Shape Optimization of Rubber Bushings Using Differential Evolution Algorithm

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Abstract: *The main functions of a rubber bushing are basically to join the elements between rigid structures in the vehicles, isolate vibrations through to the chassis and avoid the transmission of noise. Due to the increasing interest of multibody simulations of complete vehicles or subsystems, it is important to develop and effective models to represent the static stiffness of these rubber bushings. During the vehicle development process, shape optimization of rubber products is also need to have desired stiffness curves. The objective of this study is to optimize rubber bushings to meet the stiffness curve requirements. A Fortran code has been developed to find the optimum shape parameters of 3D bushing models using differential evolution algorithm. Abaqus software has been used to create the hyperelastic finite element model corresponding to the geometric variations. Stiffness curves were obtained from the finite element analysis results and chi-square values were used as an objective function for shape optimization.*

Keywords: *Rubber, hyperelasticity, optimization, differential evolution algorithm.*

1. Introduction

Isolators produced from rubber materials play an important role in the noise and vibration control in vehicles. They isolate the car body from the vibration source such as road irregularities, wheel and tire unbalances, powertrain, etc. In particular, rubber bushings are the object of this study. They consist of a rubber tube bonded on their outer and inner surfaces to rigid metal layers. Their high axial and torsion flexibility, large radial and conical rigidity and their inherent damping make them interesting for being used in primary suspensions, pivot arms and several types of mechanical linkage in vehicles. The main function of a rubber bushing is basically to join the elements between rigid structures in the system, isolate vibrations through to the chassis and avoid the transmission of noise. Due to the increasing interest in predictions performed by multibody simulations of complete vehicles or subsystems, it is important to develop and effective models to represent the static stiffness of these rubber bushings. During the vehicle development process, shape optimization of rubber products is also need to have desired stiffness curves.

2. Hyperelastic Material Model

Hyperelasticity is used to describe the behaviour of rubber-like materials. This behaviour is characterized by a strain energy function which defines the strain energy stored in the material per unit of reference volume as a function of the strain at that point in the material. Several hyperelastic material models have been developed for the behaviour of rubber materials which are Mooney-rivlin, Polynomial form, Neo-hookean, Ogden, Yeoh, Arruda-boyce etc. (Karen et al, 2010).

Successful modelling and design of rubber components relies on both the selection of an appropriate strain energy function and an accurate determination of material constants in the function using physical tests. Material constants in the strain energy density functions are generally determined from curve fitting of experimental stress-strain data. There are several different types of experiments, but in general, a combination of simple tension, simple compression, and pure shear tests are used to determine the material constants (Kim et al, 2004).

Stress-strain data acquired from physical tests are used to define material models using curve fitting method in Abaqus software. In this study, experimental physical tests are carried out at Bayrak Plastik A.Ş which produces rubber parts in Bursa, Turkey. Uniaxial tension, shear and compression tests are necessary for modelling the behaviours of rubber material but the fundamental one is the uniaxial tension test. In this study, uniaxial tension and planar tension materials test have been performed in order to get the rubber materials behavior. After the curve fitting in Abaqus software using test data, Ogden N=5 hyperelastic model is selected as material model for the rubber part.

3. Geometry and finite element model

Geometric model of rubber bushing consists of two steel tubes and rubber section between them as shown in Figure 1. Two shape parameters are selected as T and H for rubber section. Limit values for these parameters are: $0^o \leq T \leq 20^o$ and $0 \leq H \leq 12$ mm.

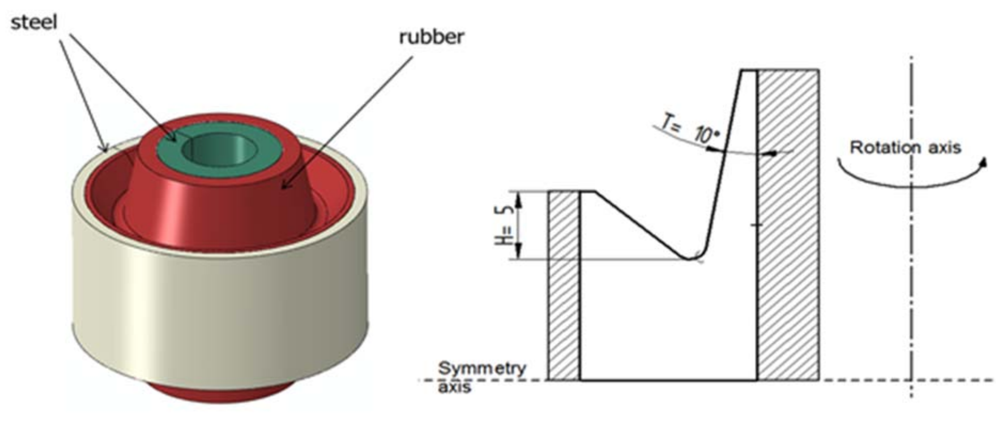

Figure 1. Rubber bushing model and shape parameters

Four possible rubber sections are shown in Figure 2 for extreme values of shape parameters.

