

Simulation of Hydraulic Fracturing of Unconsolidated Sands using Fully Coupled Poro-Elastoplastic Models

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Abstract: *Many deep water reservoir rocks are soft and consist of unconsolidated or weakly cemented sands. Fractures may have to be created in these soft reservoirs for a variety of purposes, such as the injection of fluids to maintain reservoir pressure, constructing Frac-Pack sand control mechanisms, etc. It is now recognized that fracturing in unconsolidated sands involves complex failure mechanisms and is very different from fracturing in hard rocks. The simulation of these complex failure mechanisms can only be achieved with suitable models that account for the coupling of inelastic soft rock deformation and fluid pressure changes. In this paper, we discuss the implementation in Abaqus of a poro-elastoplastic constitutive model for soft rock and its application to some practical problems. First we present a critical-state based constitutive model for soft rock, which was specifically designed to describe the evolution of sand and weakly cemented rock from the intact state to the fully damaged state. It includes pressure dependency of the elastic response as well as the yield surface. A softening regularization scheme is also introduced to minimize mesh sensitivity due to strain localization effects. The model is implemented in Abaqus/Standard and Abaqus/Explicit using an implicit backward-Euler integration scheme. Numerical examples are presented in the second part of the paper in the context of applications of practical interest to the Oil and Gas industry. First, we demonstrate the model's ability to predict shear band formation and localization in a borehole stability analysis. Second, we investigate the evolution of a fracture due to injection of fluids in unconsolidated sand.*

Keywords: *Plasticity, Geomechanics, Shear bands*

1. Introduction

Planning Oil & Gas exploration and development operations often requires predictive modeling of the behaviors of rocks under loading, such as the loads applied or induced during drilling, fracturing or production. Finite element methods, or other numerical methods, are often used with rock mechanical constitutive models to simulate the behaviors of rocks so that critical information can be obtained and analyzed to ensure operational success. In particular, weakly cemented rocks

exhibit complex deformation behaviors, such as localized deformation associated with borehole breakout and formation liquefaction during injection. Simulating such complex behaviors necessitates constitutive models that are able to reproduce a physically realistic evolution of the material from the intact state to the fully damaged state. In this paper, a critical-state based soft rock plasticity model that follows the work of Crook et al. (2006) is discussed. The model is referred to as the soft rock plasticity model in this document.

The paper is organized as follows: the governing equations of the soft rock plasticity model are summarized in Section 2. The model is first used to analyze shear band localization in a borehole using Abaqus/Explicit in Section 3. Next, in Section 4, the model is used to study hydraulic fracturing of unconsolidated sands using Abaqus/Standard. The paper ends with some concluding remarks in Section 5.

2. Constitutive model for soft rocks

The governing equations of the soft rock plasticity model are described in this section. This poro-elastoplastic constitutive model is based on the theory of critical state and assumes additive decomposition of strain rate ($\dot{\boldsymbol{\varepsilon}}$) into elastic ($\dot{\boldsymbol{\varepsilon}}^{el}$) and plastic parts ($\dot{\boldsymbol{\varepsilon}}^{pl}$).

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{el} + \dot{\boldsymbol{\varepsilon}}^{pl} \quad (1)$$

The elastic response is described by,

$$\boldsymbol{\sigma} = \mathbf{D}^{el} : \boldsymbol{\varepsilon}^{el} \quad (2)$$

where $\boldsymbol{\sigma}$ is the effective stress tensor, $\boldsymbol{\varepsilon}^{el}$ is the elastic strain tensor and $\mathbf{D}^{el}$ is the fourth-order elasticity tensor defined by Young's modulus (E) and Poisson's ratio (ν). Experimental data on soft rocks indicate that E and ν are dependent on effective mean stresses. The following empirical relations are used to define the evolution of E and ν with effective pressure,

$$E = \begin{cases} E_{ref} \left[\frac{p+p_o}{p_{ref}+p_o} \right]^n & p > 0 \\ f E_{ref} & p \leq 0 \end{cases} \quad (3)$$

$$f = \left[\frac{p_o}{p_{ref}+p_o} \right]^n \quad (4)$$

$$\nu = \begin{cases} \nu_{max} + (\nu_{min} - \nu_{max})(1 - e^{-mp}) & p > 0 \\ \nu_{max} & p \leq 0 \end{cases} \quad (5)$$

where E_{ref} is the reference Young's Modulus at pressure p_{ref} ; $p = -(\sigma_{11} + \sigma_{22} + \sigma_{33})/3$ is the effective mean stress; f is used to define the elastic modulus for tensile effective pressure as a function of E_{ref} ; p_o , n and m are material constants; ν_{max} is the value of Poisson's ratio at $p = 0$, and ν_{min} is the value of Poisson's ratio at $p = \infty$. The above description of elastic constants is shown schematically in Figure 1.

