# Parameter Survey for Implicit Dynamic Time Integration under Contact Condition

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*Abstract: Dynamic problems involving contact are of great importance in industry related to mechanical and civil engineering, but also in environmental and medical applications. Regarding the implicit dynamic structural analysis, Abaqus/Standard has the HHT (Hilber-Hughes-Taylor) method which is a family of unconditionally stable one step methods for the direct time integration. This method is extended from well known Newmark beta method. The HHT method has three parameters, the parameters of beta and gamma are from Newmark beta method, and the parameter of alpha is additional one. While Abaqus/Standard classifies the implicit dynamic analysis applications into three types, only "TRANSIENT FIDELITY" and "MODERATE DISSIPATION" are related to the HHT method. In this paper, we clarified the spectral radius of integration approximation operator for various time integration methods including Newmark beta family and the central difference method. Also we investigated a variety of HHT's three parameters settings in contact problem, furthermore illustrated the practical application regarding impact analysis of a ratchet device.*

*Keywords: Implicit Dynamics, Impact, Direct Time Integration*

## 1. Introduction

Some Structural phenomena have to be analyzed under the law of inertia, so called dynamic analysis. There are two principal methods in the dynamic analysis procedure of Finite Element Methods. One is implicit method, the other is explicit method. Explicit method is useful to analyze the severe non linear boundary condition problems including impact, dropping and plastic forming, but its result involves some vibration. While we need the precise result on occasion, for example investigating the peak value of buckling structure like the rubber switch. In such a case, an implicit procedure is preferable to obtain the exact transient value without oscillation.

Several implicit procedures were developed in the past. As a representative of implicit procedures, there was Classical Newmark beta method, which had the robustness with unconditionally stable and no damping characteristics in linear problems. But it is necessary to have high frequency damping in presence of contact condition. For the sake, generalized alpha methods like as HHT-α (Hilber, 1977) and WBZ-α (Wood, 1980) were developed.

In this paper, we summarize the implicit time integration method and exhibit the spectral radius which shows the characteristics of time integration. Some numerical experiences are presented to reveal the influence of the HHT-α method parameters. Furthermore, we illustrate the practical application with impact analysis of a ratchet device.

## 2. Generalized alpha method

### 2.1 Generalized alpha method

Newmark algorithm is one of the representative time integration methods for implicit dynamic procedure. Its characteristic is the unconditionally stability with proper parameter values in linear problems. This procedure is enhanced by additional alpha parameter in (Hilber, 1977) (Wood, 1981) (Chung, 1993).

The basic form of the generalized alpha method is as follows. To begin with, the displacement **d**, the velocity **v** and the acceleration **a** are related to Equation (1) and (2) within the manner of Newmark beta.

$$\mathbf{d}_{n+1} = \mathbf{d}_n + \Delta t \mathbf{v}_n + \Delta t^2 \left[ \left( \frac{1}{2} - \beta \right) \mathbf{a}_n + \beta \mathbf{a}_{n+1} \right] \tag{1}$$

$$\mathbf{v}_{n+1} = \mathbf{v}_n + \Delta t \left[ (1-\gamma) \mathbf{a}_n + \gamma \mathbf{a}_{n+1} \right] \tag{2}$$

The matrix equation of structural dynamics is

$$\mathbf{M}\mathbf{a}_{n+1}^{\alpha_B} + \mathbf{C}\mathbf{v}_{n+1}^{\alpha_H} + \mathbf{K}\mathbf{d}_{n+1}^{\alpha_H} = \mathbf{F}\left(t_{n+1}^{\alpha_H}\right) \tag{3}$$

where **M**, **C,** and **K** are the mass, damping and stiffness matrices, respectively, **F** is the vector of applied loads. This modified equation is effectively a combination of the HHT-α (Hilber, 1977) and WBZ-α (Wood, 1980) method. Each variable and time of load are expanded to

$$\mathbf{a}_{n+1}^{\alpha_B} = (1-\alpha_B)\mathbf{a}_{n+1} + \alpha_B \mathbf{a}_n \tag{4}$$

$$\mathbf{v}_{n+1}^{\alpha_H} = (1-\alpha_H)\mathbf{v}_{n+1} + \alpha_H \mathbf{v}_n \tag{5}$$

$$\mathbf{d}_{n+1}^{\alpha_H} = (1-\alpha_H)\mathbf{d}_{n+1} + \alpha_H \mathbf{d}_n \tag{6}$$

$$t_{n+1}^{\alpha_H} = (1-\alpha_H)t_{n+1} + \alpha_H t_n . \tag{7}$$

We can apply the effective numerical dissipation by using adequate value of these alpha parameters. The following condition must be satisfied to maintain unconditional stability (Chung, 1993).

