

Elastoplastic Simulations with a Tangential Plasticity Constitutive Model for a Thin Wall Bridge Pier Subjected to Various Non-proportional Cyclic Loading Conditions

Hideto Momii¹, Seiichiro Tsutsumi², Riccardo Fincato²

¹⁾ ESTECH Corporation

²⁾ Joining and Welding Research Institute, Osaka University

Abstract: Various elastoplastic constitutive models have been proposed to predict the structural stiffness; most of them use an associated flow rule, which defines a plastic flow direction normal to the yield surface. These models, based on this assumption, have contributed to the predictions of elastoplastic deformations of solid structures in several applications, however, in some cases, they tend to overestimate the structural stiffness. In order to overcome this defect, we have adopted an unconventional elastoplastic model capable of taking into account the generation of the inelastic strain rate not only along the direction normal to the yield surface but also along the tangential one. In this paper the aforementioned model has been studied by applying a series of non-proportional loading paths to a thin wall pier and comparing the results obtained with the ones derived by neglecting the tangential contribution.

Keywords: Non-proportional Cyclic Loading, Elasto-plastic FE Analysis, Tangential Plasticity, Low Cycle Fatigue Life, Constitutive Equation.

1. Introduction

Thin wall steel bridge are widely used in highway bridges, buildings, and offshore structures. In Japan, on 17th January 1995, the Hyogoken-Nanbu earthquake hit huge region around Kobe and Osaka and its magnitude, 7.3 degree, was higher than that assumed level. Many bridge piers of the highway were severely damaged due to these cyclic excitations. Seismic solicitations are naturally quite complex, the amplitude changes during earthquake as well as its direction, which usually is not uni-directional but rather bi-directional and non-proportional in lateral direction. Therefore, in order to correctly design the structural stiffness for those conditions, several cyclic non-proportional loadings have been investigated in this work, trying to understand the reliability of the model to catch a realistic behavior for the bridge pier. Moreover, bridges structural parts, buildings and offshore piers may not experience seismic actions only one time, therefore, in addition to the previous considerations, the design method must be able to take account also the cumulative damages due to loading history.

On the other hand, numerous experimental studies have shown that fatigue life depends not only on the strain amplitude but also on the strain path as well, in fact, it changes if proportional or non-proportional cyclic loading conditions are applied (c.f. Itoh, 2013; Hoshide, 2011; Reis, 2011). The base of cantilever structures including bridge piers, for instance, is one of the typical site where the principal stress/strain directions changes under the macroscopic unidirectional loading conditions, as well as for more complex operational loading conditions. Therefore, it is important to predict accurately the plastic deformations under general loading conditions and to reflect them in a damage counting law for fatigue life prediction.

The conventional elastoplasticity models (Drucker, 1988), based upon the premise that the interior of yield surface is a pure elastic domain, have contributed to the predictions of elastoplastic deformation of solid materials such as metals, geomaterials and concrete. They, however, cannot describe cyclic loading behavior below the yield stress of materials due to the neat distinction into two separate domains: elastic and plastic ones. An additional drawback is the unrealistically stiff response that might be generated in multiaxial loading problems, especially for those where a non-proportional loading path is considered. The main reason for this is the assumption that the inelastic strain is not induced by the stress rate component tangential to the yield surface but it is uniquely directed outward along the normal to the plastic potential surface, underestimating the real development of irreversible contributions.

In order to overcome these defects various elastoplasticity models have been proposed up to the present (Hashiguchi, 2009). Many of them however, except the one adopted in this study (Hashiguchi, 2001, 2011, 2014; Tsutsumi, 2005), may not be applicable for a general deformational behavior including cyclic loading conditions.

In this paper an unconventional elastoplastic constitutive model based on the Extended Subloading Surface model with additional modifications to catch the so-called “Tangential Plasticity” (Tsutsumi, 2005), has been adopted for the numerical analyses. This theory allows to take into account the contributions of the plastic deformations, even within the elastic domain of conventional plasticity theories, and the tangential inelastic strain induced by a stress rate component tangential to the plastic potential surface. Experimental results have been compared with the numerical ones carried out including or not the effect of tangential plasticity.

2. Elastoplastic constitutive equation

2.1 Basic description of extended subloading surface model

In the present work the extended subloading surface model (Hashiguchi, 2009), in the form of the cutting-plane return mapping method (Simo, 1985; Hashiguchi, 2011, 2014), has been adopted for a fast and accurate computation.

