Non-linear elastic models for composite materials

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Abstract*: Modern composite materials exhibit nonlinear behavior even under the conditions of elastic deformation. Usually shear properties have essentially more non-linearity than the tensile longitudinal or transverse ones almost for all reinforced plastics. For dry textile composites, it is especially important to take into account the shear nonlinearity for the prediction of the deformation characteristics, shape and structure of composite structural members during the draping and the performance of a layup process. Two approaches to take into account composite elastic nonlinearity effect are developed. First one can be used to model different elastic module behavior in the dependence on the type of external forces. Second one is proposed to take into account the nonlinear dependency of shear stress - strain curves. It is demonstrated that different dependencies of shear properties can be used. The elastic potentials and corresponding constitutive equations for both approaches are proposed. Numerical implementation for non-linear shear model has been performed. One of the possible methods for textile preform draping process simulation is considered.*

Keywords*: Composite materials, Nonlinear shear diagram, Different module behavior, Anisotropic nonlinear elastic model, Micro-heterogeneous materials, Stress state dependent properties, Constitutive Model, Elasticity.*

1. Introduction

Composite materials play a significant role at present time for different structural industries. Many approaches to the stress analysis of structural elements made of this type of material exist and they are well developed. Nevertheless for the characterization of some effects that composite materials exhibit, still there are not exist standard and well working methods to take them into account during engineering simulation. This research work is dedicated to one of the set of the examples of noted effects, which are the nonlinear elastic material response to the action of external forces. It is very common for fiber reinforced materials to have different modulus in the dependence on the type of loadings; also it is usual for this type of materials to display nonlinear diagram in plane shear experiments. Common approach is based on the use of linear elastic material model with averaged properties. There are no generally accepted methods to model the noted effects, and the problem of developing of appropriate material model is still competitive one. This work proposes two possible approaches: the first one is for simulating of stiffness property dependence on type of loading and the second one is for taking into account nonlinearity of plane shear diagram. Also by means of second approach, the example of possible application is demonstrated on the base of the problem of a dry preform layup process.

2. Different modulus material model

According to the work [1], the constitutive relations for anisotropic material of stress state dependant properties could be formulated as elastic equations with potential written in following form:

$$\Phi = \frac{1}{2} A_{ijkl}(\xi)\sigma_{ij}\sigma_{kl}, \tag{1}$$

where $= \sigma/\sigma_0$, $\sigma = \sigma_{ii}/3$, $\sigma_0 = \sqrt{3/2 S_{ij}S_{ij}}$, $S_{ij} = \sigma_{ij} - \sigma\delta_{ij}$.

Parameter ξ, used for representing the dependency on stress state, could be found in the literature under the name of stress triaxiality. The main idea is that this parameter can recognize different types of loadings and in dependence on its value, the different sets of coefficients could be purposed for material stiffness.

In case of plane stress conditions, the constitutive equations with proposed potential could be reduced to the following ones:

$$\begin{aligned}
\varepsilon_x &= a_{11}(\xi)\sigma_x + a_{12}(\xi)\sigma_y + \left[\left(\frac{1}{3\xi} + \frac{3}{2}\xi\right)\sigma - \frac{3}{2}\xi\sigma_x\right]\Phi_1\sigma_0^{-2},\\
\varepsilon_y &= a_{12}(\xi)\sigma_x + a_{22}(\xi)\sigma_y + \left[\left(\frac{1}{3\xi} + \frac{3}{2}\xi\right)\sigma - \frac{3}{2}\xi\sigma_y\right]\Phi_1\sigma_0^{-2},\\
\gamma_{xy} &= \left[a_{66}(\xi) - \frac{3}{2}\xi\Phi_1\sigma_0^{-2}\right]\tau_{xy},\\
\Phi_1 &= \frac{1}{2}[a'_{11}(\xi)\sigma_x^2 + a'_{22}(\xi)\sigma_y^2 + 2a'_{12}(\xi)\sigma_x\sigma_y + a'_{66}(\xi)\tau_{xy}^2].
\end{aligned} \tag{2}$$

