Effect of Tolerance Analysis in Simulations Using Finite Element Methods

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Abstract: *This paper explores the effects of tolerance analysis in finite element simulations. Simulations in bending, tension, and torsion were run for a beam. For each situation, 100 different simulations were run using slight variations in the dimensions and material properties of the beam. In doing so, the tolerances of these properties were captured; all were within two percent of the mean value. The computer-generated data for maximum stress and maximum deflection was compared to the theoretical values. The goal was to determine if Abaqus/CAE can capture variations in the maximum deflection and maximum stress. Probability density functions were generated to see if the data followed a normal distribution. This research reaffirms the importance of designing products while considering tolerances. Variations in the dimensions and material properties impact the maximum stress and maximum deflection, which could be catastrophic when working close to failure regions. Additionally, designed parts may not fit together if tolerances are not considered. The results mimic what is expected in industry and the tolerances are indeed captured in the finite element*.

Keywords: *Abaqus/CAE, Finite Element Analysis, Tolerance Analysis, Bending, Tension, Torsion*

1. Introduction

In manufacturing, despite efforts to maintain accuracy, no two parts are ever identical. Manufacturers work within certain tolerance levels typically determined by statistics. Tolerances are given for dimensions, material properties, and other measurable quantities. If a part is not within the determined tolerance levels, it is considered “out of spec”. Manufactures keep careful track of the means and standard deviations in their processes by testing sample sizes of their products. It is important for design engineers to understand how tolerances will ultimately affect the final product and to obtain the means and standard deviations from the manufacturer ahead of time.

The effects of tolerances can be explored using finite element simulations. By randomly generating dimensions and material properties within certain tolerance limits, the resulting stresses and deflections can be explored. This computer generated data is essential for understanding how stresses and deflections may change as the dimensions and material properties change. Without this understanding of tolerances, failures may occur due to unpredicted results. By taking tolerances into account, engineers can more accurately determine if stresses will be above yield or ultimate.

Three situations were considered: bending, tension, and torsion. For each case, the maximum stress and deflection were compared to the theoretical values. Additionally, the mean and standard deviation were calculated for both values. Probability density functions were generated for the actual and theoretical results to display the distribution of the data. The end goal was to determine if Abaqus/CAE would capture the variability and produce results following a normal distribution.

2. Methods

2.1 Nomenclature

δ_{max}	Maximum deflection in *mm*
σ_{max}	Maximum stress in *MPa*
P	Load in *N*
L	Length in *mm*
E	Modulus of Elasticity in *MPa*
I	Moment of inertia in mm^4
c	Half the width in *mm*
M	Maximum moment in *Nmm*
A	Cross-sectional area in mm^2
T	Torque in *Nmm*
ϕ	Angle of twist in *radians*
G	Modulus of Rigidity in *MPa*
J	Polar Moment of Inertia of an Area in mm^4
a	Cross-section width in *mm*
b	Cross-section height in *mm*

2.2 Equations

For the bending simulations, the theoretical maximum deflection and max stress were calculated using Equations 1 and 2 below.

$$\delta_{max} = \frac{PL^3}{3EI} \tag{1}$$

$$\sigma_{max} = \frac{Mc}{I} \tag{2}$$

To calculate the theoretical maximum deflection and maximum stress for tension, equations 3 and 4 were used.

$$\delta_{max} = \frac{PL}{EA} \tag{3}$$

$$\sigma_{max} = \frac{P}{A} \tag{4}$$

For torsion, the theoretical angle of twist and maximum shear stress were calculated using equations 5 and 6.

$$\phi = \frac{TL}{GJ} \tag{5}$$

$$\tau = \frac{T}{0.601 \cdot a \cdot b^2} \tag{6}$$

2.3 Mesh and MATLAB Code

The beam's mean dimensions are 10 mm x 10 mm x 100 mm. The mean modulus of elasticity is 210 GPa, and the Poisson's ratio is 0.3. As shown in Figure 1, The beam was meshed with C3D8I elements and the same mesh was used for every simulation. The beam was fixed on one end and loaded with three separate conditions: bending, tension, and twisting (Figure 2). For each condition, the mean load applied was 50 N. The mean twisting torque was defined as 5000 N-mm. For bending and twisting, skins were defined on the surface of the beam to better capture the stresses and deformation in the loaded beam. These skins were meshed with S4R elements.