For sake of brevity, the extended subloading surface constitutive equations and its return mapping formulation will not presented in the present paper; the reader is referred to the lecture notes (Hashiguchi, 2009) for a full and detailed explanation of the theory. As a general description of the model it can be mentioned here that the smooth transition from elastic and plastic domains is achieved by adding a new loading surface (i.e. subloading surface), which is created by means of a

similarity transformation from the conventional yield one. The insertion of a mobile similarity center enriches further the formulation, allowing to a more realistic description of the irreversible strain accumulation during cycling loading.

2.2 Extension to the tangential plasticity

Most of the conventional plasticity models, and unconventional one as well, adopt an associate flow rule for the definition of the plastic strain rate/increment. Under this assumption it is possible to catch a realistic response in case of proportional loading paths, where the ratio among the principal stresses and their directions are kept constant. However, whenever a non-proportional loading condition is imposed, they tend to overestimate the material stiffness.

In order to take into account the tangential stress rate component some preliminary hypotheses (Hashiguchi, 2001; Tsutsumi, 2005) are needed:

1. the stress rate is linearly related with the tangential stretch (where A is a stress function):

$$\mathbf{D}^t = A \dot{\boldsymbol{\sigma}}^* \quad (1)$$

2. the additive decomposition of the stretching holds:

$$\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p + \mathbf{D}^t \quad (2)$$

3. the tangential stress component, and the inelastic stretch associated, are purely deviatoric (Rudnicki, 1975);
4. no hardening can be generated by a stress rate component tangential to the yield surface.

The main idea of this new numerical algorithm is to adapt the return mapping, formulated with an associate flow rule, for taking into account the tangential relaxation in order to keep the advantages of a fast but accurate computation.

In particular, the last of the assumptions (i.e. number 4), allows us to split the normal and tangential stress rate component effects, evaluating the first with the cutting-plane method and the second with a sort of ‘radial return mapping’.

After the stress is brought back to lie on the correct plastic surface (in Figure 1) through a series of local convergences on cutting-planes, the tangential stress rate component can be subtracted from the total stress state. The definition of the rate involves some mathematical passages starting from the substitution of Equation 1 into Equation 2 and inverting the relationship to express the stress rate as function of the strain one.

In order to split the aforementioned contributions, the generic stress function A is given as:

$$A = \left(\frac{T}{2G - 2GT} \right); \quad T = \xi R^b \quad (3)$$

Where T is an exponential function that depends on two material constants ξ and b, and on the similarity ratio R. Finally it is possible to write the stress rate as in Equation 4, where the tangential one is expressed in the lower expression:

$$\begin{aligned}\dot{\boldsymbol{\sigma}} &= \left\{ \mathbf{E} - \frac{\mathbf{E}\mathbf{N} \otimes \mathbf{E}\mathbf{N}}{M_p + tr\mathbf{N}\mathbf{E}\mathbf{N}} \right\} \mathbf{D} - \dot{\boldsymbol{\sigma}}_t^* \\ \dot{\boldsymbol{\sigma}}_t^* &= T \left(\mathbf{E} - \frac{\mathbf{E}\mathbf{N} \otimes \mathbf{E}\mathbf{N}}{M_p + tr\mathbf{N}\mathbf{E}\mathbf{N}} - \frac{1}{3} \mathbf{E} \otimes \mathbf{I} - \frac{\mathbf{E}\mathbf{N} \otimes \mathbf{N}}{M_p + tr\mathbf{N}\mathbf{E}\mathbf{N}} M_p \right) \mathbf{D}\end{aligned}\quad (4)$$

However, once the subtraction is performed, the new stress state will not satisfy anymore the local equilibrium, and then a correction must be introduced as shown in the following system of equations:

$$\begin{aligned}\boldsymbol{\sigma}_{n+1}^{(k+1),2correct} &= \boldsymbol{\sigma}_{n+1}^{(k+1),1} - (\dot{\boldsymbol{\sigma}}_t^* + correction) \\ \mathbf{N}^{correct,2} &= \dot{\boldsymbol{\sigma}}_{n+1}^{*(k+1),2correct} / |\dot{\boldsymbol{\sigma}}_{n+1}^{*(k+1),2correct}| \\ \lambda_{n+1}^{tangential} &= \sqrt{3/2} |\dot{\boldsymbol{\sigma}}_{n+1}^{*(k+1),2correct}| - \sqrt{3/2} |\dot{\boldsymbol{\sigma}}_{n+1}^{*(k+1),1}| \\ \boldsymbol{\sigma}_{n+1}^{(k+1),final} &= \boldsymbol{\sigma}_{n+1}^{(k+1),2correct} - \sqrt{2/3} \lambda_{n+1}^{tangential} \mathbf{N}^{correct,2}\end{aligned}\quad (5)$$

It is worth mentioning that this strategy does not require an iterative procedure, since the distance between $\boldsymbol{\sigma}_{n+1}^{(k+1),2correct}$ and the final stress state point can be easily evaluated without slowing down excessively the computational time.