The obtained equations have a difficult configuration but from engineering applications point of view, this form is practically convenient. The coefficients $a_{11}(\xi), a_{22}(\xi), a_{12}(\xi)$ and $a_{66}(\xi)$ could be presented as a piecewise linear functions of ξ; and the data used for determination of these functions should be obtained experimentally. Using the approximation of real experimental curves by the straight lines gives some advantages for this model. In this case under conditions of uniaxial experiments the parameter ξ equals to 0.33 for tension and -0.33 for compression, and this gives, as a sequence, that the second parts of described equations are equal to zero and one can obtain values of coefficients from simple linear equations:

for tension:

$$\varepsilon_x = a_{11}(0.33)\sigma_x + a_{12}(0.33)\sigma_y,$$

$$\varepsilon_y = a_{12}(0.33)\sigma_x + a_{22}(0.33)\sigma_y,$$

for compression:

$$\varepsilon_x = a_{11}(-0.33)\sigma_x + a_{12}(-0.33)\sigma_y,$$

$$\varepsilon_y = a_{12}(-0.33)\sigma_x + a_{22}(-0.33)\sigma_y.$$

Second gain of the considering relations is their simplicity of obtaining values of coefficients for biaxial tests in the case of regular orientation of specimens, where forces applied in directions of fiber orientation and transverse one. In this case if loading time parameter is referred as *t* the proposed equations (2) could be reduced to the following form:

$$\varepsilon_x = (a_{11}(\xi) + a_{12}(\xi)Const_1 + Const_2)t,$$

$$\varepsilon_y = (a_{12}(\xi) + a_{22}(\xi)Const_3 + Const_4)t,$$

where $Const_i$ are known from ratio of applied loads values.

This form for determination of a set of coefficients $a_{11}(\xi), a_{22}(\xi)$ and $a_{12}(\xi)$ is also linear one. Considering the different set of the applied forces, that keeps the same value of ξ, but changes values of σ_x and σ_y, it is possible to prove by means of Betti's theorem that the system of four equations with three unknown coefficients $a_{11}(\xi), a_{22}(\xi)$ and$a_{12}(\xi)$ is possible to resolve in unambiguous way.

In case of plane shear test the connection of shear stress and strain could be reduced to the following form:

$$\gamma_{xy} = a_{66}(0)\tau_{xy},$$

which could be easily resolved.

Thus, the proposed relations for anisotropic material under conditions of plane stress can guarantee a satisfactory to any experimental test results for uniaxial, biaxial and plane shear experiments in the case of regular orientation of specimens. All experiments with specimen cut outs that have orientation with nonzero angle to fiber direction could be considered as verification for proposed model.

3. Nonlinear shear material model

To formulate the potential and constitutive relations for materials displaying nonlinear response under shear conditions, the approach can be used, which is similar to one presented in the previous section. To describe this nonlinearity the potential can be written in the following form:

$$U = \frac{1}{2}E_{ijkl}(q)\varepsilon_{ij}\varepsilon_{kl}, \tag{3}$$

where $q = D_{ij}\varepsilon_{ij}$. Coefficients D_{ij} have the following representation in coordinate system coincident with the orientation of anisotropy axis:

$$D_{ij} = \begin{bmatrix} 0 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In regular coordinate system the parameter $q = \varepsilon_{12}$, which characterizes the degree of shear deformation. The tensor D_{ij} could be considered as a structural parameter of material anisotropy. Strain components were chosen in equation (3) instead of stress ones to be closer to classic anisotropic linear elastic model.

Using the potential given by equation (3), the constitutive equations could be written as:

$$\sigma_{ij} = \frac{\partial U}{\partial \varepsilon_{ij}} = \frac{\partial E_{mnkl}(q)\varepsilon_{mn}\varepsilon_{kl}}{\partial \varepsilon_{ij}} = \frac{\partial E_{mnkl}(q)}{\partial \varepsilon_{ij}}\varepsilon_{mn}\varepsilon_{kl} + E_{ijkl}(q)\varepsilon_{kl} \,. \tag{4}$$

Considering plane stress conditions, and putting the dependency on the parameter q only into shear modulus G, it is possible to show that the first part of the equation (4) could be reduced to the next form:

$$\frac{\partial E_{mnkl}(q)}{\partial \varepsilon_{12}}\varepsilon_{mn}\varepsilon_{kl} = \frac{dG(q)}{dq}\frac{dq}{d\varepsilon_{12}}\varepsilon_{12}\varepsilon_{12} = G'\varepsilon_{12}^2 \,, \tag{5}$$

where prime denotes the derivative with respect to parameter q.

Thus the equation (4) could be written for σ_{12} component as:

$$\sigma_{12} = (G'\varepsilon_{12} + G)\varepsilon_{12} \,. \tag{6}$$

One can assume that the shear modulus function $G(q)$ is an arbitrary polynomial:

$G(q) = \sum_n C_n q^n$.