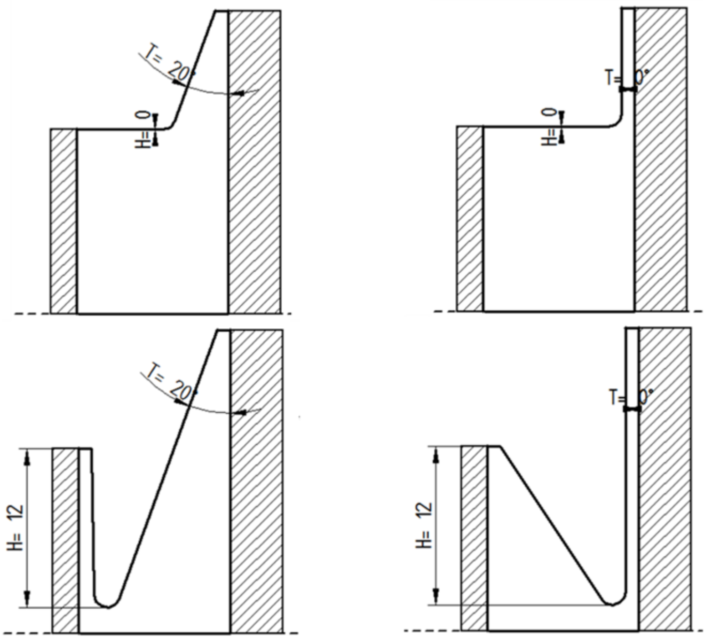

Figure 2. Rubber bushing section views for extreme values of shape parameters

The rubber part was modeled using 8-node hybrid hexahedral solid elements in Abaqus finite element software. Steel and rubber finite elements model consists of 22036 nodes and 16207 elements. Regarding the contacts in the rubber part, the self contact was considered. Tied connection is employed between steel parts and rubber. Inner and outer steel tubes were also modeled with hexahedral solid elements. Material properties for steel are E=210000 Mpa and poisson ratio=0.3.

As shown in Figure 3, the displacement of 4 mm was applied from the center of the inner steel tube using rigid connection and deformed model includes self contact. During the deformation, reaction forces are stored for each iteration and then used for drawing stiffness curves which are non-linear.