2.3 Consideration of stress plateau response

As an additional feature, for a more realistic description of the material behavior, a special numerical procedure has been added in order to freeze the material hardening after the yielding, obtaining the strain plateau visible in Figure 2.

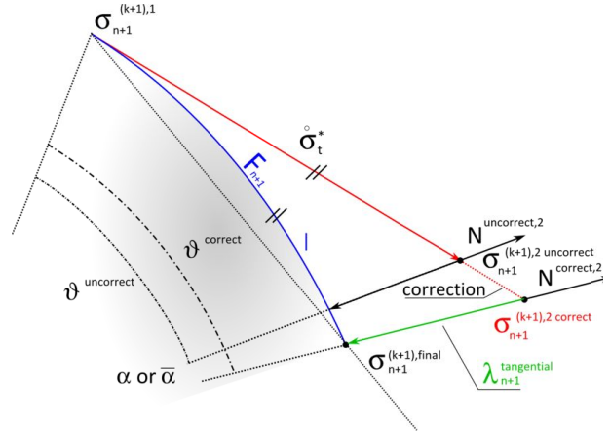


Figure 1. Tangential stress correction procedure.

Table 1. Material constants

Elastic moduli	$E = 210$ [GPa], $\nu = 0.3$
Initial value of yield surface	$F_0 = 294.1$ [MPa]
Isotropic hardening $F = F_0[1 + h_1\{1 - \exp(-h_2 H)\}]$	$h_1 = 0.567, h_2 = 30.0$
Stress plateau threshold	$H_p = 0.01$
Kinematic hardening $\dot{\alpha} = a_\alpha \left(r_\alpha F \frac{\bar{\sigma}'}{\ \bar{\sigma}'\ } - \sqrt{\frac{2}{3}} \alpha \right) \ \mathbf{D}^p\ $	$a_\alpha = 34.0, r_\alpha = 0.233$
Evolution of normal yield ratio	$Re = 0.0, u = 500.0, u_s = 1.67$
Translation of similarity center	$c = 200.0, \chi = 0.9$
Tangential plasticity $\mathbf{D}' = A \dot{\sigma}_i^*, A = \left(\frac{T}{2G - 2GT} \right), T = \xi R^b$	$\xi = 0.9, b = 2.0$

Outlined squared: Experimental results (Nakajima, 2000)

Lined: Simulation results

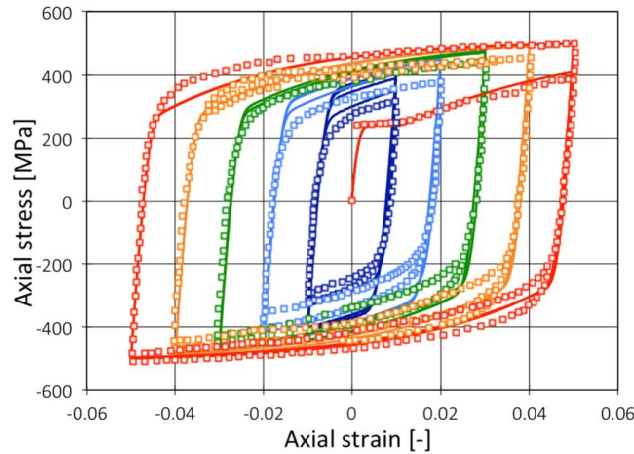


Figure 2. Stress-strain responses of SS400 for the strain ranges =0.01, 0.02, 0.03, 0.04, 0.05 (the responses of the models with $(T=0)$ and $(T \neq 0)$ exhibit the same results under uniaxial stress cycles.)

Briefly, the user can set the value of the cumulative isotropic hardening variable (i.e. H in the subloading surface model) as a threshold beyond which an expansion of the yield surface is possible. Below that value the plastic deformations can occur without contributing to the material hardening, as it happens in a perfect plasticity case.