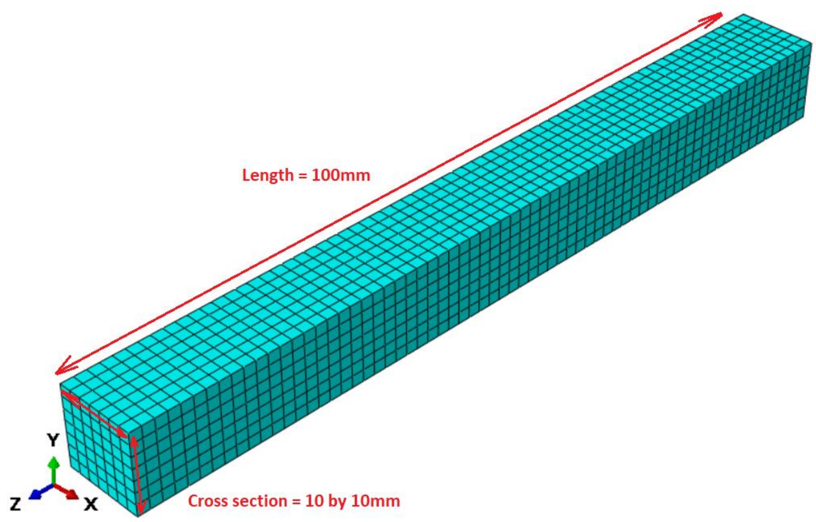

Figure 1. Mesh of the beam.

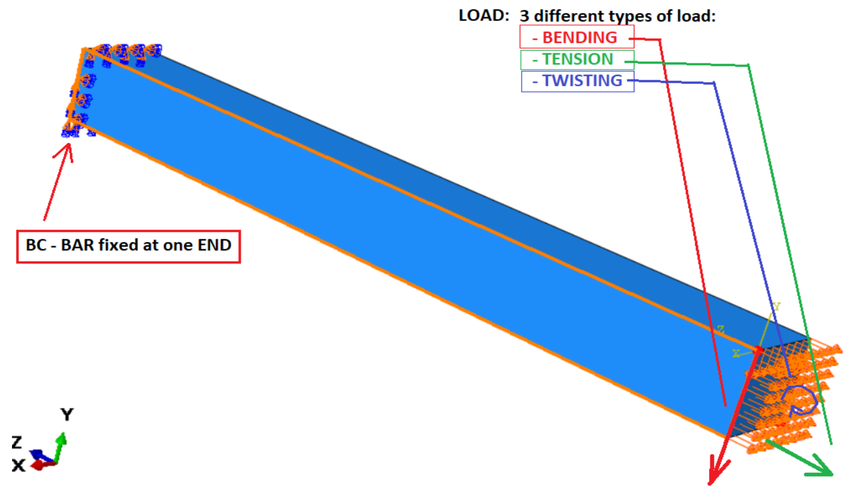

Figure 2. Visual description of the three loading conditions.

One hundred input files were generated for each loading case with variations in load, cross-section, length, and young's modulus. For each measurement, the standard deviation was 2% of the mean value. A Matlab code was developed to make the file generating and analysis process easier. For brevity the Matlab program used to generate the input files and analyze the data is not included in this report. A sample annotated input file is shown in Figures 3-5.

INPUT FILE

```
*Heading
** Job name: 445_Team_Project Model name: BAR
** Generated by: Abaqus/CAE 2016
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**
** PARTS
**
*Part, name=Part-1
*Node
      1,          10.,          10.,         100.
      2,          10.,           8.,         100.
      3,          10.,           6.,         100.
      4,          10.,           4.,         100.
      5,          10.,           2.,         100.
```

Part created and coordinates for each node

X Y Z

Figure 3. Sample of the Abaqus Input File.

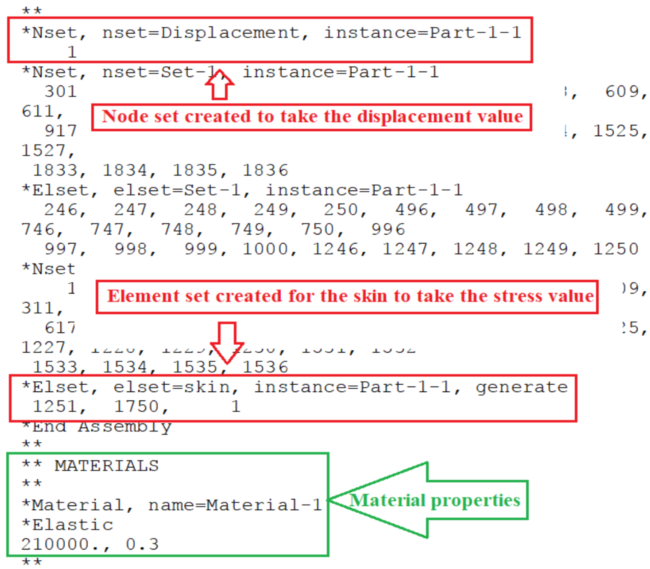

Figure 4. Abaqus input file node sets and material properties.

