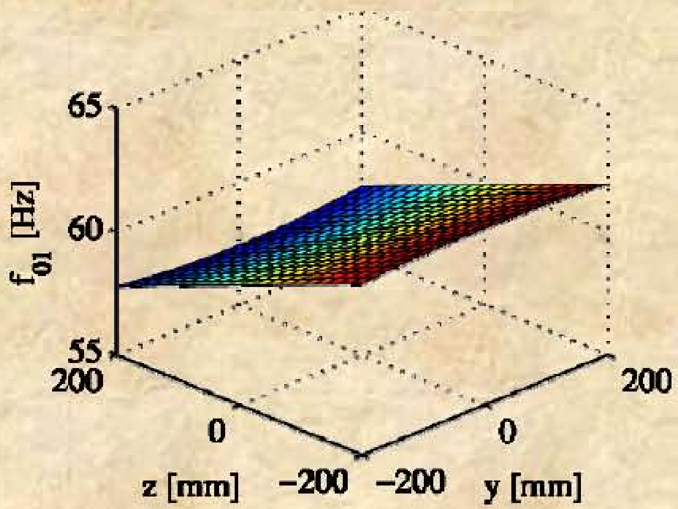
modal reduction of strut model (9 dof)

modal reduction of complete model for each pose in workspace

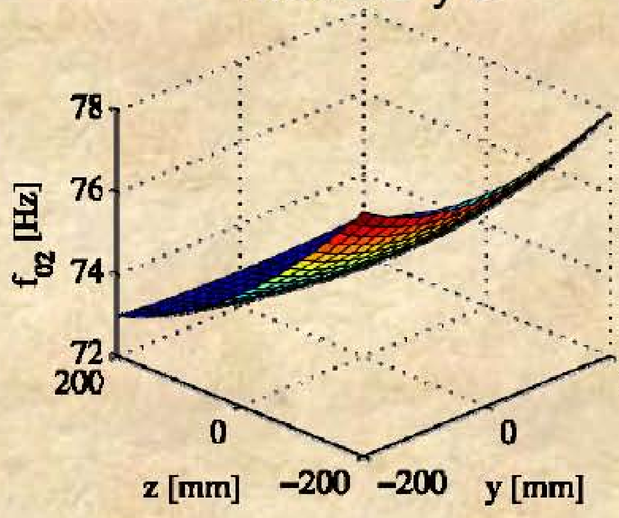
Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