3. Comparison with test result

3.1 Description of benchmark experiments

The experimental results, compared against the numerical simulations, were carried out by Nishikawa, 1998. Figure 3a shows the sketch of the specimen, fixed at the base and subjected to a constant axial load and quasi-static cyclic lateral displacements at the top of the column. Geometrically the sample consists of a thin wall bridge pier, with a circular cross section of 9 mm thickness, which is assumed to be made of structural steel SS400 (JIS) with a stress plateau domain just after the yielding. As shown in Table 1 and Figure 2, the material constants are fitted using the test results performed by Nakajima, 2000. The geometric properties of the specimen are shown in Table 2, where h is the height from the base of the column, $2r$ is the outside diameter of

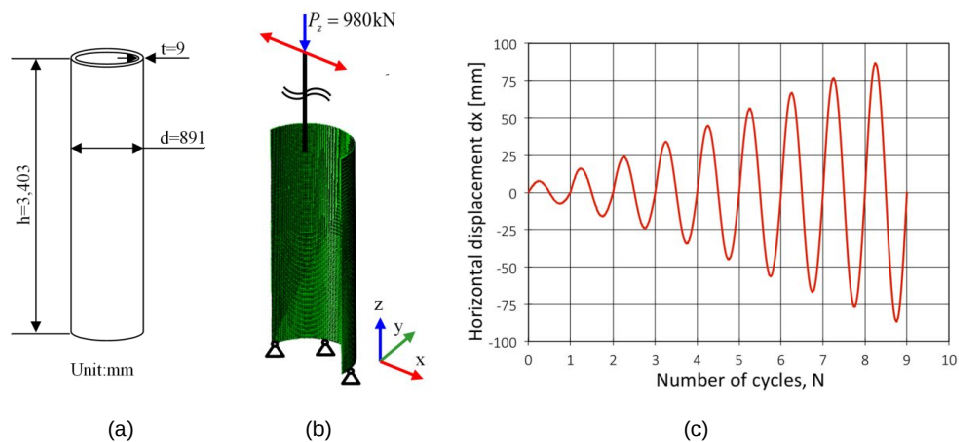


Figure 3. Schematic representation of experimental system and its finite element model.

Table 2. Geometric properties of the specimen.

h (mm)	$2r$ (mm)	t (mm)	A_0 (mm ²)	P_z (kN)	$P_z / (\sigma_y A_0)$	Grade (JIS)
3403	891.0	8.9	24663	880.1	0.121	SS400

the cross section, t is the wall thickness, A_0 is the cross section area and P_z is the constant axial load.

3.2 Description of finite element model and boundary conditions

The finite element analyses were conducted by means of the commercial finite element code Abaqus/Standard ver.6.13. Figure 3b displays the finite element model and boundary conditions, which reproduce the ones experimentally realized by Nishikawa, 1998. In order to reduce the calculation costs, the upper half of the column is modeled by beam elements (B31), the remaining part by brick elements (C3D8) and by a rigid body, which is used to connect the solid with the beam elements at the interface. Six elements have been used for the thickness discretization and the mesh density is higher in the lower part of the column, where local buckling is expected to happen.

The subloading surface model with tangential stress rate effect has been implemented into Abaqus via user subroutine UMAT. In order to identify the material constants, we have used the measurement results under the monotonic and cyclic stress-strain curves (Nakajima, 2000). The results of the calibration, and the SS400 steel material constants, are shown in Figure 2 and Table 1 respectively. On the other hand the material constants for tangential plasticity, reported in Table 1, cannot be obtained by fitting the uniaxial stress-strain curve, therefore they were defined through a trial-and-error approach, comparing the outcome of the FE simulations against the

Black thin line: Experimental result (Nishikawa, 1998)
Colored bold line: Simulation results