$$\beta = \frac{1}{4}\left(1+\alpha_H - \alpha_B\right)^2 \tag{8}$$

In addition, the following equation is required for the second order accuracy, stability, and energy momentum conservation.

$$\gamma = 0.5 + \alpha_H - \alpha_B \tag{9}$$

Note that the sign of $\alpha_H$ is reverse to $\alpha$ of Abaqus, for the reason that this equation is described with the different style of Abaqus. Besides Abaqus does not adopt the WBZ-α method, therefore Equations (8) and (9) have to be modified as following brief equations.

$$\beta = \frac{1}{4}\left(1-\alpha\right)^2 \tag{10}$$

$$\gamma = 0.5 - \alpha \tag{11}$$

For frictionless contact problems, we should also consider conservation of momentum and energy during impact. The energy and momentum conservation are only satisfied when $\beta$ is given by (Czekanski, 2001)

$$\beta = \frac{1}{4}\frac{-2\alpha_B{}^2 + \alpha_B\left(3+2\alpha_H\right)-2}{\alpha_B - 1}. \tag{12}$$

By the relation between Equation (8) and (12), we could derive the following equation between $\alpha_B$ and $\alpha_H$ for satisfying energy momentum conservation criterion and high frequency damping.

$$\alpha_H = \frac{\alpha_B{}^2 - \alpha_B + 1 - \sqrt{2\alpha_B{}^2 - 3\alpha_B + 2}}{\alpha_B - 1}. \tag{13}$$

If we substitute zero for $\alpha_B$, Equation (13) shows the alpha value of “Moderate Dissipation” in Abaqus.

$$\alpha_H = \sqrt{2} - 1. \tag{14}$$

Note again, the sign of $\alpha_H$ is reverse to Abaqus’s $\alpha$ .

## 2.2 Spectral radius

Considering the stability of time integration methods, we can rewrite dynamics equation (3) in the succinctly recursive form with the applied loads dropped

$$\mathbf{X}_{n+1} = \mathbf{A}\mathbf{X}_n , \tag{15}$$

where $\mathbf{X}_{n+1}$ and $\mathbf{X}_n$ are vectors storing the solution quantities (displacements, velocities and accelerations) and the matrix **A** is named the integration approximation operator. Let $\rho(\mathbf{A})$ be the spectral radius of **A** defined as

$$\rho(\mathbf{A}) = \max|\lambda_i| \tag{16}$$

where $\lambda_i$ are eigenvalues of **A** in the complex plane. In order to maintain the stability through any time step, we must have

$$\rho(\mathbf{A}) \leq 1 . \tag{17}$$

In other words, if the spectral radius is smaller than 1, the integration operator has numerical dissipation. Figure 1 shows spectral radius plots of representative time integration methods depending on time step ratio $\Delta t/T$, where $\Delta t$ is the time step (called time increment in Abaqus) of numerical integration, and $T$ is the cycle (period) time for one structural mode.

Figure 1 reveals various time integration characteristics.

1. The central difference method and the linear acceleration method show the conditional stability characteristics.
2. The classical Newmark-beta shows the unconditional stability and accuracy without numerical damping.
3. The Houbolt method shows the strong numerical damping.
4. The Wilson-theta method has the variable damping ratio depending on the time ratio.
5. The HHT with alpha -0.05 shows the unconditional stability and slight numerical damping. The damping ratio is about 10 percent. This method is marked as “Transient Fidelity” in Abaqus.
6. If the parameter of alpha is -0.41421 according to Equation (14), the HHT method shows the unconditional stability and stronger numerical damping than alpha -0.05. This method is marked as “Moderate Dissipation” in Abaqus. The concave shape means the eigenvalue turning point between spurious root and principal root. About 30 percent numerical damping is expected in high frequency (large $\Delta t/T$) area.