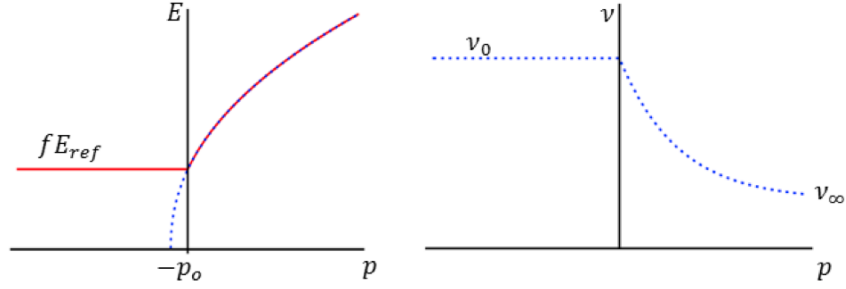


Figure 1. Variation of elastic constants with mean effective stress.

The plastic response of the material follows the critical state based constitutive model proposed by Crook et al. (2006), modified to introduce an eccentricity parameter (e_0). The yield function depends on the three stress invariants and is defined in Equation (6). It intersects the hydrostatic axis in both tension and compression, as shown in Figure 2.

$$\Phi(\boldsymbol{\sigma}, \varepsilon_v^{pl}) = \sqrt{(e_0 p_c^0 \tan \beta)^2 + (g(\boldsymbol{\sigma})q)^2} - (p - p_t) \tan \beta \left(\frac{p - p_c}{p_t - p_c} \right)^{\frac{1}{n_y}} - e_0 p_c^0 \tan \beta = 0 \quad (6)$$

where β is a material parameter related to the friction angle at low confining pressures, $q = \sqrt{\frac{3}{2} \mathbf{S} : \mathbf{S}}$ is the effective Mises stress, $\mathbf{S} = \boldsymbol{\sigma} + p\mathbf{I}$ is the deviatoric effective stress, p_c is the intercept of the yield surface with the positive p -axis in the $q - p$ plane, p_t is the intercept of the yield surface with the negative p -axis in the $q - p$ plane, p_c^0 is the initial value of p_c , n_y is a material constant that controls the shape of the yield function in the $q - p$ plane, and $g(\boldsymbol{\sigma})$ captures the observed transition of the yield surface in the Π -plane from rounded-triangular shape to circular shape with an increase in pressure. $g(\boldsymbol{\sigma})$ is defined as

$$g(\boldsymbol{\sigma}) = \left[\frac{1}{1-f(p)} \left(1 + f(p) \frac{r^3}{q^3} \right) \right]^\alpha \quad (7)$$

$$r = \sqrt[3]{\frac{9}{2} \mathbf{S} : \mathbf{S}} \quad (8)$$

$$f(p) = f_0 \exp \left(f_1 p \frac{p_c^0}{p_c} \right) \quad (9)$$

where r is the third stress invariant, and f_0, f_1 and α are material constants.

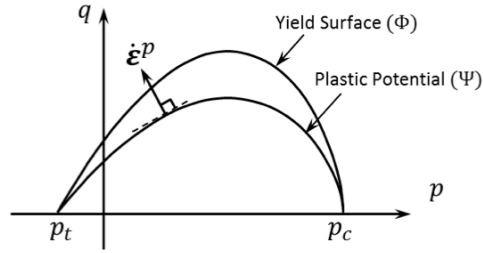


Figure 2. Yield Surface and plastic potential of the soft rock plasticity model in the $q - p$ plane.