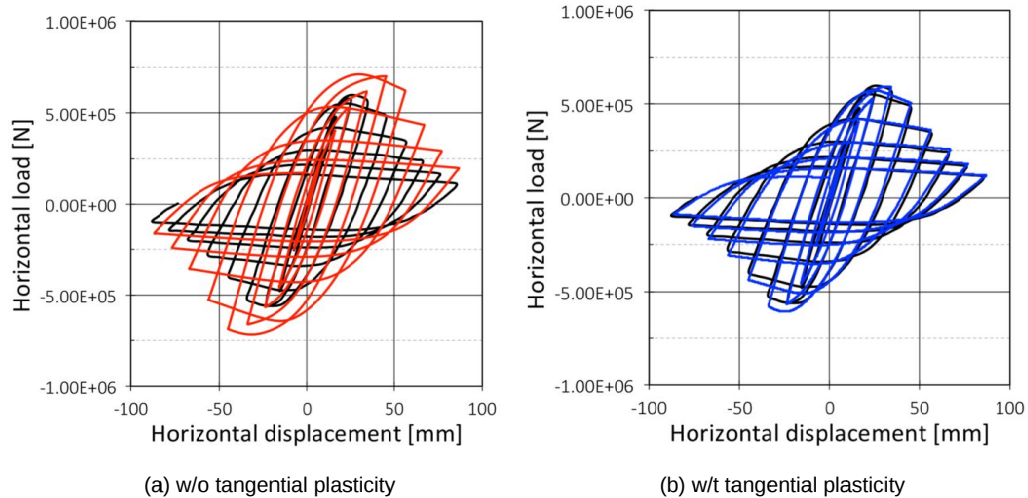


Figure 4. Relationship between horizontal displacement and horizontal load at loading point on top: (a) elastoplasticity without tangential stress rate effect and (b) elastoplasticity with tangential stress rate effect.

experimental results.

As shown in Figure 3b, the base of specimen is constrained, whereas a double boundary condition is applied on top: the first one is a compressive axial load P_z , constant through all the analysis, and the second one is a series of cyclic lateral displacements d_x along the x-direction, whose amplitude is increased during the simulation. The axial load P_z is set at 12.1% of the full-section yield load, which is calculated from the nominal yield stress at cross section area. The horizontal displacements d_x are calculated by:

$$\begin{aligned} d_x &= n d_0 \\ d_0 &= \frac{H_y h^3}{3EI} \\ H_y &= \left(\sigma_y - \frac{P_z}{A_0} \right) \frac{z}{h} \end{aligned} \quad (6)$$

where n is the number of loading cycles, σ_y is the yield stress of material, z is the section modulus of the specimen, I is the second moment of area. The d_0 is the loading point displacement when the lower part of the specimen yields. The history of the lateral displacement dx is given in Figure 3c.

3.3 Results and Discussions

The comparison between the hysteresis responses of the column obtained in the FE simulation,

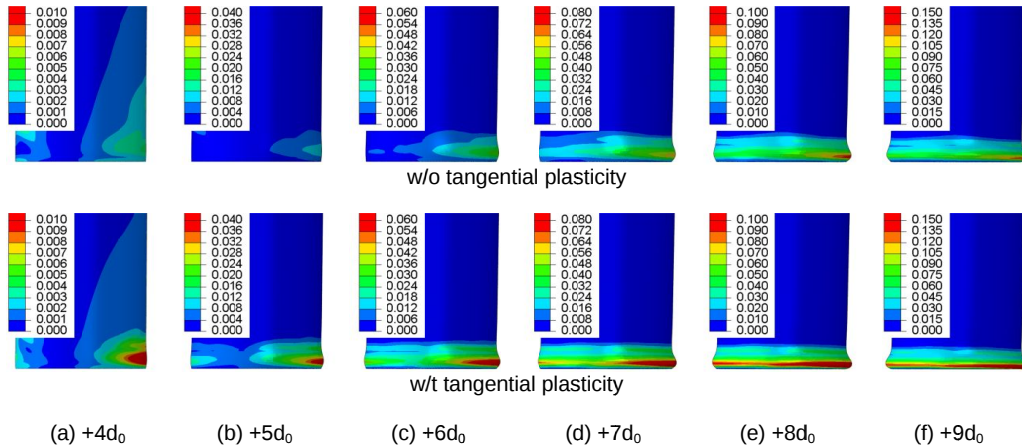


Figure 5. Evolution of the local buckling at the bottom part of the specimen and maximum principal strain distributions: the upper figures are calculated by elastoplasticity without tangential plasticity and the lower figures are calculated by elastoplasticity with tangential plasticity.

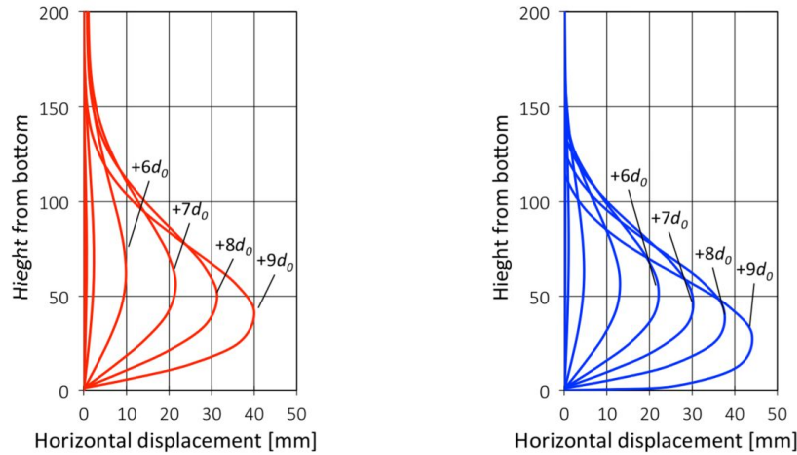


Figure 6. Comparison of the local deformations, from $+1d_0$ to $+9d_0$, at the bottom of column between (a) w/o tangential plasticity and (b) w/t tangential plasticity.