Eigenfrequencies of Machine Structure in Workspace

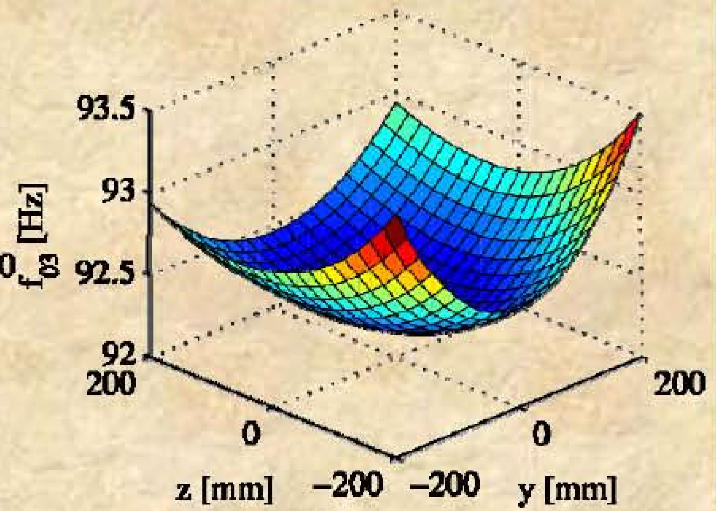
Mode 1 y-z



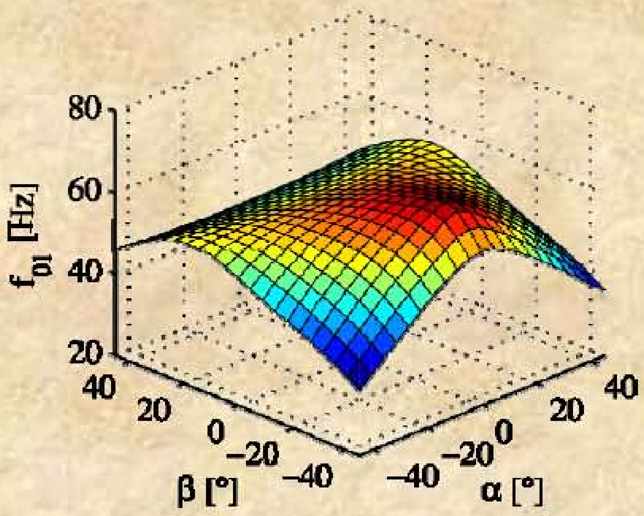
Mode 2 y-z



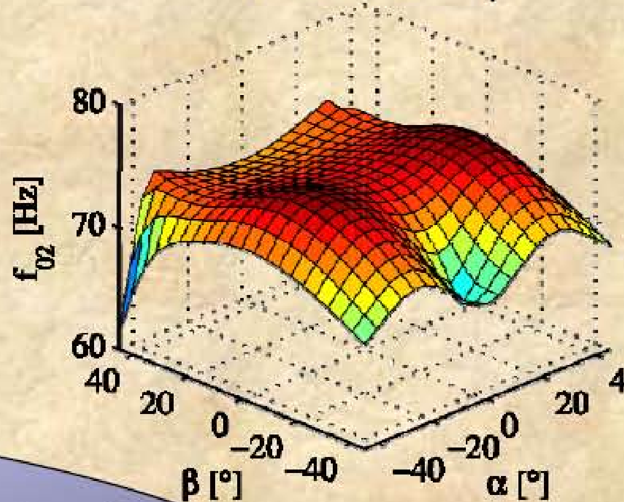
Mode 3 y-z



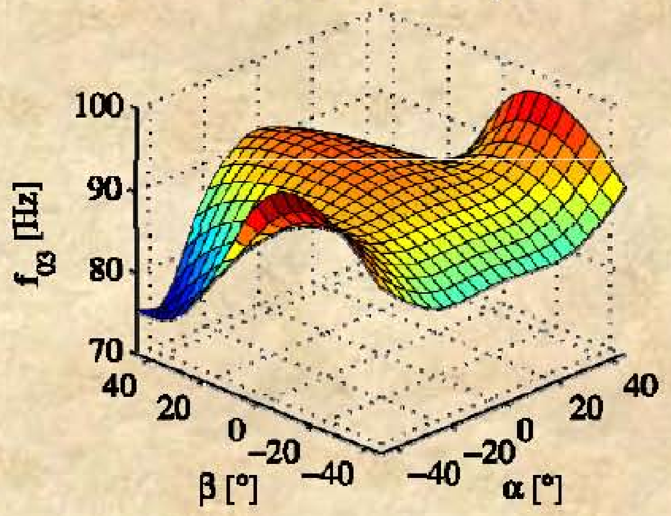
Mode 1 α-β



Mode 2 α-β



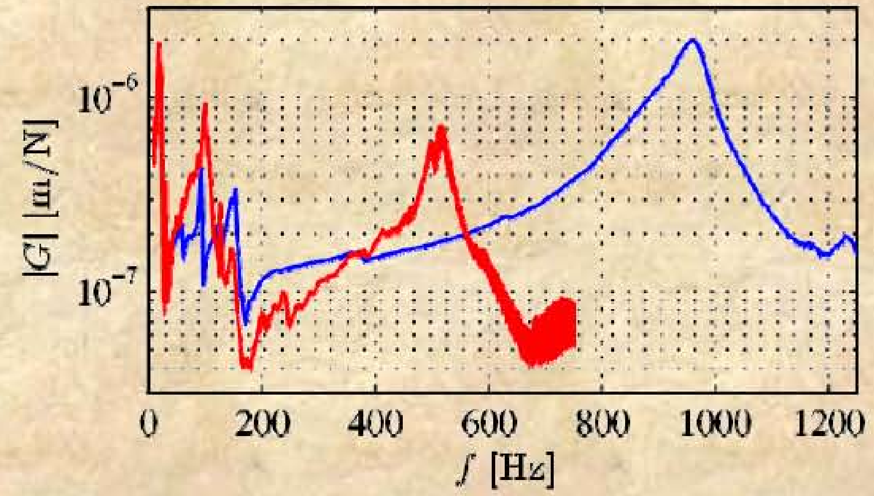
Mode 3 α-β

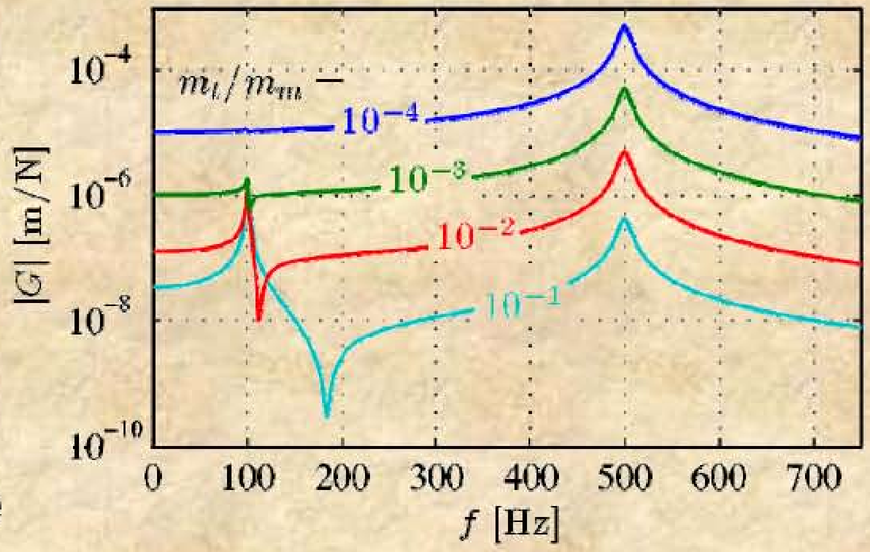


Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

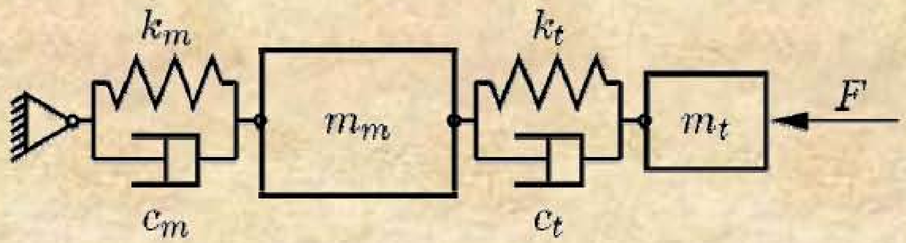
Role of Machine Structure in Tool-Tip Dynamics