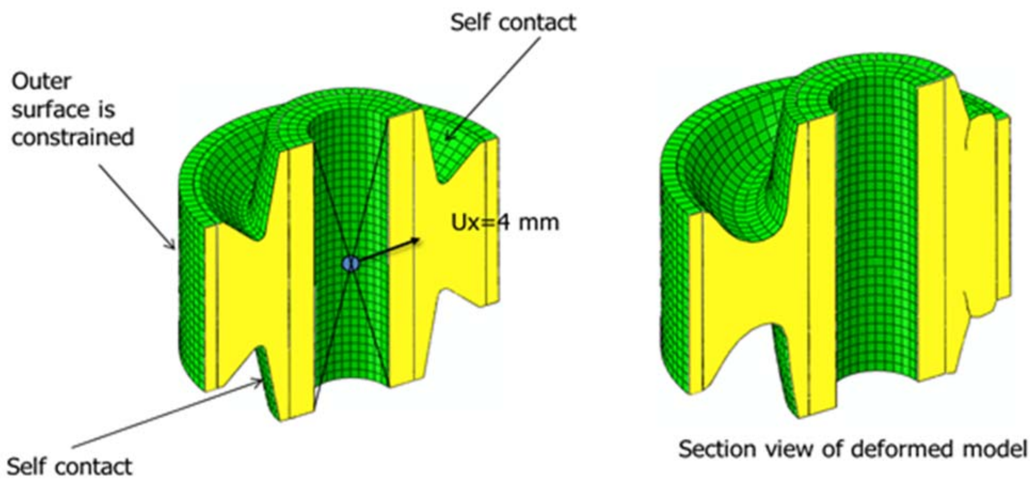

Figure 3. Finite element section model and boundary conditions

4. Shape Optimization

One of the main shortcoming of classical optimization methods is to stuck into local optimum instead of global optimum. Genetic Algorithm (GA) and Differential Evolution (DE) algorithms are evolutionary optimization algorithms, they developed for finding the global optimum of the optimization problems. DE is a relatively new evolutionary optimization algorithm. Many studies demonstrated that DE converges fast and is robust, simple in implementation and use, and requires only a few control parameters (Kim et al, 2007). DE is an improved version of Genetic Algorithm for faster optimization. Unlike simple GA that uses binary coding for representing problem parameters, DE uses real coding of floating point numbers. Among the DE's advantages are its simple structure, ease of use, speed and robustness. The key parameters of control in DE are: NP-the population size, CR-the crossover constant (0.0 -1.0), and F - scaling factor that controls the amplification of differential variations (0.0 – 2.0). In this study, optimum shape parameters of rubber section are determined using differential evolution optimization algorithm.

Design of experiment method (DOE) is used for constructing response surface for shape optimization. A full factorial sampling in DOE method is employed to decide on the analysis points in order to obtain enough information for the approximation. Three parameters which are T,H and displacement are selected for DOE study. Five levels for T, 6 levels for H and 4 levels for displacement (x) are adopted to obtain analysis points. 30 analyses have been solved to obtain the response surfaces for stiffness curves. All finite element models are solved in Abaqus software and DOE table is generated as shown in Table 1.

Table 1. Reaction force values recpect to parameters

Displacement (mm)	T=0 H=0	T=0 H=2.4	T=0 H=4.8	T=0 H=7.2	T=0 H=9.6	T=0 H=12	T=5 H=0	T=5 H=2.4	T=5 H=4.8	T=5 H=7.2	T=5 H=9.6	T=5 H=12
1	1862	1593	1259	908	589	300	1937	1681	1345	969	613	307
2	4446	3785	2938	2067	1314	671	4652	4044	3158	2233	1368	672
3	9051	7611	5772	3921	2396	1194	9549	8268	6381	4472	2681	1266
4	18878	15791	11828	7957	4755	2283	20089	17466	13557	9574	5775	2769

Displacement (mm)	T=10 H=0	T=10 H=2.4	T=10 H=4.8	T=10 H=7.2	T=10 H=9.6	T=10 H=12	T=15 H=0	T=15 H=2.4	T=15 H=4.8	T=15 H=7.2	T=15 H=9.6	T=15 H=12
1	1991	1766	1030	908	648	323	2076	1850	1502	1084	676	344
2	4792	4255	2404	2067	1465	692	4976	4464	3605	2546	1529	737
3	9828	8837	4962	3921	3005	1356	10275	9283	7555	5328	3160	1431
4	20688	18874	10917	7957	6751	3165	21453	19857	16554	11872	7185	3321

Displacement (mm)	T=20 H=0	T=20 H=2.4	T=20 H=4.8	T=20 H=7.2	T=20 H=9.6	T=20 H=12
1	2126	1936	1564	1107	686	373
2	5118	4708	3728	2608	1549	813
3	10523	9843	7812	5452	3094	1442
4	21824	21049	17088	12016	6818	3054

Curve fitting technique is applied to find the response equation (F_{DOE}) respect to shape parameters of 3D bushing model. Following force equation is constructed using curve fitting technique:

DOEForce=-0.070 $\mathbf{t^3}$ + 4.50 $\mathbf{h^3}$ + 380 $\mathbf{x^3}$ + 5.56 $\mathbf{t^2}$ - 55.93 $\mathbf{h^2}$ - 954.10 $\mathbf{x^2}$ - 0.064 $\mathbf{t^2h}$ - 2.07 $\mathbf{t^2 x}$ - 1.52 $\mathbf{h^2t}$ - 7.88 $\mathbf{h^2x}$ + 21.48 $\mathbf{x^2t}$ - 148.02 $\mathbf{x^2h}$ + 23.14 **th** + 8.81 **tx** + 412.71 **hx** - 3.68 **htx** - 89.07 **t** - 274.38 **h** + 2234.04 **x** + 563.70

Coefficient of determination (R^2) which is an overall measure of the accuracy of a regression was calculated as 0.9972. It is a good approximation and this function can be used for stiffness curve calculation at optimization process instead of finite element analyses.