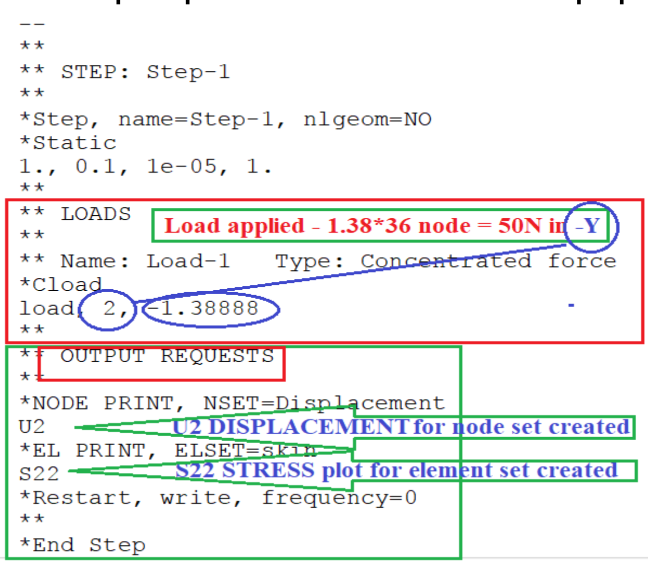

Figure 5. Abaqus input file loads and output.

2.4 Tolerance Analysis Results

Figures 6-8 show how the data from maximum deflection and maximum stress were collected for each loading condition. For all three conditions, maximum deflection was determined using the solid beam. The maximum stress in bending and torsion was found using skins, while the maximum stress for tension was determined using the solid beam.

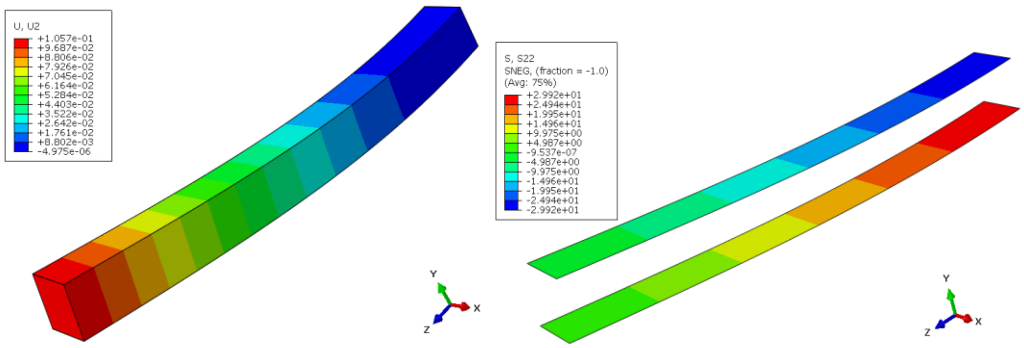

Figure 6. Results of the bending analysis on the beam and skins respectively.

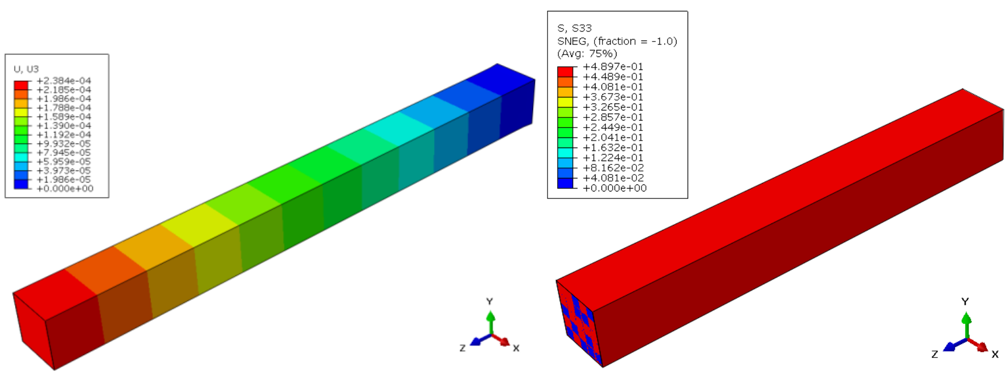

Figure 7. Results of the tension analysis on the beam and the fixed end respectively.