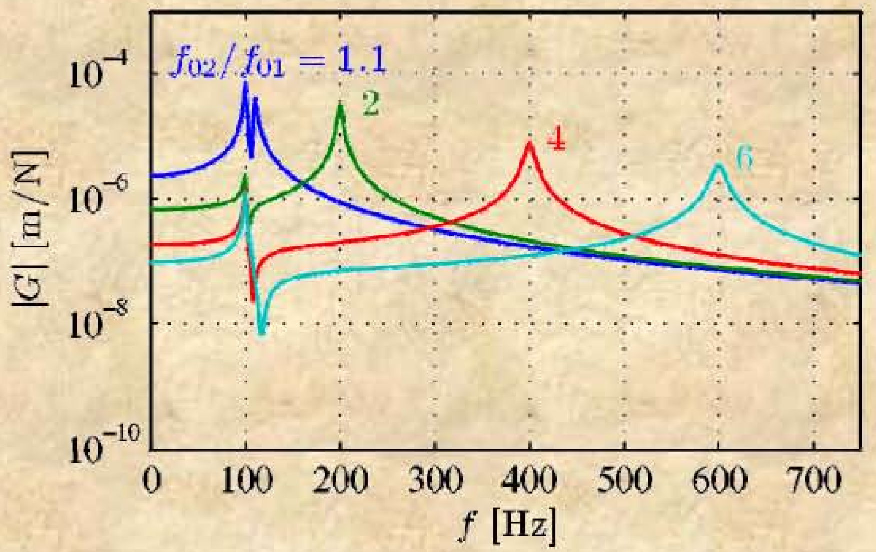
measured tool-tip frequency repsonse





bimodal system representing
machine structure and tool-toolholder-spindle

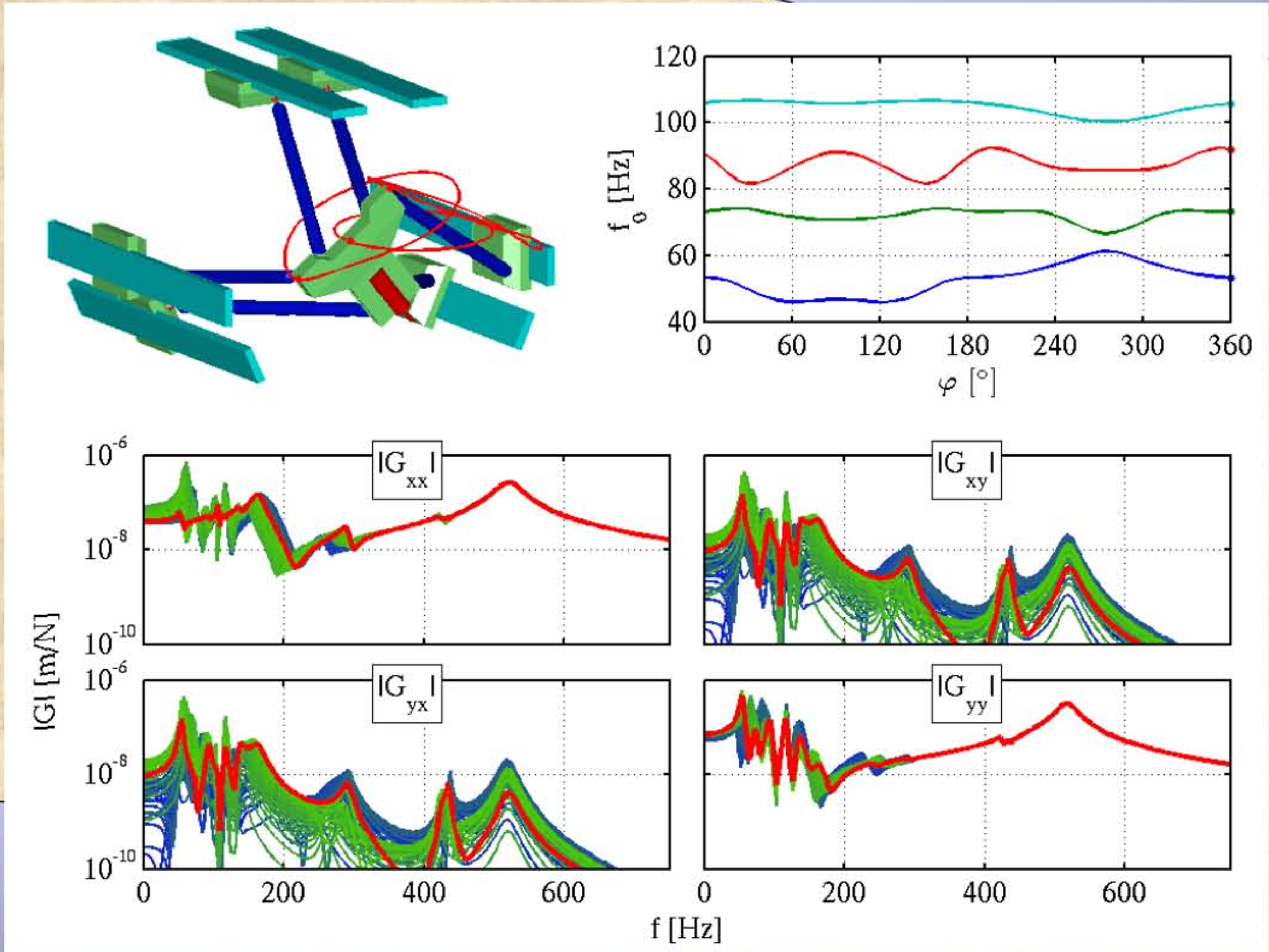




Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

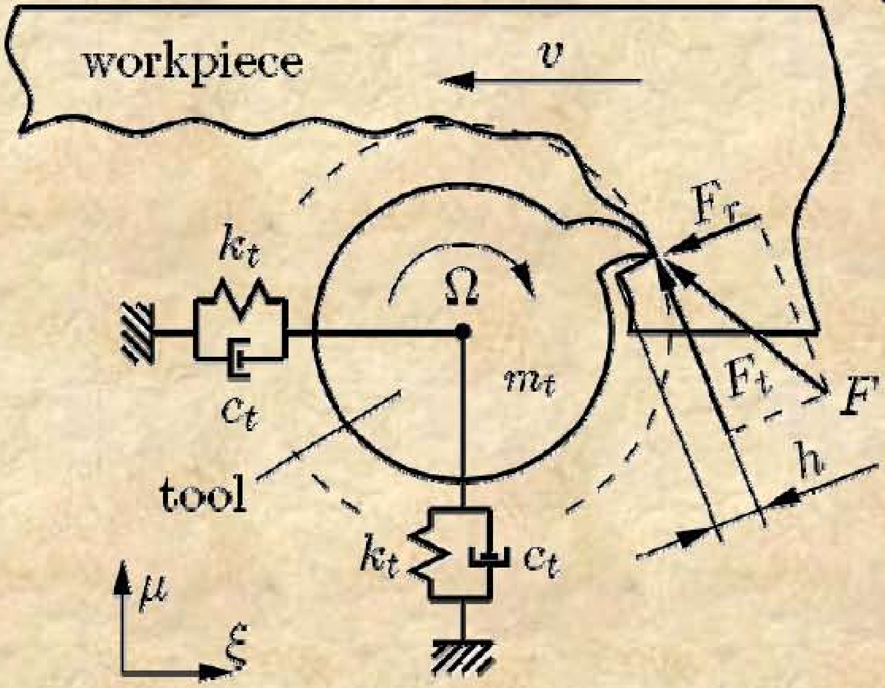
Pose-dependency of Dynamic Behaviour

influence on eigenfrequencies
and tool-tip frequency response



Modeling of Milling Process

two-dimensional milling process



linear cutting force model

$$F_t = F_{t0} + k_{ct} bh \qquad F_r = F_{r0} + k_{cr} bh$$

dynamic depth of cut with regenerative effect

$$h = h_0(\varphi) + g(\varphi)\big[\big(\xi(t) - \xi(t-\tau)\big)\sin(\varphi) + \big(\mu(t) - \mu(t-\tau)\big)\cos(\varphi)\big]$$

dynamic process force equation

$$\mathbf{F} = \mathbf{F}_0(t) + b\mathbf{K}_c(t) \cdot \big(\mathbf{y}(t) - \mathbf{y}(t-\tau)\big)$$

overall system considering machine structure and process dynamics

$$\mathbf{M} \cdot \ddot{\mathbf{y}}(t) + \mathbf{D} \cdot \dot{\mathbf{y}}(t) + (\mathbf{K} + \mathbf{K}_c) \cdot \mathbf{y}(t) - \mathbf{K}_c \cdot \boxed{\mathbf{y}(t-\tau)} = \mathbf{F}_0(t)$$

delay-differential equation (DDE) with periodic coefficients

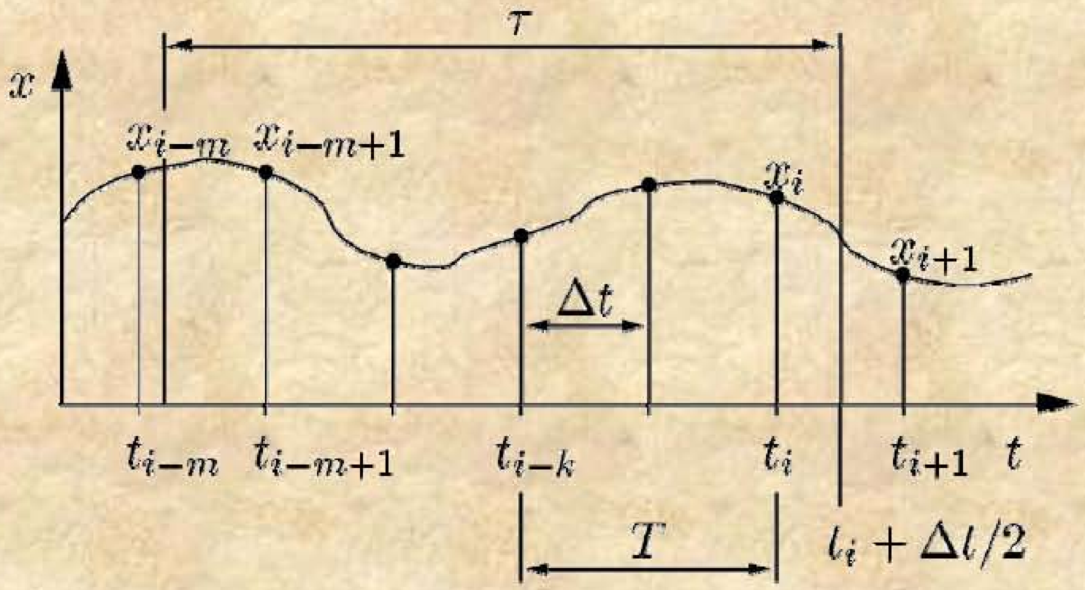
dynamic stability?