Then equation (6) could be reformulated as:

$$\sigma_{12} = \left[\left(\sum_n C_n n q^{n-1}\right) q + \sum_n C_n q^n\right]\varepsilon_{12} = \left[\sum_n (C_n n + 1) q^n\right]\varepsilon_{12} \,.$$

Using the substitution of $B_n = C_n n + 1$, it is possible to obtain the relation:

$$\sigma_{12} = \left[\sum_n B_n q^n\right]\varepsilon_{12}. \tag{7}$$

Eventually the equation (7) shows that the dependency between shear stress and shear strain could be arbitrary function that could be approximated by means of polynomial, and what is practically important it could be a piecewise linear function.

Thus the constitutive relation with proposed potential for plane stress condition can be written in the following form:

$\varepsilon_x = \frac{\sigma_x}{E_1} - \nu\frac{\sigma_y}{E_1}$,

$\varepsilon_y = -\nu\frac{\sigma_y}{E_1} + \frac{\sigma_y}{E_2}$,

$\gamma_{xy} = \frac{\tau_{xy}}{G(q)}$,

where $G(q)$ characterizes the nonlinearity of shear stress – shear strain diagram.

Therefore the proposed constitutive equation can describe any plane shear test data in the case of regular orientation of a specimen. Similar to the previous material model, all experiments with specimens cutouts with orientation at some angle to the fiber orientation should be considered as a verification ones. The form of the developed material model is very close to the classical laminate theory that makes this approach mostly convenient for practical engineering implementation.

4. Abaqus implementation

Abaqus software has perfect tools for implementation of special user materials UMAT, VUMAT, USDFLD and VUSDFLD procedures.

First model with stress state dependency is formulated in terms of strains as functions of stress, therefore these equations should be resolved for stress components, that seems problematic to do in analytical form, and thus it could be done numerically by any type of Newton method. Jacoby matrix from strain to stress components could be found analytically, and then inverse matrix should be obtained.

Second model could be realized by means of USDFLD or VUSDFLD interfaces, because it is close to classical laminate theory, which is already built into Abaqus. Thus the simple program which realizes user field with parameter q should be written and corresponded variation of shear modulus G is supposed to be used.

5. Textile composite draping modeling

The development of infusion type manufacturing of composite parts raised an interest in modeling of dry preform layup process. Dry preform has a significant nonlinearity under conditions of elastic deformations. Practical experience shows that the most critical property in material draping modeling is the shear one. The shear modulus of dry preform is usually small in comparison with tensile one, but shear modulus increases with increase of shear deformation and reaches considerable values at approximately of 30^0 of deformed angle of dry material. Under these high values of shear modulus, the preform wrinkles can take place. Figure 1 shows typical diagrams for considered materials.

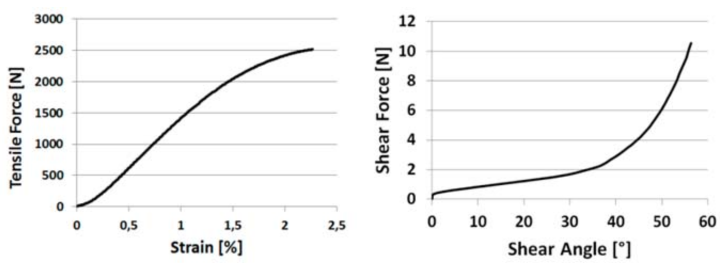

Figure 1. Typical diagram of dry preform [2].

It is possible to have qualitative study of shear nonlinearity by means of meso-level preform model. Figure 2 shows common view of initial and deformed state of developed model. Figure 3 shows the obtained diagram of the dependence of shear forces versus value of fiber's angular deformation.