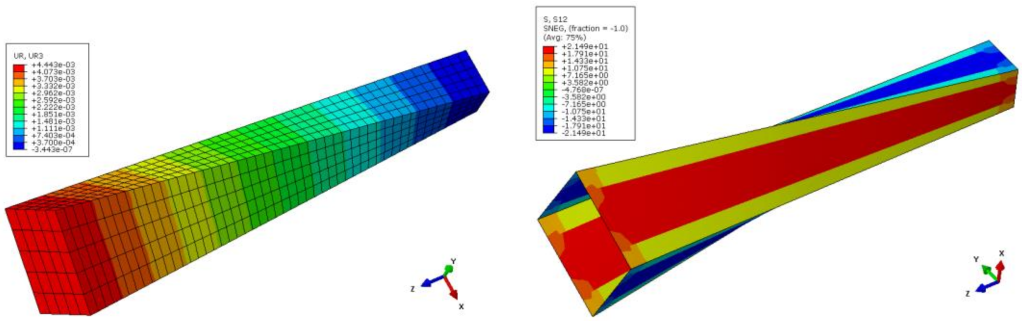

Figure 8. Results of the torsion analysis on the beam and the skins respectively.

2.5 Numerical Results

To keep track of the variations in the data, Tables 1-3 were developed for bending, tension, and torsion, respectively. Each table lists the dimensions, material properties, computer-generated results, theoretical results, and percent error for the first 10 simulations of each condition. The theoretical values were calculated using Equations 1 and 2 for bending, Equations 3 and 4 for tension, and Equations 5 and 6 for twisting. With the addition of skins for the bending and twisting conditions, there was very little error in the data.

Table 1. Numerical results for first 10 bending simulations.

	Data from INPUT Files						Results		Theoretical		Error %	
	X	Y	L	E	Load/ node	TOTAL Load	δ_{max}	σ_{max}	δ_{max}	σ_{max}	δ_{max}	σ_{max}
1	10.1080	10.108	103.67	2.01E+05	1.4128	50.8608	0.1082	29.9300	0.1083	30.6380	0.10%	2.31%
2	10.0640	10.064	97.385	2.08E+05	1.3984	50.3424	0.0870	28.2700	0.0871	28.8600	0.07%	2.04%
3	10.7160	10.716	105.54	2.04E+05	1.4732	53.0352	0.0925	26.7100	0.0926	27.2940	0.08%	2.14%
4	10.1450	10.145	99.874	2.13E+05	1.3832	49.7952	0.0879	27.9700	0.0879	28.5770	0.08%	2.12%
5	9.9752	9.9752	102.98	2.16E+05	1.4283	51.4188	0.1049	31.2600	0.1051	32.0070	0.15%	2.33%
6	10.1340	10.134	97.585	2.13E+05	1.4342	51.6312	0.0854	28.4500	0.0854	29.0440	0.06%	2.05%
7	10.0980	10.098	102.07	2.13E+05	1.3805	49.6980	0.0953	28.8900	0.0954	29.5590	0.11%	2.26%
8	10.0590	10.059	98.425	2.14E+05	1.357	48.8520	0.0851	27.7500	0.0852	28.3470	0.08%	2.11%
9	9.7862	9.7862	98.381	1.98E+05	1.4288	51.4368	0.1080	31.6700	0.1081	32.3970	0.07%	2.24%
10	10.0650	10.065	98.49	2.16E+05	1.3413	48.2868	0.0833	27.3900	0.0833	27.9860	0.08%	2.13%

Table 2. Numerical results for first 10 tension simulations.