Institute of Engineering and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

Analysis of Dynamic Stability by Semi-Discretization Method

Insperger & Stépán, 2003:

discretization of system history



homogeneous periodic time-variant DDE

$$\dot{\mathbf{x}} = \mathbf{A}(t)\cdot\mathbf{x}(t) + \mathbf{Q}(t)\cdot\mathbf{x}(t-\tau)$$

approximations on $[t_i, t_{i+1}]$

$$\mathbf{x}(t-\tau) \approx \mathbf{x}(t_i + \Delta t/2 - \tau)$$

$$\approx w_b \mathbf{x}_{i-m} + w_a \mathbf{x}_{i-m+1}$$

$$\mathbf{A}(t) \approx \mathbf{A}(t_i + \Delta t/2) =: \mathbf{A}_i$$

$$\mathbf{Q}(t) \approx \mathbf{Q}(t_i + \Delta t/2) =: \mathbf{Q}_i$$

approximating ODE on $[t_i \quad t_{i+1}]$

$$\dot{\mathbf{x}} = \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{Q}\cdot\left(w_b \mathbf{x}_{i-m} + w_a \mathbf{x}_{i-m+1}\right)$$

$$\int \rightarrow \mathbf{x}_{i+1} = P_i\cdot\mathbf{x}_i + w_a \mathbf{R}_i\cdot\mathbf{x}_{i-m+1} + w_b \mathbf{R}_i\cdot\mathbf{x}_{i-m}$$

with $\mathbf{P}_i = \exp(\mathbf{A}_i \Delta t)$

$\mathbf{R}_i = (\exp(\mathbf{A}_i\Delta t) - \mathbf{I})\cdot\mathbf{A}_i^{-1}\cdot\mathbf{Q}_i$

Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

Stability Analysis

transition matrix for Δt

$$\begin{bmatrix} \mathbf{x}_{i+1} \\ \tilde{\mathbf{x}}_i \\ \tilde{\mathbf{x}}_{i-1} \\ \tilde{\mathbf{x}}_{i-2} \\ \vdots \\ \tilde{\mathbf{x}}_{i-m+1} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_i & \mathbf{0} & \cdots & \mathbf{0} & w_a \mathbf{R}_i \mathbf{C}^T & w_b \mathbf{R}_i \mathbf{C}^T \\ \mathbf{C} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x}_i \\ \tilde{\mathbf{x}}_{i-1} \\ \tilde{\mathbf{x}}_{i-2} \\ \vdots \\ \tilde{\mathbf{x}}_{i-m+1} \\ \tilde{\mathbf{x}}_{i-m} \end{bmatrix}$$

vector of delay-active states $\tilde{\mathbf{x}} = \mathbf{C} \cdot \mathbf{x}, \quad \mathbf{C} \in \mathbb{R}^{d \times n}, \quad d < n$

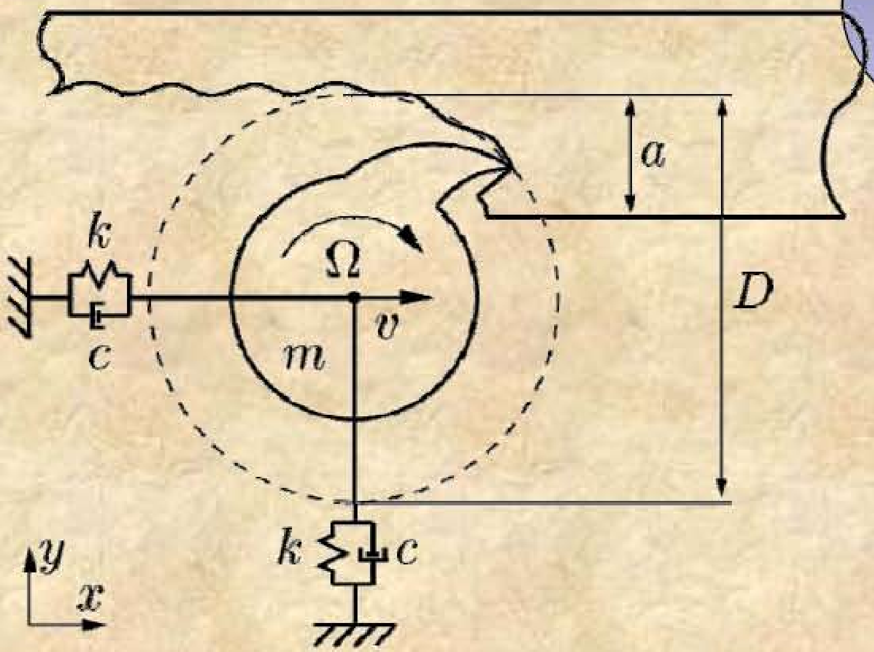
transition matrix for system period T $\quad \boldsymbol{\Phi} = \boldsymbol{\Phi}_{k-1} \cdot \boldsymbol{\Phi}_{k-2} \cdots \boldsymbol{\Phi}_1 \cdot \boldsymbol{\Phi}_0$

Floquet theory for stability of
periodic time-variant linear systems

$$\max\left(\left|\operatorname{eig}(\boldsymbol{\Phi})\right|\right) \begin{cases} > 1 & \text{unstable} \\ = 1 & \text{stability boundary} \\ < 1 & \text{stable} \end{cases}$$

Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

Stability Boundary Diagram

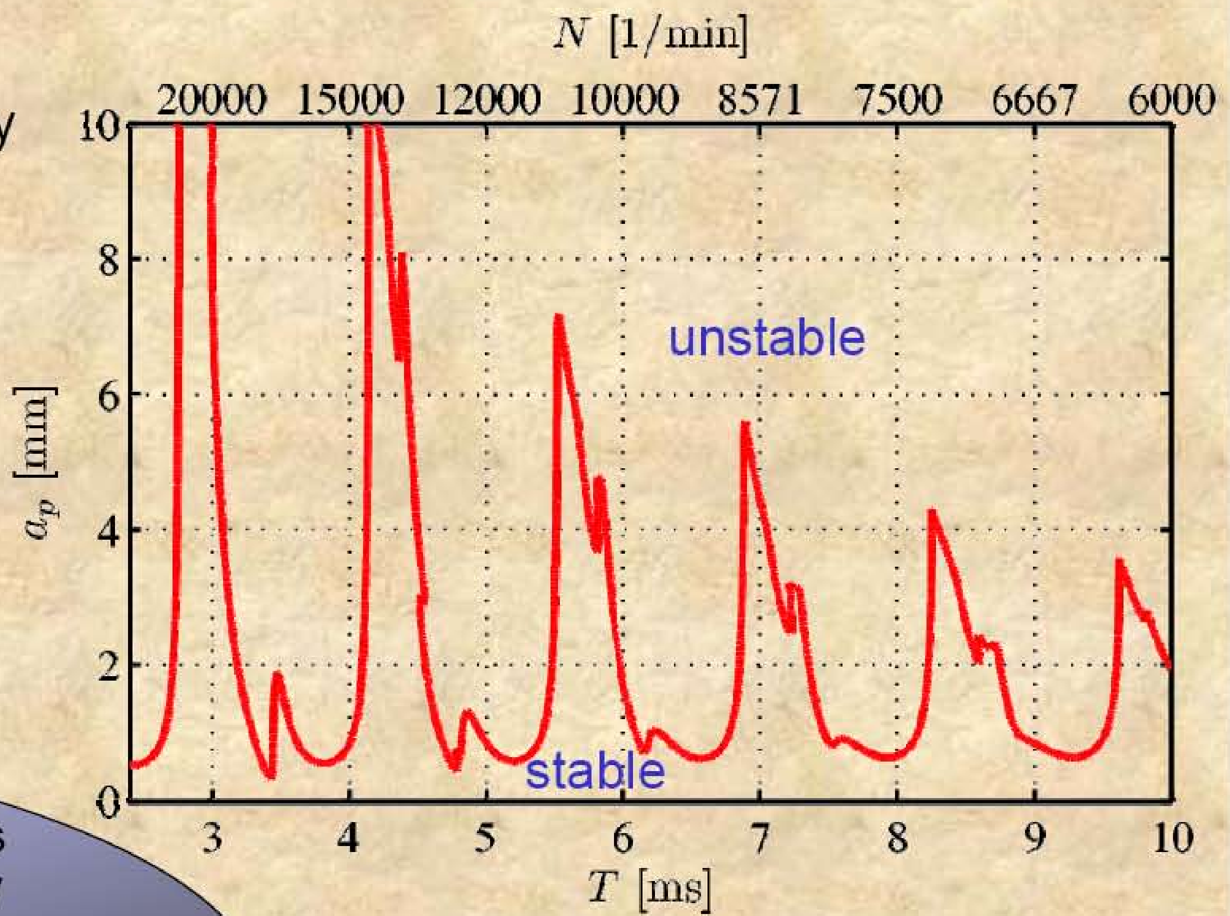


up-milling process of aluminum alloy with single-bladed cutter, a/D=0.05

f_0 [Hz]	D [-]	m [kg]
722	0.0087	0.02

k_{ct} [MPa]	k_{cr} [MPa]
644	238

dynamic stability depending on process parameters spindle speed N and axial depth of cut a_p



Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

Efficient Computation of Stability Diagrams

implicitly defined stability boundary

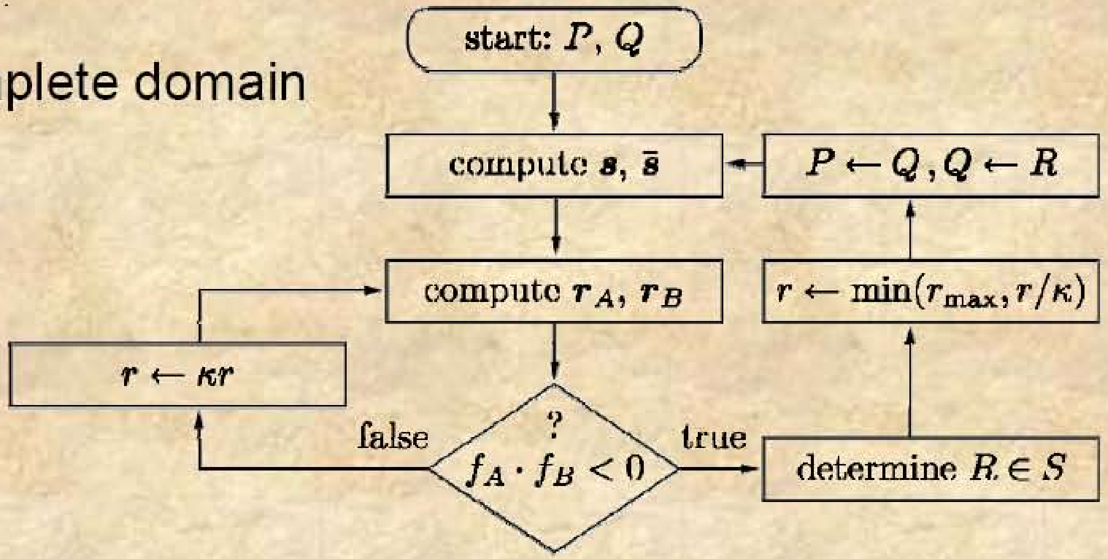
$$S = \{(N, a_p) \mid \max(|\mathrm{eig}(\boldsymbol{\Phi}(N, a_p))|) = 1\}$$

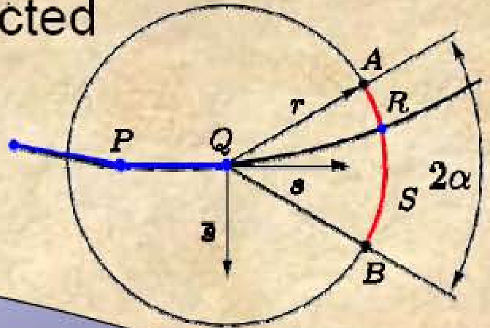
full discretization

- equidistant discretization of complete domain
- computationally expensive

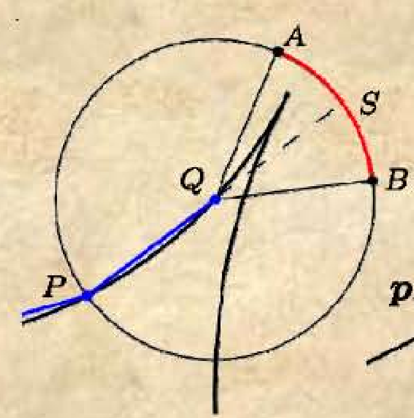
curve tracking

- local search method
- adaptive discretization
- high computational efficiency
- extra treatment of cusps and near-branch zones necessary
- discretization of disconnected regions not possible