The function $f(p)$ should satisfy the following constraints to ensure convexity of the yield surface (Bigoni and Piccolroaz, 2004).

$$\begin{aligned} 0 \leq f(p) &\leq \frac{1}{9\alpha-1} & \alpha &\geq \frac{3}{11} \\ 0 \leq f(p) &\leq \left[\frac{4-13\alpha}{4(\alpha-1)(9\alpha^2-1)} \right]^{1/2} & 0 < \alpha < \frac{3}{11} \end{aligned} \quad (10)$$

The evolution of the yield surface is governed by the plastic volumetric strain, ε_v^{pl} , via hardening/softening relations in the form

$$p_c = p_c(\varepsilon_v^{pl}) \quad (11)$$

$$p_t = p_t(\varepsilon_v^{pl}) \quad (12)$$

Typical curves of $p_c(\varepsilon_v^{pl})$ and $p_t(\varepsilon_v^{pl})$ are shown in Figure 3. The initial size of the yield surface ($p_c|_0$ and $p_t|_0$) is determined from the initial volumetric plastic strain $\varepsilon_v^{pl}|_0$.

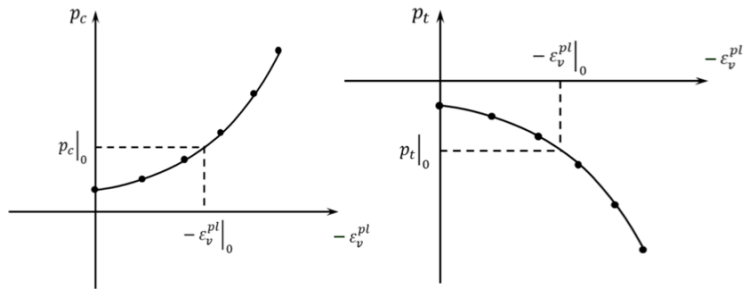


Figure 3. Typical piecewise linear hardening/softening curves of the soft rock model.

2.1. Flow rule

The evolution of the plastic flow is governed by a non-associated flow rule so that the plastic strain rate is defined as

$$\dot{\boldsymbol{\varepsilon}}^{pl} = \dot{\lambda} \frac{\partial \Psi}{\partial \boldsymbol{\sigma}} \quad (13)$$

where $\dot{\lambda} \geq 0$ is the plastic multiplier and Ψ is the plastic potential defined as (see Figure 2)

$$\Psi(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}_v^{pl}) = \sqrt{(e_0 p_c^0 \tan \psi)^2 + (g(\boldsymbol{\sigma})q)^2} - (p - p_t) \tan \psi \left(\frac{p - p_c}{p_t - p_c} \right)^{\frac{1}{n_y}} - e_0 p_c^0 \tan \psi \quad (14)$$

where ψ is a material parameter related to the dilation angle at low confining pressures. It should be noted that adding eccentricity to the model ensures uniqueness of plastic flow at $p = p_c$ and $p = p_t$, as shown in Figure 4.

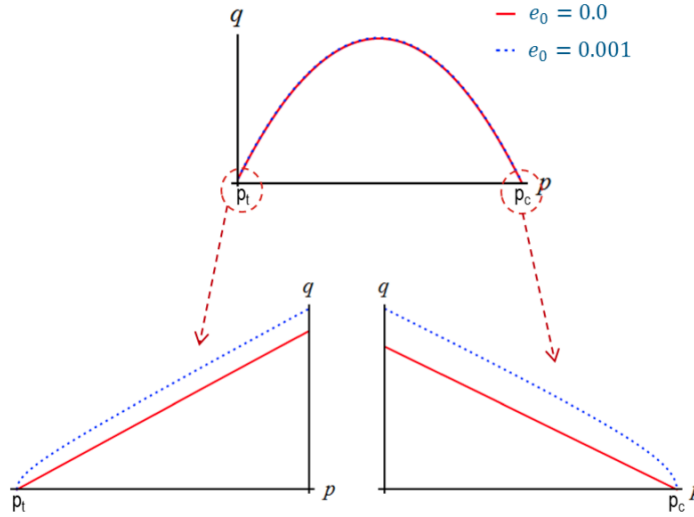


Figure 4. Effect of eccentricity on the shape of the plastic potential in the q - p plane.