	Data from INPUT Files					Results		Theoretical		Error %	
	X	Y	L	E	Load	δ_{max}	σ_{max}	δ_{max}	σ_{max}	δ_{max}	σ_{max}
1	10.1080	10.108	103.67	2.01E+05	50.862	1.08E-01	29.9300	1.08E-01	30.6380	0.10%	2.31%
2	10.0640	10.064	97.385	2.08E+05	50.343	8.70E-02	28.2700	8.71E-02	28.8600	0.07%	2.04%
3	10.7160	10.716	105.54	2.04E+05	53.035	9.25E-02	26.7100	9.26E-02	27.2940	0.08%	2.14%
4	10.1450	10.145	99.874	2.13E+05	49.795	8.79E-02	27.9700	8.79E-02	28.5770	0.08%	2.12%
5	9.9752	9.9752	102.98	2.16E+05	51.417	1.05E-01	31.2600	1.05E-01	32.0070	0.15%	2.33%
6	10.1340	10.134	97.585	2.13E+05	51.63	8.54E-02	28.4500	8.54E-02	29.0440	0.06%	2.05%
7	10.0980	10.098	102.07	2.13E+05	49.697	9.53E-02	28.8900	9.54E-02	29.5590	0.11%	2.26%
8	10.0590	10.059	98.425	2.14E+05	48.853	8.51E-02	27.7500	8.52E-02	28.3470	0.08%	2.11%
9	9.7862	9.7862	98.381	1.98E+05	51.438	1.08E-01	31.6700	1.08E-01	32.3970	0.07%	2.24%
10	10.0650	10.065	98.49	2.16E+05	48.288	8.33E-02	27.3900	8.33E-02	27.9860	0.08%	2.13%

Table 3. Numerical results for the first 10 twisting simulations.

	Data from INPUT Files						Calculated		Results		Theoretical		Error %	
	X	Y	L	E	G	T	r	J	Angle of twist, θ	τ_{max}	Angle of twist, θ	τ_{max}	Angle of twist, θ	τ_{max}
1	10.108	10.11	103.67	2.01E+05	80667	5032	7.1474	1467.99	4.44E-03	22.730	4.41E-03	23.429	0.83%	2.98%
2	9.739	9.739	99.133	2.11E+05	84975	5277	6.8862	1264.83	4.90E-03	26.640	4.87E-03	27.471	0.72%	3.03%
3	9.730	9.73	106.07	2.13E+05	79200	5072	6.8801	1260.42	5.02E-03	25.680	5.39E-03	26.470	6.81%	2.98%
4	9.959	9.959	99.752	2.16E+05	81535	5142	7.0421	1383.33	4.30E-03	24.270	4.55E-03	25.028	5.51%	3.03%
5	10.134	10.13	97.585	2.13E+05	81886	5049	7.1658	1483.15	3.91E-03	22.600	4.06E-03	23.323	3.67%	3.10%
6	10.207	10.21	101.45	2.09E+05	79766	4921	7.2174	1526.35	3.93E-03	21.580	4.10E-03	22.251	4.23%	3.02%
7	10.178	10.18	97.706	2.06E+05	78016	4706	7.1969	1509.08	3.72E-03	20.800	3.91E-03	21.460	4.87%	3.08%
8	10.288	10.29	100.65	2.07E+05	81473	4829	7.2747	1575.38	3.74E-03	20.670	3.79E-03	21.323	1.29%	3.06%
9	9.980	9.98	99.517	2.11E+05	79796	4914	7.0566	1394.81	4.16E-03	23.050	4.39E-03	23.770	5.38%	3.03%
10	9.994	9.994	99.67	2.13E+05	81034	5111	7.0668	1402.88	4.28E-03	23.870	4.48E-03	24.618	4.50%	3.04%

2.6 Means and Standard Deviations

Table 4, Table 5 and Table 6 list the propagated mean and standard deviation values for bending, tension, and twisting respectively.

Table 4. Mean and standard deviations for 100 bending simulations.

	Results		Theoretical	
	δ_{max}	σ_{max}	δ_{max}	σ_{max}
Mean	0.0960	29.2400	0.1026	29.9205
Stdev	0.011134	0.975807	0.008055	1.014698

Table 5. Mean and standard deviations for 100 tension simulations.

	Results		Theoretical	
	δ_{max}	σ_{max}	δ_{max}	σ_{max}
Mean	0.0960	29.2400	0.1026	29.9205
Stdev	0.011134	0.975807	0.008055	1.014698

Table 6. Mean and standard deviations for 100 torsion simulations.