Fort the optimization purpose, two desired stiffness curves (F1 and F2) are given in Figure 5 and 6. The most rigid and elastic stiffness curves are also shown in these Figures. In order to have such a desired stiffness curves, shape parameters are found by differential evolution (DE) optimization algorithm which is written in Fortran programming language.

Optimization problem has been defined as follows:

$$\text{Objective function: } \min \text{chi-square} = \sum_{i=1}^{n} \frac{\left(F_{DOE\,i} - \mathrm{F1}_i\right)^2}{\mathrm{F1}_i}$$

Subject to : $0 \leq \mathrm{T} \leq 20$

$0 \leq \mathrm{H} \leq 12$

where F_{DOE} : Force function which is generated from DOE table, F1: Desired stiffness curve, T and H: shape parameters.

A Fortran code is developed to find the global minimum of functions using DE algorithm. DE parameters are selected as; population size: 25, number of generation: 100, scale factor: 0.75, crossover rate: 0.85. Convergence graph of the optimization algorithm for the objective function F1 is shown in Figure 4.

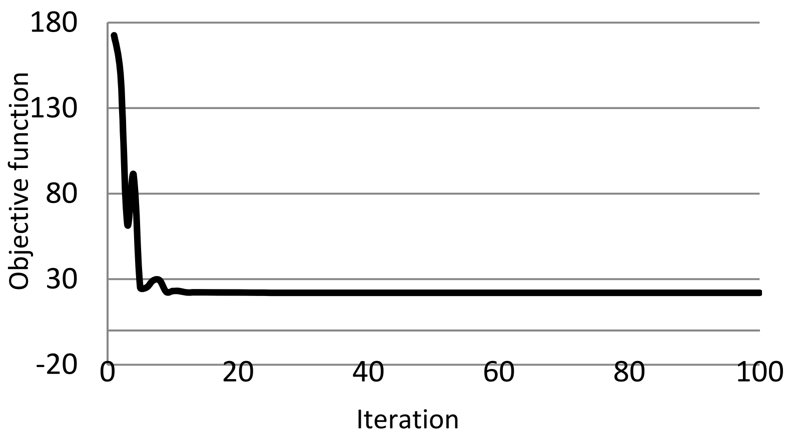

Figure 4. Iteration history graph for the first optimization problem

Optimum shape parameters are found as T=8.26 mm and H=6.61 mm after solving the optimization problem for F1 using DE. As seen in Figure 5, optimum curve is very close to desired curve. DE algorithm has been successfully applied to this shape optimization problem.

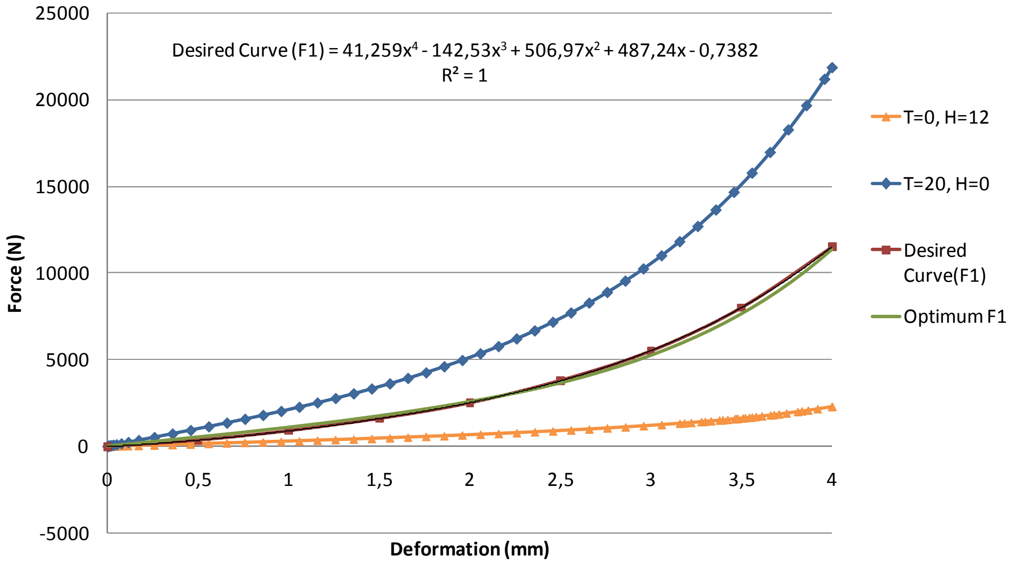

Figure 5. Desired curve (F1) and optimum curve

As a second example, second desired curve (F2) has been shown in Figure 6. This time the stiffness curve is a little different compared to other stiffness curves. The same methodology has been used to obtain the optimum shape parameters which are T=20 mm, H=9.95 mm.