with ($T \neq 0$) and without tangential stress rate effect ($T=0$), and the experimental ones are presented in Figure 4. The curves in Figure 4b are in better agreement with the experiment than the ones obtained with the conventional plasticity ($T=0$). The tangential plasticity model can, in fact, capture both: the ultimate load and the decreasing of the strength for the each of the post cycle peaks during the cycles.

Figure 5 depicts the deformations carried out by using the conventional and the tangential plasticity models in the FE simulations, whereas Figure 6 compares the horizontal profiles of the sample obtained by the nodal displacements around the bottom of the column. Both of them indicate that the localization carried out by means of the tangential plasticity algorithm tends to occur at an earlier stage, moreover it is enhanced with respect to the one resulting from the conventional plasticity model.

4. Various bi-directional loading conditions in the cross section

4.1 Loading paths

In this section seven loading paths (Ucak, 2014) are applied to the model in order to investigate differences in response under bi-directional loading conditions. The schematic representations of these seven loading processes are shown in Figure 7(a)~(g). Paths BS and BA trace a butterfly shape, symmetrically BS and asymmetrically BA, in the X-Y displacement space. Both XU and XP trace a cross shape path, however the number of changes of the loading direction in each cycle is different. Paths DM and SQ have the same shape, but the direction at the maximum displacement in each cycle is different. Case CR traces a simple circular path.

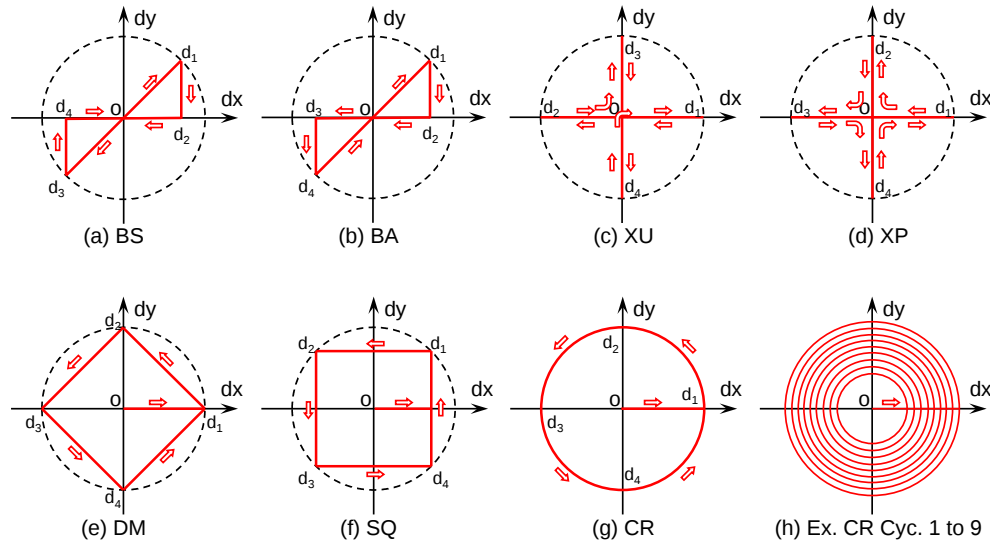
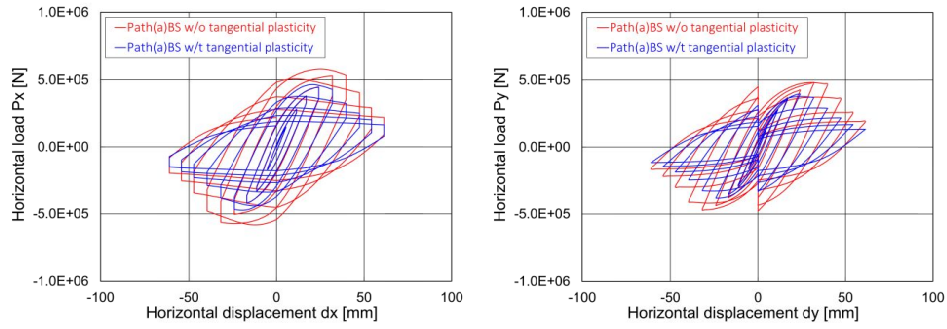


Figure 7. Schematic representation of the loading conditions.