	Results		Theoretical	
	δ_{max}	τ_{max}	δ_{max}	τ_{max}
Mean	0.0043	23.1050	0.0045	23.8195
Stdev	0.000353	0.53033	0.000121	0.55225

2.7 Probability Density Functions

Once the data was collected, four comparative Probability Density Functions, or PDFs, were developed for each loading condition. The first PDF set for each condition compares experimental and theoretical data (Figure 9, Figure 11 and Figure 13). The second set of PDFs compares random and theoretical data (Figure 10, Figure 12 and Figure 14). These random variables are a set of values that was used to compare to the experimental values that were found in each of the loading conditions. It is clear the data follows a normal distribution.

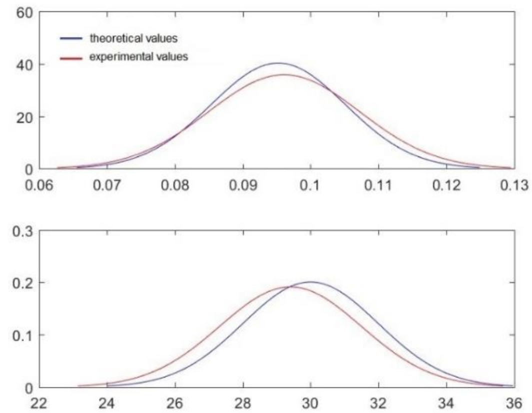

Figure 9. PDFs for the experimental and theoretical bending values.

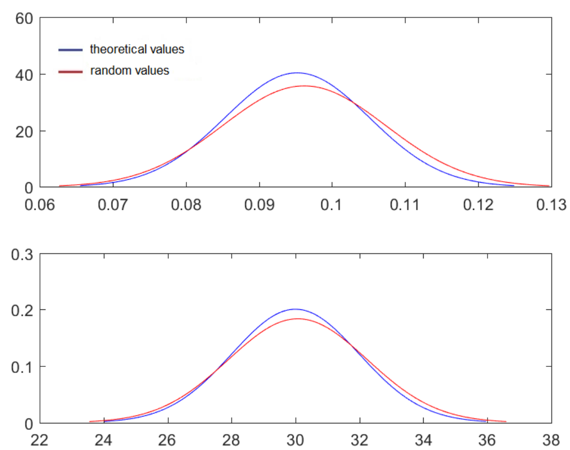

Figure 10. PDFs for the random and theoretical bending values.

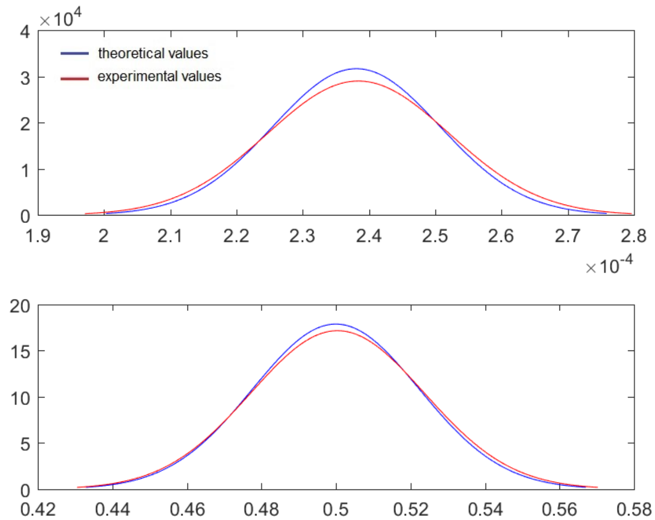

Figure 11. PDFs for the experimental and theoretical tension values.

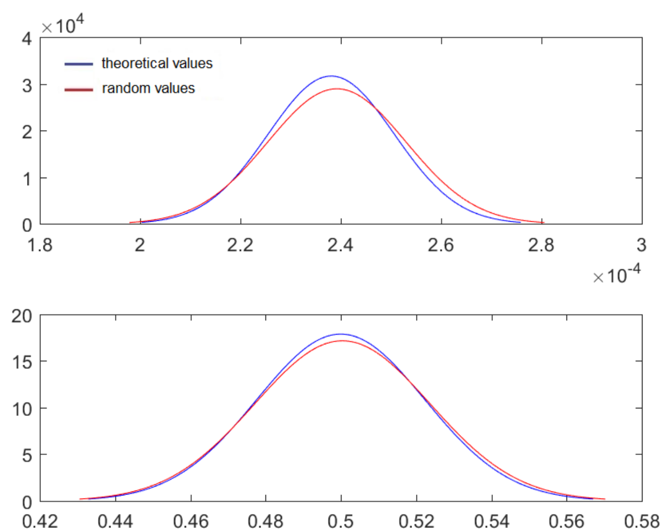

Figure 12. PDFs for the random and theoretical tension values.