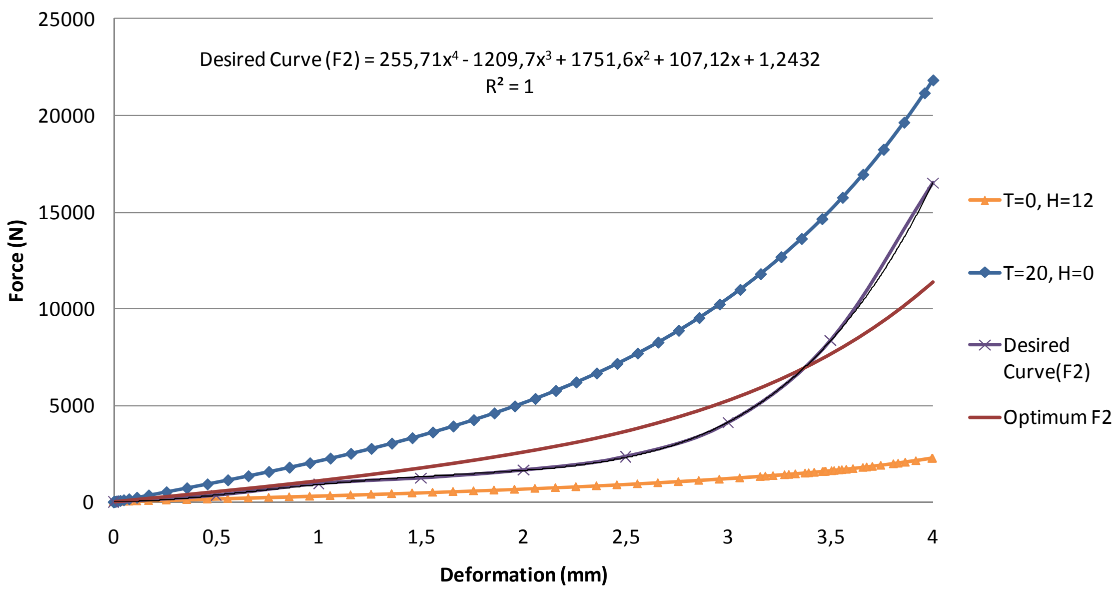

Figure 6. Desired curve (F2) and optimum curve

After solving the second optimization problem, it is seen from the graph that optimum curve does not well match to our desired curve (F2). The reason is that this bushing model cannot give such a stiffness curve, the first region of curve looks not stiff, but from the 3 mm displacement beyond, the slope is suddenly increased. This type of curve can be seen at engine mounts which have some holes. During radial deformation, the slope of stiffness curve slowly increased, after closing the holes by self contacting, it suddenly increased. Therefore, in this case study, optimum curve is the best one that matches the desired curve.

5. Conclusions

In this work, a Fortran code based on DE algorithm is developed to solve shape optimization problems. DE algorithm is successfully applied to shape optimization of rubber parts to obtain desired stiffness curves. DOE and curve fitting technique is applied to find the response equation respect to shape parameters of 3D bushing model. All non-linear finite element models are solved by Abaqus software.

6. Acknowledgements

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7. References

1. Karen, İ., Kaya, N., Öztürk, F., (2010) 'Optimisation of vehicle engine mount system using simulation-based design approach', Int. J. Vehicle Design, Vol. 52, Nos. 1/2/3/4.

2. Kim, W.D., Kim, W.S., Woo, C.S. and Lee, H.J. (2004) 'Some considerations on mechanical testing methods of rubbery materials using non-linear finite element analysis', Polymer International, Vol. 53, pp.850–856.

3. Kim, H-K.,Chong, J-K., Park, K-Y., Lowther, D.A., (2007), 'Differential Evolution Strategy for Constrained Global Optimization and Application to Practical Engineering Problems', IEEE Transactions on Magnetics, Vol. 43, No. 4.

4. Abaqus v6.11 Software, 2012.