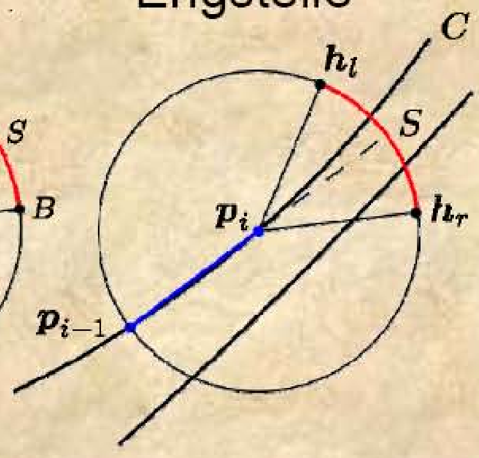




Spitzkehre

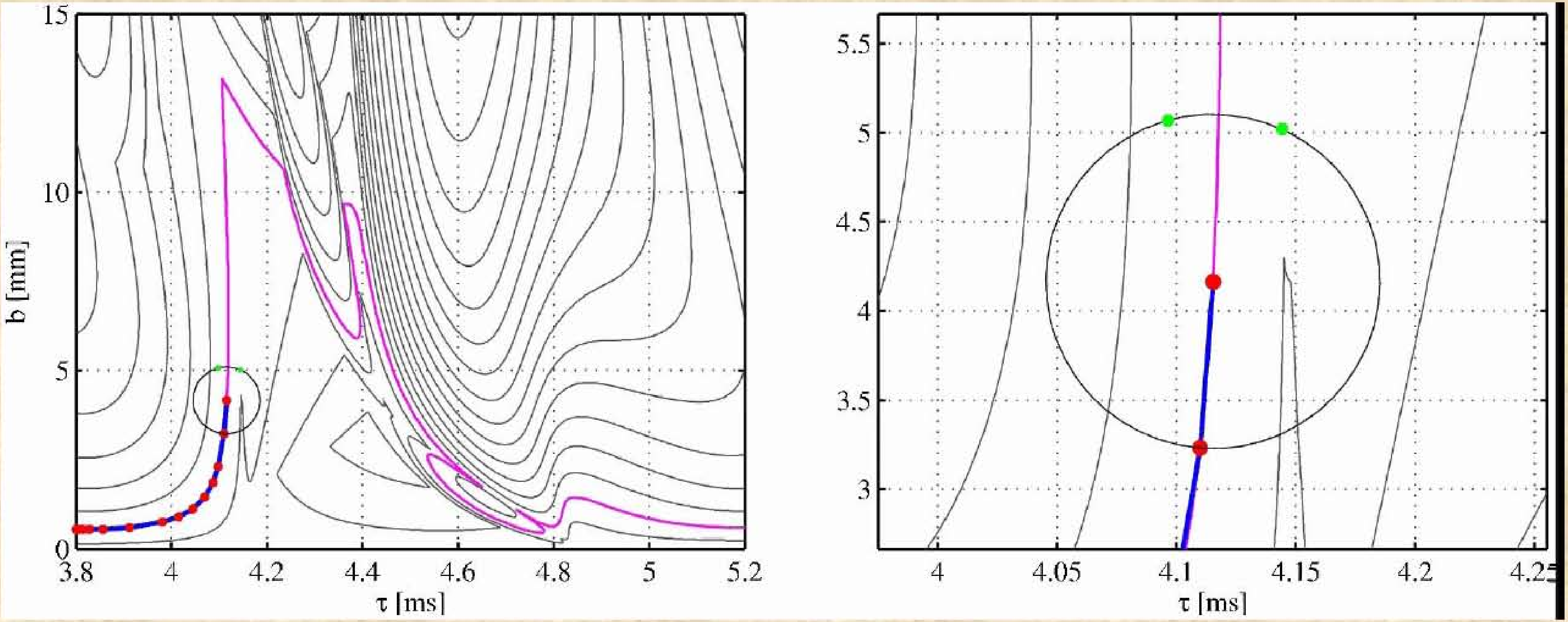


Engstelle



Institute of Engineering and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

Curve Tracking



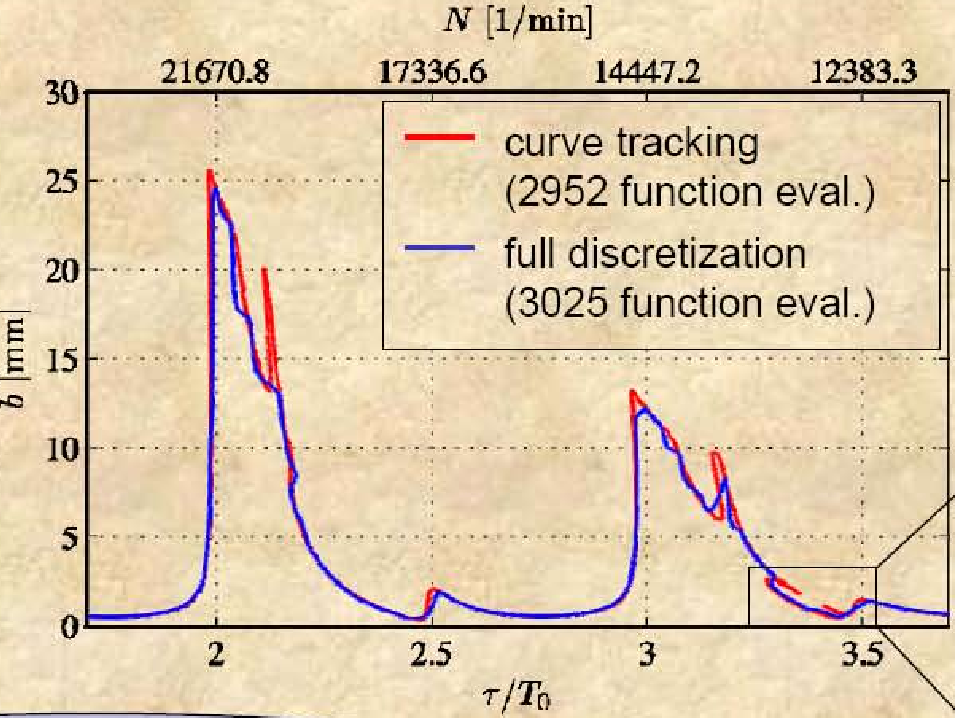
Comparison: Curve Tracking vs. Full Discretization