2.2. Regularization of the softening process

Granular materials often exhibit strain localization with increasing plastic deformation. Post-failure solutions from conventional finite element methods can be strongly mesh-dependent. In order to mitigate mesh-dependence of the solutions, a regularization method is often used to introduce a micro-structural length-scale into the constitutive formulation. Let $l_c^{(m)}$ denote the characteristic width of a shear band or a crack band, $l_c^{(e)}$ the characteristic length of the element,

and $(\varepsilon^p)^{(e)}$ the inelastic strain for the element; then the inelastic strain in the localization band, $(\varepsilon^p)^{(m)}$, is defined as

$$(\varepsilon^p)^{(m)} = (\varepsilon^p)^{(e)} \min \left(\left(\frac{l_c^{(e)}}{l_c^{(m)}} \right)^{n_r}, f_{max} \right) \quad (15)$$

where n_r is a material parameter and f_{max} is a positive number used for bounding the magnitude of regularization. The strain regularization method is strictly only valid when the characteristic length of the element is greater than the width of the localization band, i.e., $l_c^{(e)} \geq l_c^{(m)}$.

2.3. Implementation in Abaqus

The poro-elastoplastic constitutive model, described in the previous subsections, was implemented in Abaqus/Standard and Abaqus/Explicit using an implicit backward-Euler integration scheme. This implementation was used for the numerical analyses described next.

3. Wellbore stability-thick walled cylinder test

Borehole collapse can lead to significant economic consequences, and therefore, its prediction and prevention becomes important for the success of oil and gas projects. The behavior of rock along the periphery of a borehole depends on its elastic, plastic, as well as failure properties. The failure of rocks is often associated with the formation of shear bands and the localization of strains along those bands. Reliable prediction of the formation of such shear bands is therefore important, along with an accurate computation of the stress distribution in the rock strata in the neighborhood of the borehole.

The deformation of a borehole can be studied by performing computational tests on thick cylinders subjected to external pressure. Such thick cylinder tests were performed using Abaqus/Explicit and are discussed in this section. The setup of the model is based on the numerical experiments described in Crook et al. (2003).

3.1. Problem setup

A thick walled cylinder with an internal diameter (ID) of 10 mm and outer diameter (OD) of 30 mm was considered. Sketch of a typical model is shown in Figure 5. The tangential displacement was constrained at four points on the outer boundary of the cylinder. These points are indicated by blue dots in Figure 5. Pressure was applied on the outer boundary of the cylinder and was ramped up linearly from 0 MPa to 3.52 MPa in 0.044 s.

Plane strain elements, CPE4R and CPE3, were used for discretizing the model. The effect of element size on the results was also studied. Three different meshes with 320, 480 and 640 elements on the inner circle of the annulus were used. The soft rock plasticity model with constant linear elastic properties was used in all the analyses. The material properties used in this example are listed in Table 1. Variable mass scaling was used to speed up the solution time of the Abaqus/Explicit analysis.

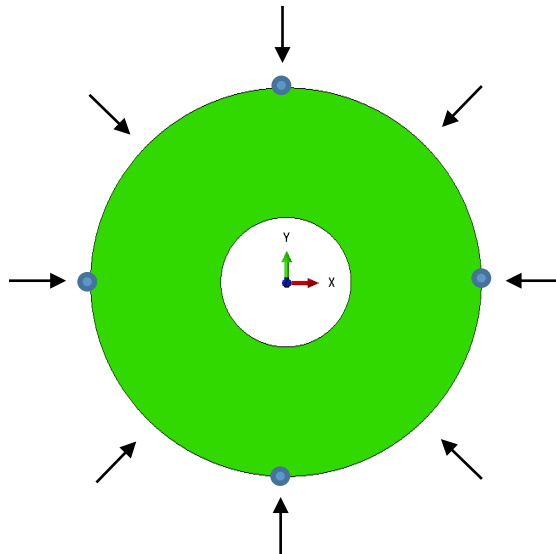


Figure 5. Sketch of the model used for testing.

Table 1. Material properties used in the wellbore stability problem.

Elastic properties

E (Pa)	ν
7×10^{10}	0.3

Plastic properties

β	ψ	n_v	f_0	f_1 (Pa $^{-1}$)	α	e_0
50°	48°	1.3	0.06	0.007	0.	0.001

Softening regularization properties

$l_c^{(m)}$ (m)	n_r	$\hat{f}_{max}$
5×10^{-5}	0.6	1.0

Hardening in hydrostatic compression

p_c (Pa)	ε_v^{pl}
2.5×10^6	0.0
2.52×10^7	0.002
1.0×10^8	0.003

Hardening in hydrostatic tension

p_t (Pa)	ε_v^{pl}
-2.5×10^4	0.
-2.7×10^4	0.002
-3.0×10^4	0.003

3.2. Results and discussion

As the analyses progress, the increasing pressure eventually leads to the formation of shear bands, as shown in Figure 6. The criterion for the onset of instability is assumed to be based on the increase in kinetic energy. For all the meshes studied, the kinetic energy increases substantially at $t=0.026$ s, which corresponds to an external load of 2.08 MPa. It was concluded from this observation that the external load at which the instability initiates is independent of the mesh. However, some variation of the localization profile with element size was observed.