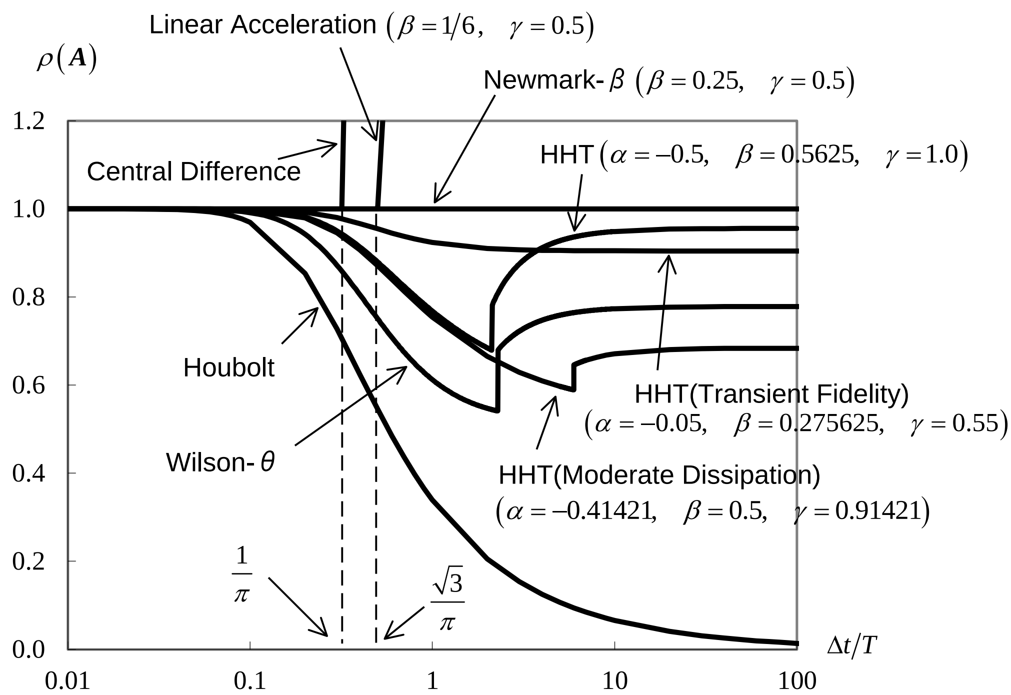


**Figure 1. Spectral radius.**

## 3. Numerical Examples

In this chapter, we performed two numerical experiments. At first, we investigated the various parameters of HHT method on the simple impact problem of two elastic bars' collision. Secondly, we tested three different time integration schemes on the ratchet device FE model.

### 3.1 Impact of two elastic bars

In accordance with past examination (Czekanski, 2001), we prepare the impact model of two elastic bars which have length L=10 UL, height H=1 UL and are located an initial gap g=0.01 UL. The FE model is shown in Figure 2, the contact region is known a priori. The material properties were assumed as linear elastic: E=1000 UF/UL$^2$, ν=0.0, and ρ=0.001UM/UL$^3$. Both bars were given initial opposing unit velocities. Two time step values were used: $\Delta t = 5\times10^{-5}$ and $\Delta t = 8\times10^{-4}$. Each bar was modeled with the plane strain element (CPE4). In this simple model, we compared the various alpha parameters through following sections.

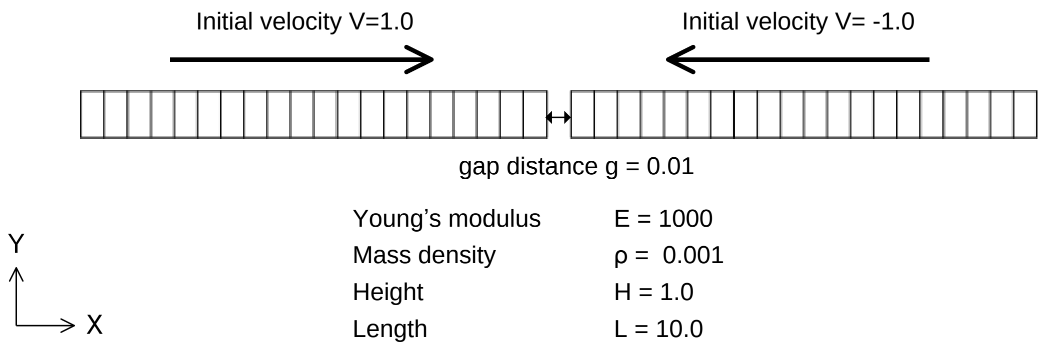


**Figure 2. Impact of two elastic bars.**

### 3.1.1 Classical Newmark parameters

At first, we studied the classical Newmark parameters, which are alpha=0.0, beta=0.25, and gamma=0.5. This case corresponds to an unconditional stability and no damping procedure. Furthermore the options of "IMPACT=NO" and "INCREMENTATION=CONSERVATIVE" were applied in order to establish the natural condition of Newmark procedure. The principal step part of input files is listed below.

```
*STEP, NAME=DYNAMIC, INC=1000
*DYNAMIC,DIRECT,ALPHA=0.0,BETA=0.25,GAMMA=0.5,
IMPACT=NO,INCREMENTATION=CONSERVATIVE
5.E-5,0.04,
```

Figure 3 shows the velocity at the tip of one of the bars obtained using the two time steps. The figure shows inaccurate result for this problem in either of these two time steps.

### 3.1.2 Newmark beta for contact

Secondly, we examined another set of Newmark parameters, which are alpha=0.0, beta=0.5, gamma=0.5. These parameters are traditionally advocated for the contact problem without numerical damping.

```
*STEP, NAME=DYNAMIC, INC=1000
*DYNAMIC,DIRECT,ALPHA=0.0,BETA=0.5,GAMMA=0.5,
IMPACT=NO,INCREMENTATION=CONSERVATIVE
5.E-5,0.04,
```

Figure 4 also shows evidently incorrect result for this problem with either of these two time steps.