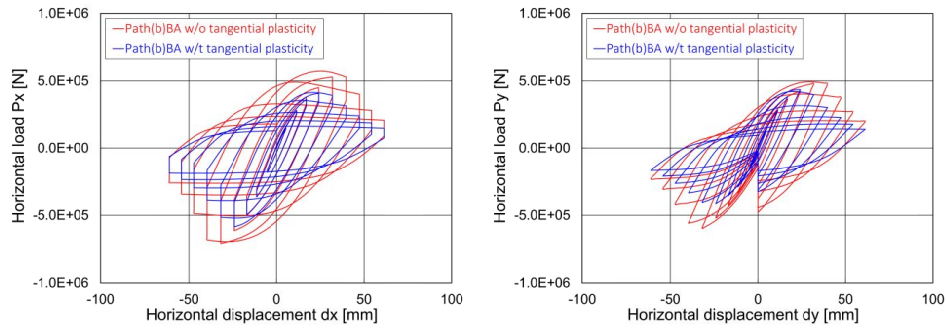
Figure 7(h) shows CR displacement history from cycle one to nine as an example. All the paths consist of nine cycles and their displacement amplitudes are increased through the cycles.

4.2 Numerical results and comparisons

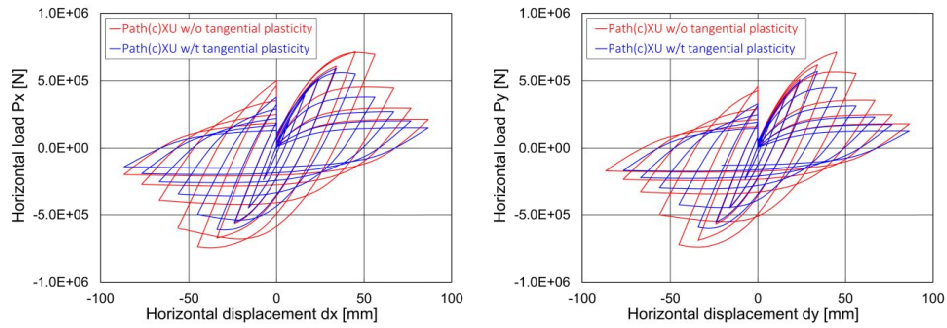
Numerical calculations have been carried out using the same material constants for both the return mapping and the tangential plasticity algorithm. Figure 8, 9 and 10 show the hysteresis responses at the top of the specimen through the cycles. As for the case shown in Figure 4, the elastoplasticity model with tangential plasticity better estimates the minimum ultimate loads compared to the ones obtained with the conventional plasticity ($T=0$). In addition, the numbers of cycles necessary to reach the critical conditions are different in the two models: the results obtained by considering the tangential plasticity tend to achieve the ultimate load at an earlier stage. This means that, under operational loading conditions, the predictions simulated by the conventional elastoplasticity models may overestimate the structural resistance, with more or less serious implications for the safety factors design.