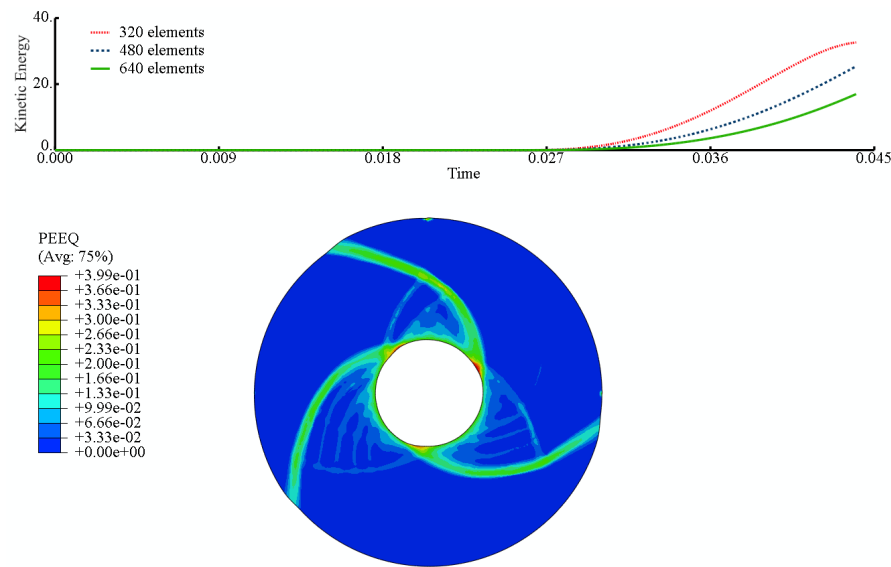


Figure 6. Localization in thick walled cylinder test; equivalent plastic strain (PEEQ) profile is shown for the mesh with 640 elements on the inner circle at an external load of 3.2 MPa.

4. Fracturing of unconsolidated sands

Unconsolidated or weakly cemented sand reservoirs experience significant compaction and this compaction can lead to reduction in permeability and failure of completion equipment. In order to mitigate these challenges, water or other fluids are injected as oil/gas is extracted to maintain reservoir pressure. The optimum design of injection program requires thorough understanding on how fractures develop in unconsolidated sand reservoirs. Limited number of experiments conducted by various researchers show that fracturing in unconsolidated sands differs from fracture growth in hard rocks because of poroelastic effects and elastoplastic material behavior. A key result from past experimental program is that fracture growth and pressure distribution in unconsolidated sands strongly depends on fluid rheology and matrix permeability. Thus the objectives of this analysis are to study:

- failure modes and evolution of breakdown pressure in unconsolidated sands, and

- the shape of fracture for different mobility of injected fluids.

4.1. Problem setup

In this problem, a borehole in a rectangular domain was considered, as shown in Figure 7. Only half of the domain was modeled due to symmetry of the geometry. Plane strain, pore pressure elements, CPE4P, were used for discretizing the model. Water was injected into the well at a prescribed flow rate using the pipe element, FP2D2, in Abaqus/Standard. Pore pressure on the left, right and top surfaces was set to zero during the analysis. The top surface was fixed in the horizontal direction and symmetric boundary conditions in the vertical direction were applied on the bottom surface at all times.

The porous elasticity model with soft rock plasticity, as described in Section 2, was used in this example (material properties are listed in Table 2). The hardening curves of $p_c(\epsilon_v^{pl})$ and $p_t(\epsilon_v^{pl})$ are shown in Figure 8. The permeability of the unconsolidated sand was assumed to vary with void ratio as shown in Figure 9. Two test cases with different values of permeability were considered: a baseline case with lower permeability and a case with increased permeability. Initial void ratio was set to 0.36 and the following initial stress state was prescribed:

$$\sigma_{xx} = -520 \text{ psi}, \sigma_{yy} = -670 \text{ psi}$$

This analysis involves three steps. In the first step, surface loads, equal in magnitude to the initial stresses, were applied on the top, left and right surfaces. The surface of the borehole was fixed in the horizontal and vertical directions, and the bottom surface was fixed in the horizontal direction. An equilibrium configuration was obtained using the *GEOSTATIC procedure. In the second step the “construction” of the wellbore was simulated by removing the displacement constraints on the borehole surface as well as the constraints in the horizontal direction on the bottom surface. In the third step, water was injected into the borehole using pipe elements by prescribing a volumetric flow rate at one end of the pipe; the pressure degree of freedom at the other end of the pipe was coupled to the surface of the borehole using the surface-to-surface tie constraints. The prescribed volumetric flow rate was ramped from zero to a constant value for the initial 1 sec of the step time and was then kept constant for another 1 second. This constant value of the prescribed flow rate is different for the two cases with different permeabilities.

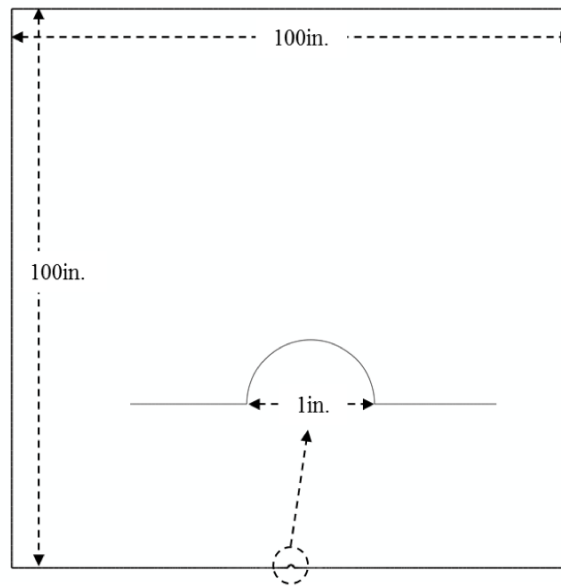


Figure 7. Sketch of the model used for testing; inset shows the geometry of the borehole.

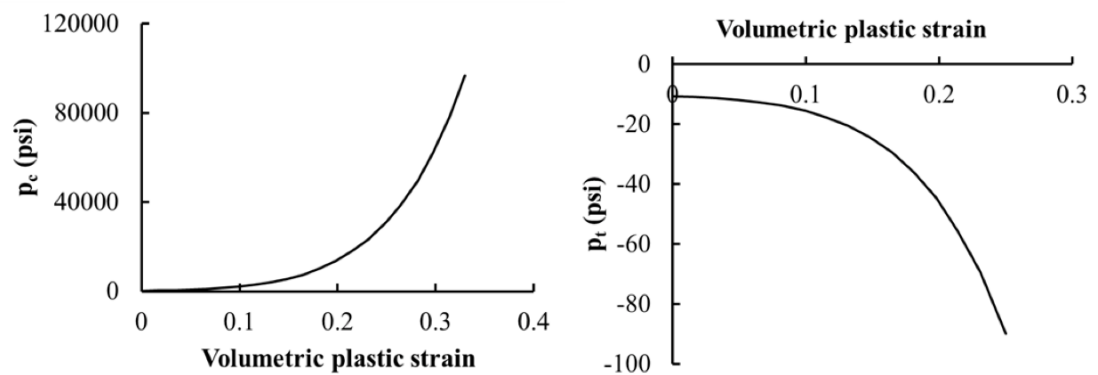


Figure 8. Hardening curves used in the fracturing problem.

Table 2. Material properties used in the fracturing of unconsolidated sands problem.

Porous Elasticity properties

E_{ref} (psi)	p_{ref} (psi)	p_o (psi)	n	v_{min}	v_{max}	m
1.5×10^5	0.0	100.0	0.01	0.2	0.28	0.003