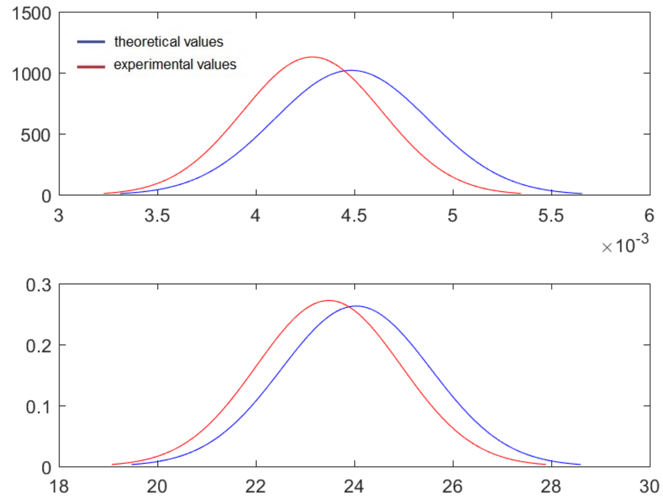

Figure 13. PDFs for the experimental and theoretical twisting values.

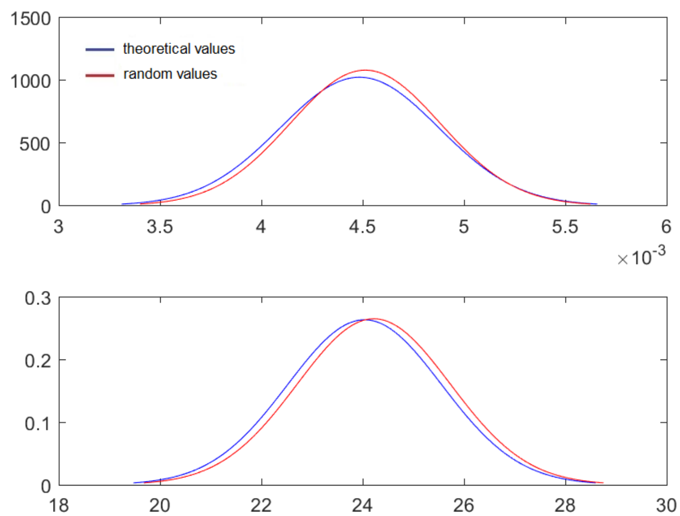

Figure 14. PDFs for the random and theoretical twisting values.

3. Analysis and Discussion

The probability density functions confirm that tolerance analysis can be verified using finite element methods. All three situations followed a normal distribution. This result was expected as it mimics what occurs in manufacturing. Slight variations in the manufactured dimensions and material properties alter the maximum stress and deflection in a part. It is important for a design engineer to be aware of the tolerances they are working with, so they can design the part according. Tolerance analysis is vital to avoid failures and ensure manufactured parts will still fit together within specifications.

For both bending and tension the error propagation was minimal. For bending, the error for displacement was about .1%, while the error for stress was around 2%. The error was around .35% for displacement and 2% for stress in tension. Twisting had slightly higher margins of error with approximately 5% for the angle of twist and 3% for shear stress. This data was originally run without the use of skins. Without skins, the error was closer to 20% for stress in bending and not even comparable to the theoretical values for shear stress in twisting. This proves the importance of using skins since stresses on the surface are often quite different than the stresses throughout the entire beam.

The data confirms that tolerance analysis using finite element methods produces legitimate results and follows a normal distribution. This demonstration further confirms that finite element analysis is a quick and inexpensive way to predict how objects will behave.

4. Conclusion

The goal of this experiment was to determine if Abaqus/CAE is capable of capturing tolerances in the maximum stress and deflection of a steel beam. In these analyses, the maximum stresses and deflections of the steel were explored, through bending, tension, and torsion. After observing these values, the mean and standard deviations of these values were also calculated and compared to their theoretical values. The curves of the probability density functions confirm the computer-generated data follows a normal distribution. This research demonstrates the importance of understanding tolerances when designing a part. Small deviations in the dimensions and material properties impact the final dimensions, deflections, and stresses. These tolerances levels must be considered in manufacturing to determine when a part is "out of specs" and should be discarded.

5. References

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