(a) BS

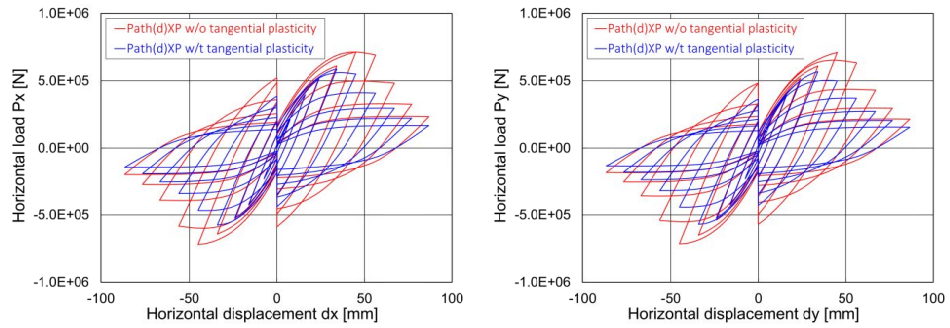


(b) BA

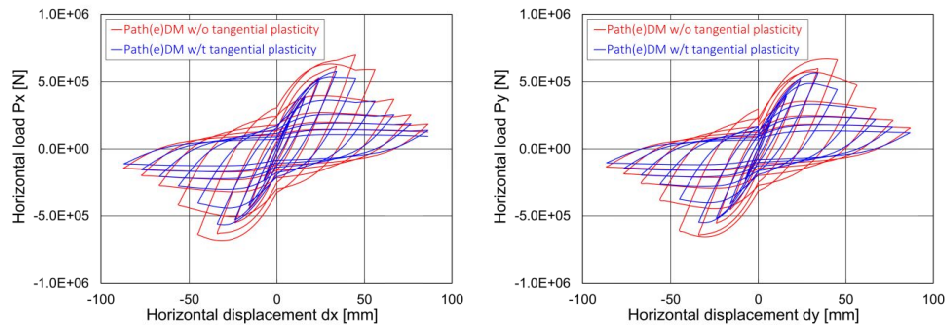


(c) XU

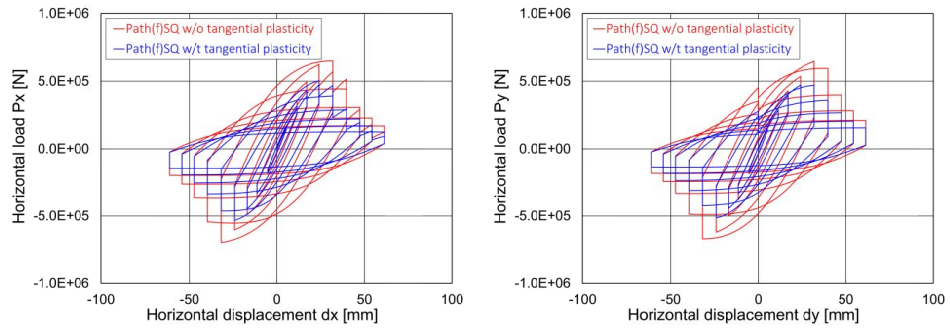
Figure 8. Comparisons of the hysteresis loops under different loading conditions (from (a) BS to (c) XU). Left figures show the responses in the x-direction, right figures show the ones along the y-direction.



(d) XP



(e) DM



(f) SQ

Figure 9. Comparisons of the hysteresis loops under different loading conditions (from (d) XU to (f) SQ). Left figures show the responses in the x-direction, right figures show the ones along in the y-direction.

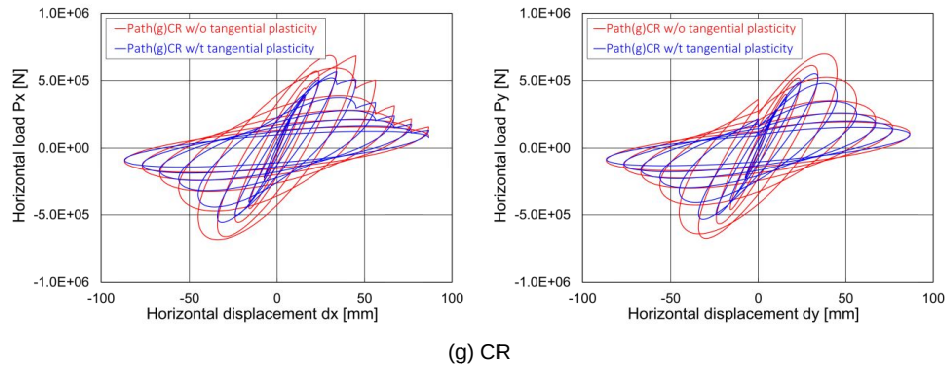


Figure 10. Comparisons of the hysteresis loops under different loading condition (g) CR. Left figure shows the responses in the x-direction, right figure shows the responses along the y-direction.

5. Conclusions

The paper presented the numerical results based on the extended subloading surface model incorporating the tangential plasticity effect in order to capture the material response under a non-proportional loading condition. The algorithm has been used to simulate the behavior of a thin wall bridge pier subjected to biaxial cyclic loadings. As a result of the comparison between the conventional plasticity model and the tangential plasticity model against experimental data, it can be concluded that the latter has higher predictive capability. The conventional plasticity, in fact, overestimated the ultimate load, whereas the tangential plasticity has been proved to be able to catch the experimental peak load. Moreover a decrease of the peak loads and an anticipation of the ultimate loads at earlier stage have been observed whenever the tangential relaxation has been taken into account for all of the seven bi-directional loading paths. This indicates the importance of considering the tangential plasticity effect in order to achieve more accurate and reliable predictions under operational loading conditions. The further validation of material constants for tangential plasticity is left as a future work.

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