Plastic properties

β	ψ	n_y	f_0	f_1 (psi ⁻¹)	α	e_0
67.1°	66.9°	1.25	0.5	1.5×10^{-6}	0.25	0.001

Softening regularization properties

$l_c^{(m)}$ (in)	n_r	f_{max}
0.05	0.6	100.0

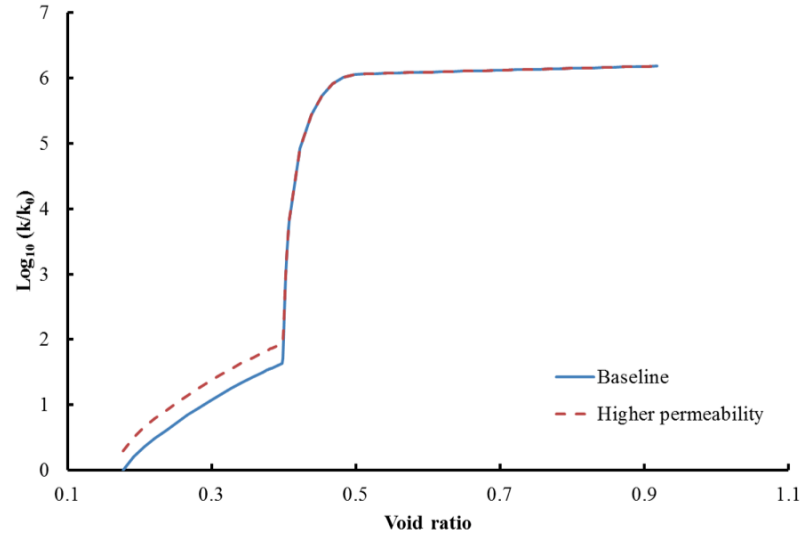


Figure 9. Permeability as a function of void ratio. k is the permeability in in/sec and k_0 is the base permeability corresponding to the void ratio of 0.18.

4.2. Results and discussion

Results for the fracturing problem are summarized in this section. Figure 10 shows the effect of permeability on the borehole breakdown pressure. It is observed that the breakdown pressure slightly increases on decreasing the permeability. The breakdown pressure (P_b) of an elastic medium with zero tensile strength is a function of initial stress state and is given by (Hubbert and Willis, 1957)

$$P_b = 3\sigma_h - \sigma_H - P \quad (16)$$

where σ_h is the in-plane minimum total stress, σ_H is the in-plane maximum total stress and P is the pore pressure. The breakdown pressure per the above equation is 890 psi. The computed breakdown pressure is higher than 890 psi for the following reasons:

- (i) Ductility of unconsolidated sands,
- (ii) Absence of explicit fracture representation (e.g. cohesive zone elements), and
- (iii) Finite amount of tensile strength prior to breakdown.

Figure 11 shows plastic strains and pore pressure at the end of injection. As can be seen, the localization profile depends on the permeability of the formation. In the higher permeability case, there is significant leak-off into the formation, whereas in the baseline case with lower permeability there is less leak-off and more energy is available to propagate “fracture”. This causes the “fracture” (represented by the localization of equivalent plastic strain, PEEQ) in the lower permeability formation to extend further than in the higher permeability formation.

The stress path of an integration point close to the fracture and the borehole was analyzed and was found that the material fails on the shear failure surface, rather than on the tensile failure surface.

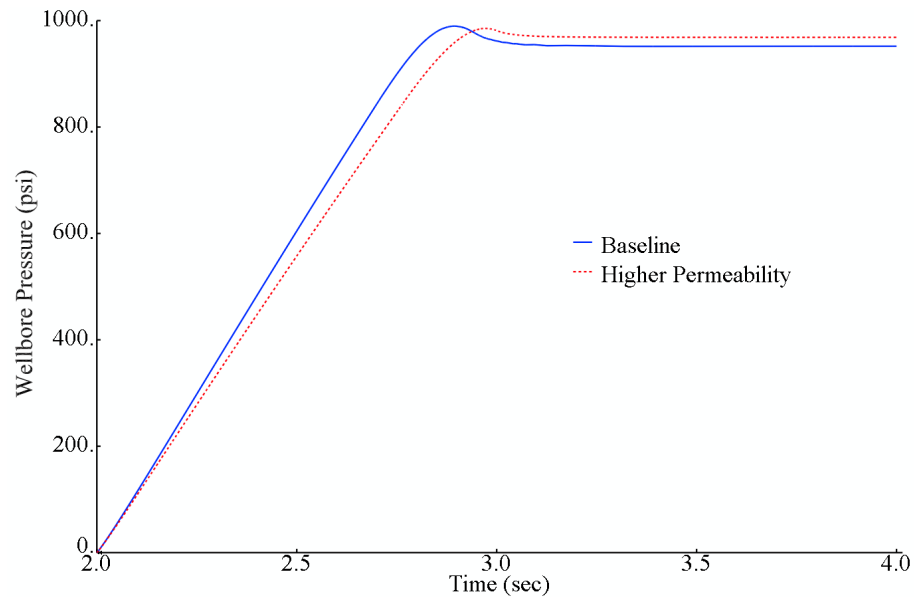


Figure 10. Variation of breakdown pressure with permeability.