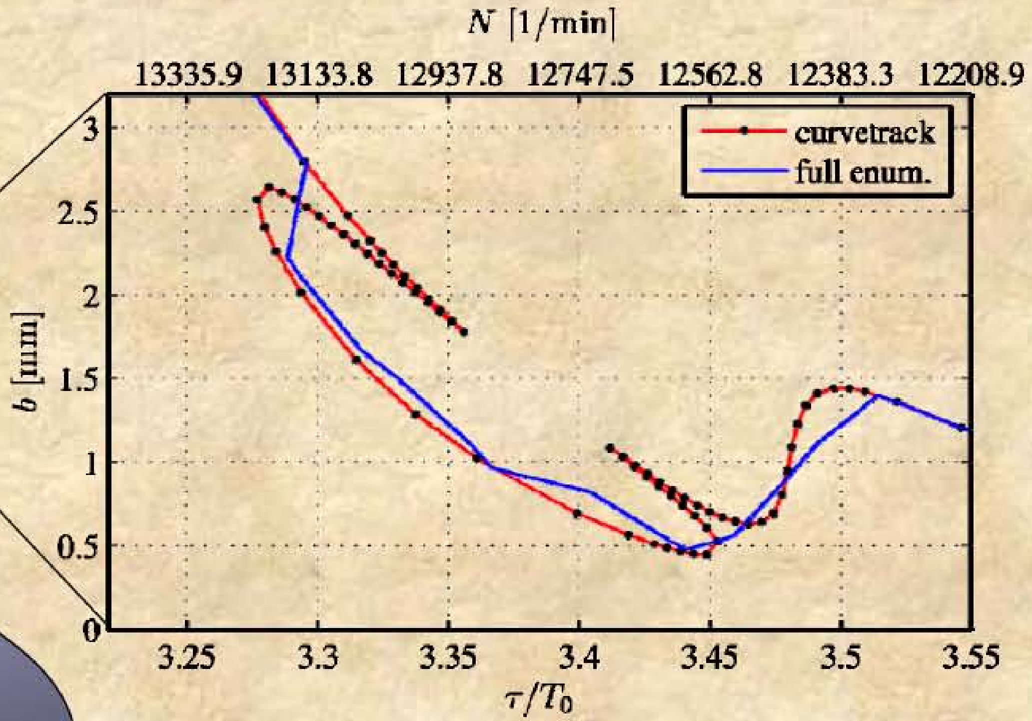
full discretization

- equidistant grid 55x55 points
- 3025 function evaluations

curve tracking

- 270 curve points
- 2952 function evaluations
- equivalent to full discretization with 250x250 = 62500 points





Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

Subspace Division

idea:

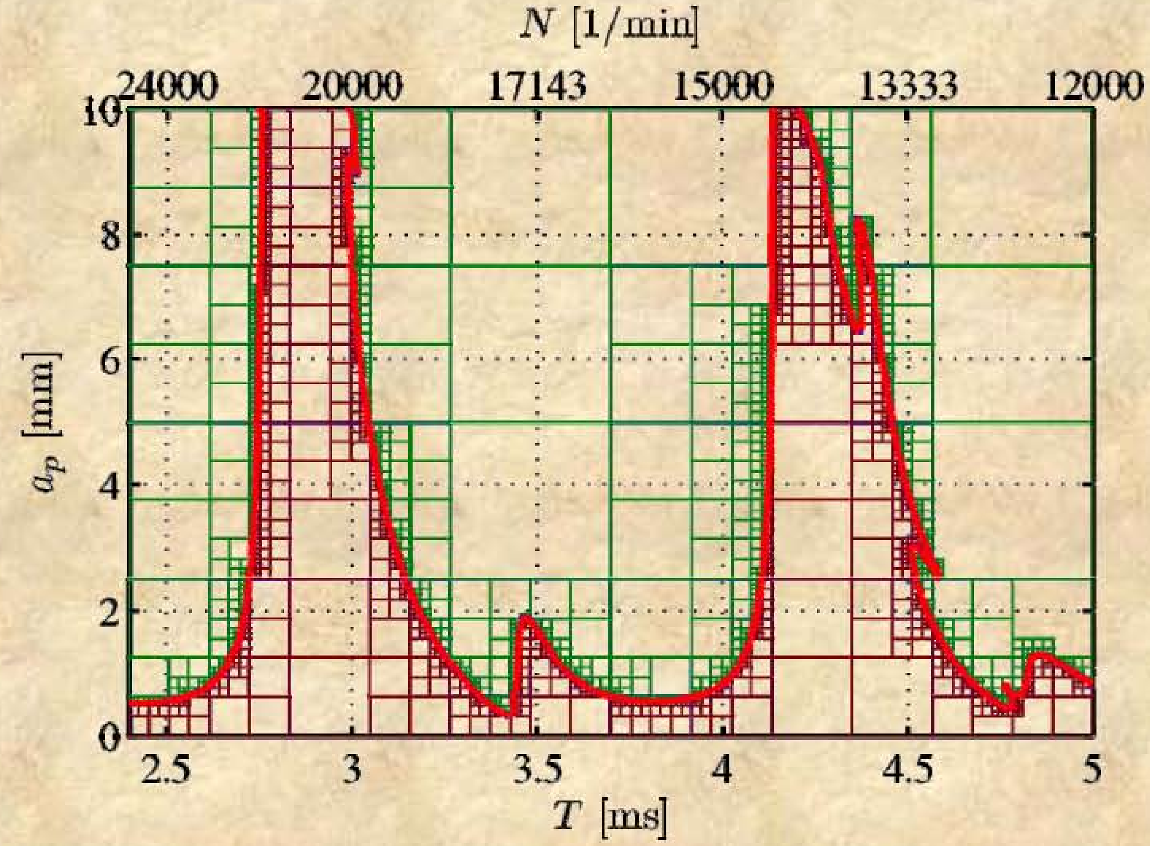
- initial discretization with coarse mesh
- successive refinement of subspaces being crossed by curve

advantages:

- robust algorithm
- discretization of disconnected curves possible

example:

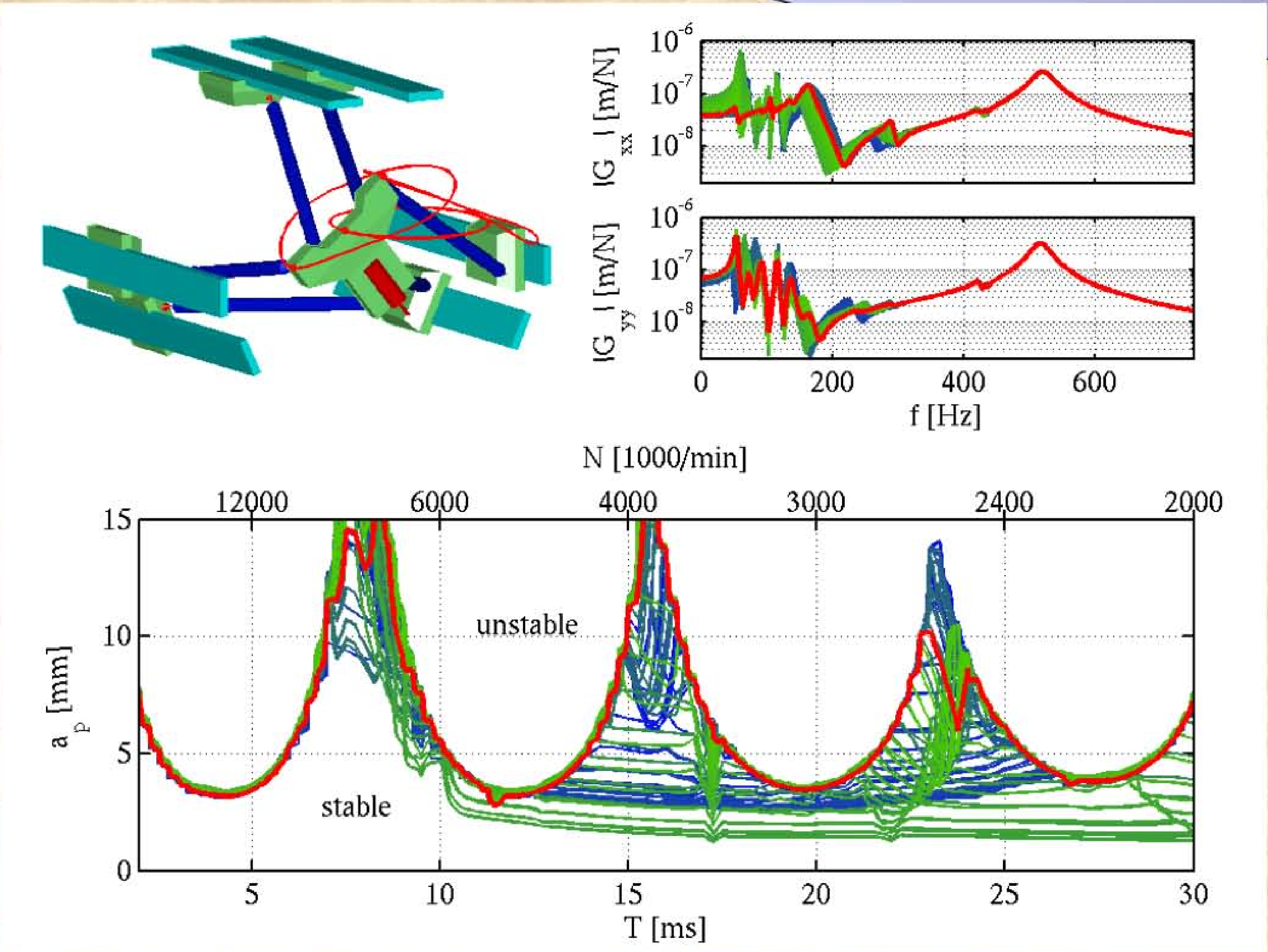
- base discretization: 5x7 points
- 7 refinement levels
- 6066 function evaluations
- equivalent grid: 257x385 points (98945 points)



Institute of Engineering and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Peter Eberhard

influence of rotation
on dynamic stability

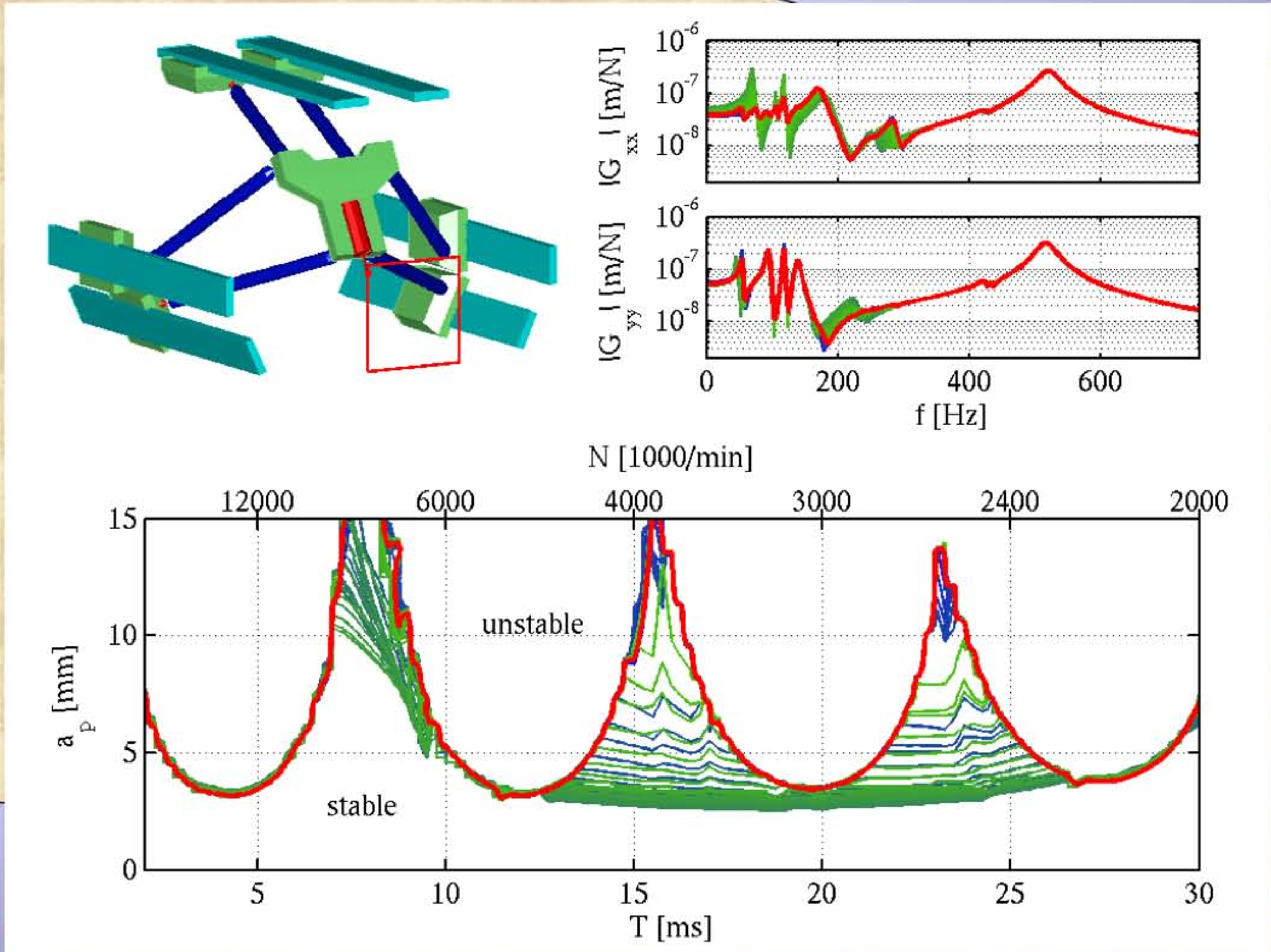
Pose-dependent Stability Charts



Prof. Dr.-Ing. Peter Eberhard

influence of translation
on dynamic stability

Pose-dependent Stability Charts



Conclusions

- strong pose-dependency of dynamic behavior at tool-tip for parallel kinematic machine tool Paralix

- multibody system of machine structure can be used for analysis of dynamic stability of cutting processes

- mass distribution between machine structure and tool-toolholder-spindle is responsible for ratio of resonance amplitudes

- methods for efficient computation of stability charts have been developed

- dynamic stability of process is influenced by pose of tool platform in workspace