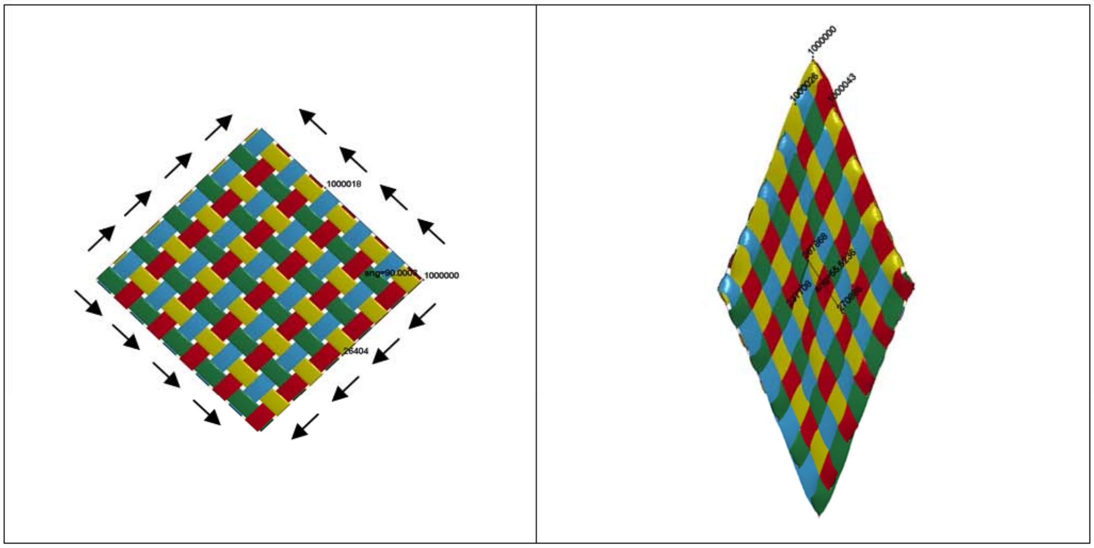

Figure 2. Meso-level preform model for plane shear test.

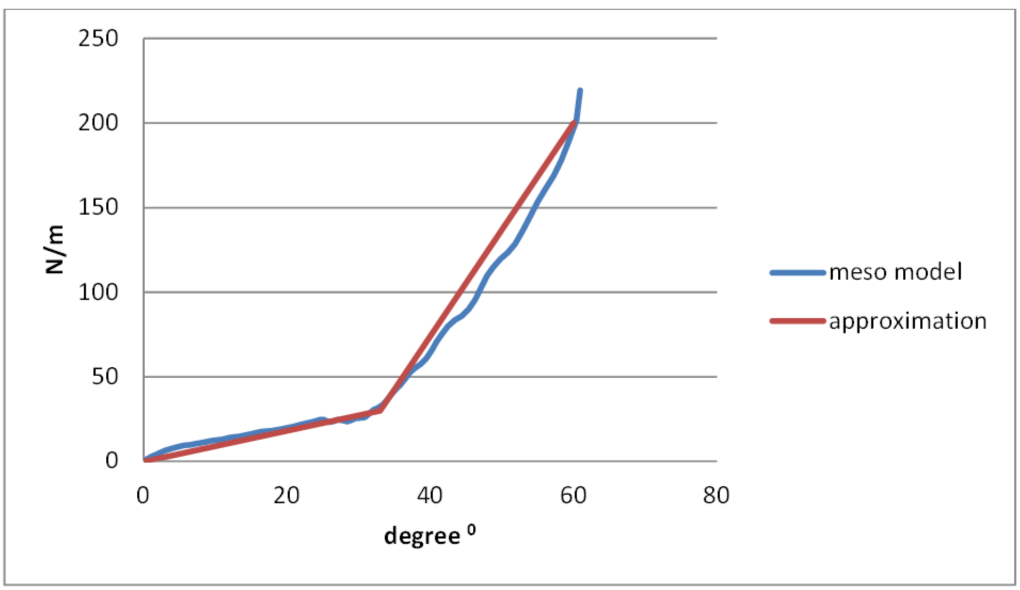

Figure 3. Deformation diagram from meso-level preform model for plane shear test.

Using the obtained shear diagram, it is possible to set a corresponding dependence for the shear modulus G as a function of parameter q. Putting this data to Abaqus by means of written VUSDFLD subroutine, it is possible to model layup process for composite manufacturing. Figure 4 shows the statement of example problem of draping simulation.

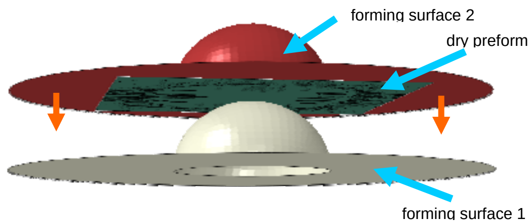

Figure 4. Example problem statement for draping simulation

Figure 5 shows the different stages of preform during layup process.