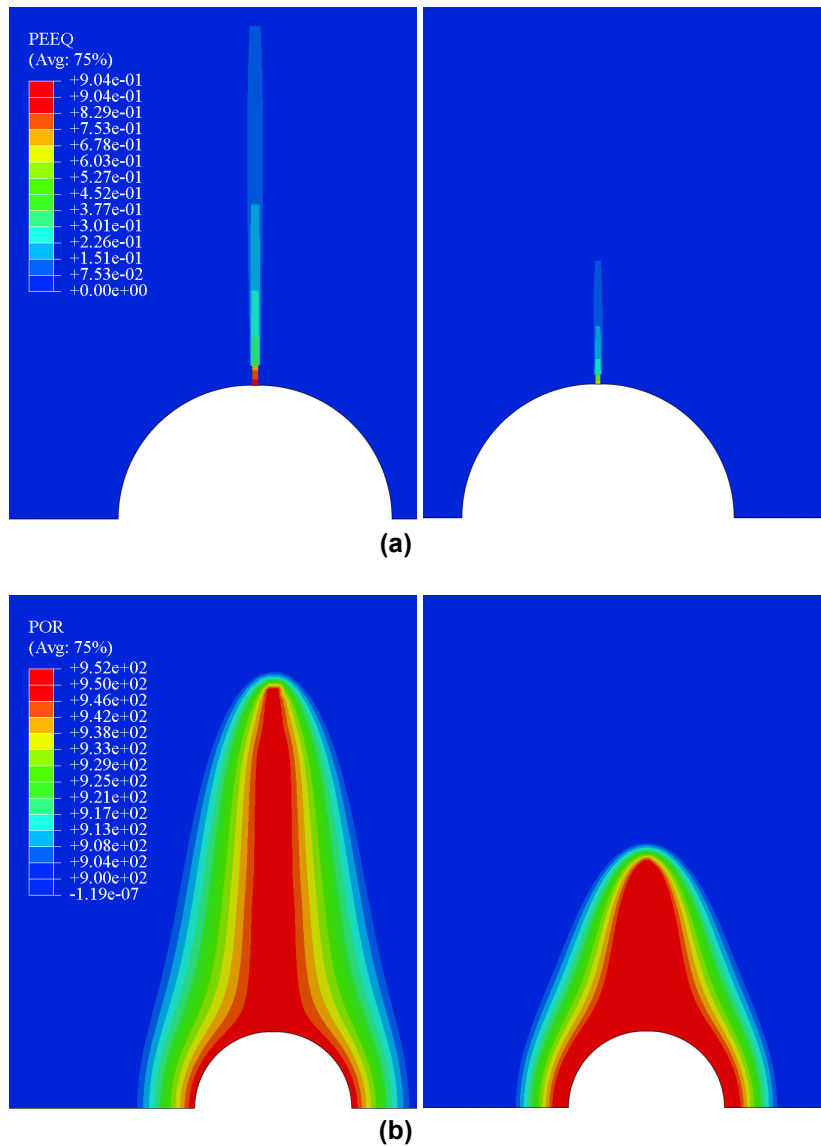


Figure 11. Equivalent plastic strain (a) and pore pressure (b) at the end of the simulation. In both, (a) and (b), figure on the left side represents the lower permeability case (baseline) and the figure on the right side represents the higher permeability case.

5. Conclusions

A poro-elastoplastic constitutive model for weakly cemented rocks was implemented in Abaqus. The model captures the variation of elastic properties and strength with mean effective stress. The implementation was verified using simple problems for which analytical solutions may be computed. The soft rock model was used to simulate wellbore stability and hydraulic fracturing applications. The borehole model was able to capture the formation of shear bands due to enhanced shear loading. The results from the simulation of fracturing in unconsolidated sands show that fracturing is dominated by shear failure as opposed to Mode I tensile failure in hard rocks. The results also show that higher ductility of unconsolidated sands leads to significantly higher net fracturing pressure compared to that of hard rocks. It was also observed that breakdown pressure in soft rocks is higher than the minimum stress, again due to ductility.

6. References

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