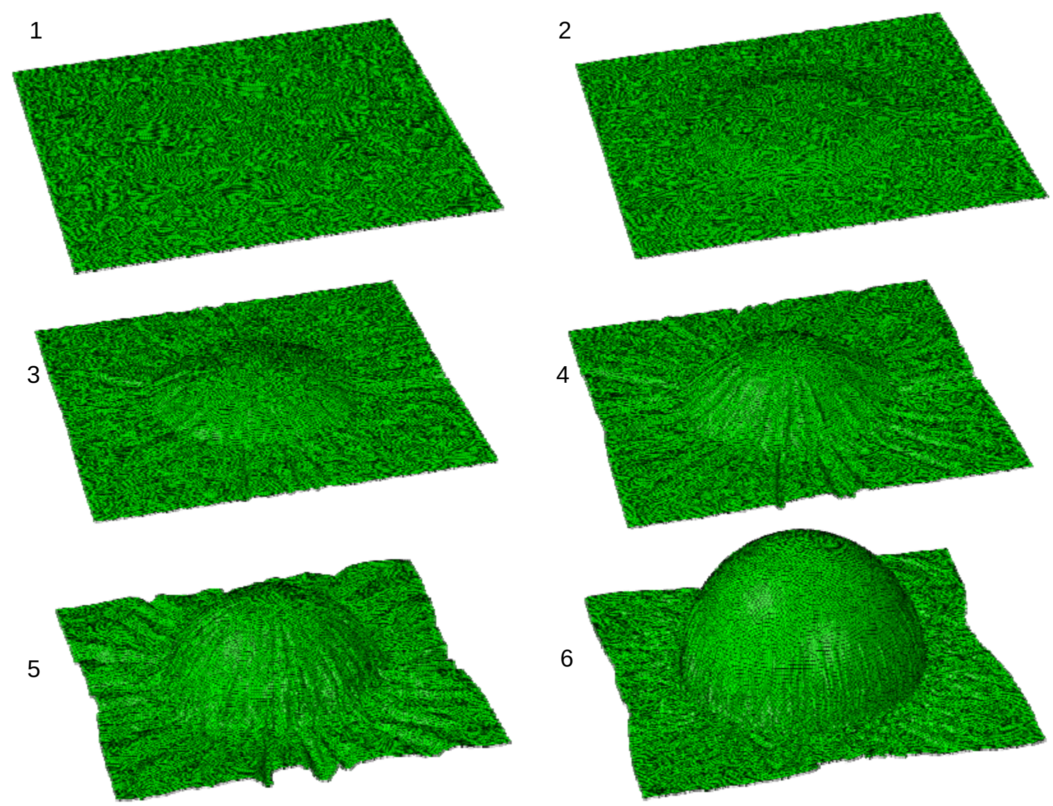

Figure 5. Draping simulation result

It should be noted that textile composites in general have a low bending stiffness. This effect can be captured by means of increasing the stiffness module with corresponding decreasing of thickness, this can keep membrane properties with capability to reduce bending stiffness to any required level.

Considering the exact way of realization of proposed method, one can say that the choice of Abaqus system for demonstrated example was mostly dictated by its good functionality and friendly usability in combination with perfectly organized manuals [3]. Also the USDFLD and VUSDFLD procedures, which let to researcher checks his ideas in simple way, without writing and developing difficult program codes, is a valuable tool for any scientist dealing with solid mechanics. The modeling time for showed simulation of material draping was less than one hour on usual computer using four cores. This gives a real opportunity for technology specialists to check different strategies for dry preform placement, and provide a comparison analysis for perfect option.

6. Conclusions

The approaches to take into account elastic nonlinearity of composite materials are presented. It is shown that the first method can guarantee satisfaction of any uniaxial and biaxial test results in the case of regular orientation of specimens. Second material model can capture any shear nonlinearity of properties of composite materials, and can be easily implemented into modern stress analysis software. The example of the use of nonlinear shear model based on special subroutine for Abaqus software is demonstrated. The application of the method based on the proposed model to the draping simulation of textile composites is demonstrated. Future task is the implementation of both models into universal one with further numerical realization.

7. References

1. Lomakin, E.V., “Constitutive Models of Mechanical Behavior of Media with Stress State Dependent Material Properties”, Mechanics of Generalized Continua. Advanced Structured Materials, Volume 7, 2011, pp 339-350
2. Döbrich O., Gereke T., Cherif Ch., “Drape Simulation: Textile Material Model for Correct Property Reproduction to Improve the Preform Development Process of Fiber-Reinforced Structures.” 12th International LS-DYNA Conference 2012.
3. Abaqus Users Manual, Version 6.12-3, Dassault Systémes Simulia Corp., Providence, RI.

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