### 3.1.3 HHT Contact with weak damping

We also investigated the parameters for contact problem with weak numerical damping. According to (Czekanski, 2001, Figure 1), if the alpha is designated for -0.5, this application turns the optimal contact condition without damping. But this is open question. Because this alpha parameter means about 5% numerical damping ratio as shown in Figure 1. In this case, other parameters have to be calculated by Equations (10) and (11).

$$\beta = \frac{1}{4}(1-\alpha)^2 = 0.5625 \tag{18}$$

$$\gamma = 0.5 - \alpha = 1.0 \tag{19}$$

```
*STEP, NAME=DYNAMIC, INC=1000
*DYNAMIC,DIRECT,ALPHA=-0.5,BETA=0.5625,GAMMA=1.0,IMPACT=NO,
INCREMENTATION=CONSERVATIVE
5.E-5,0.04,
```

This case indicated suitable result as in Figure 5. There were some spike waves at the moment of impact, nevertheless each time step could take the proper velocity transition after impact.

### 3.1.4 HHT Contact with damping (Moderate dissipation option)

In this case, the moderate dissipation application was applied. This application is default for the contact model in Abaqus/Standard. The parameter of alpha is set to $-(\sqrt{2}-1) = -0.41421$, while the other parameters of beta and gamma will be adjusted automatically accordance with the following equations.

$$\beta = \frac{1}{4}(1-\alpha)^2 = 0.5 \tag{20}$$

$$\gamma = 0.5 - \alpha = 0.91421 \tag{21}$$

```
*STEP, NAME=DYNAMIC, INC=1000
*DYNAMIC, APPLICATION=MODERATE DISSIPATION,DIRECT
5.E-5,0.04,
```

The results in Figure 6 are more accurate compared to the case of weak damping. The oscillation after the impact was reduced by the advantageous numerical damping.

### 3.1.5 Transient fidelity option

In the last case as the HHT method, we applied the transient fidelity setting. This application is default for the model without contact condition. The parameter of alpha is set to -0.05, the other parameters of beta and gamma also will be adjusted automatically accordance with the following equations.

$$\beta = \frac{1}{4}(1-\alpha)^2 = 0.275625 \tag{22}$$

$$\gamma = 0.5 - \alpha = 0.55 \tag{23}$$

```
*STEP, NAME=DYNAMIC, INC=1000
*DYNAMIC, APPLICATION=TRANSIENT FIDELITY, DIRECT
5.E-5,0.04,
```

Figure 7 apparently shows good result with either time increments, for the reason that this time integration application has the hidden default option “IMPACT=AVERAGE TIME”. This hidden option causes the cut back of time increments as in Figure 8 red box. Although the time increments after impact returned back to their values before impact in this problem, generally the “IMPACT=AVERAGE TIME” setting tends to decrease the subsequent time increment values significantly resulting in unnecessary increase in CPU time.

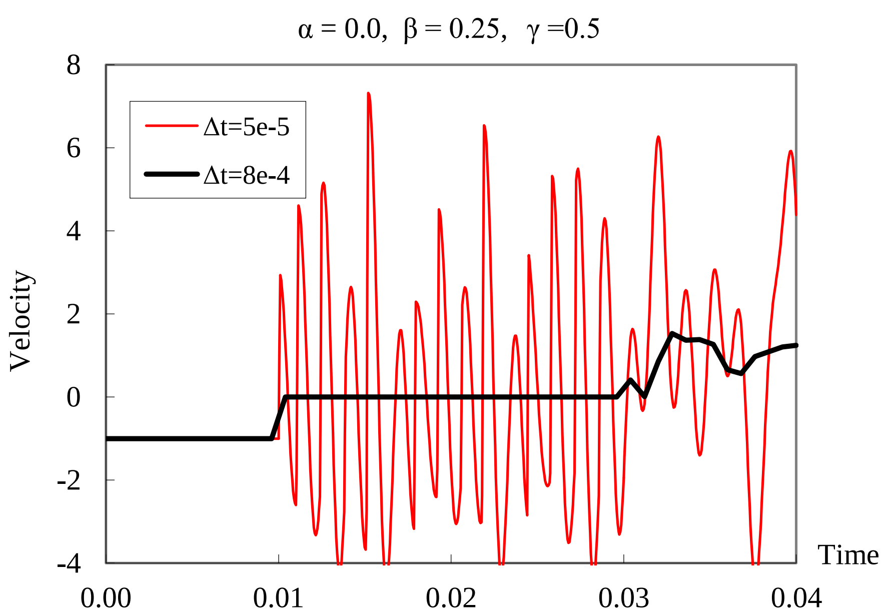


**Figure 3. Classical Newmark parameters.**

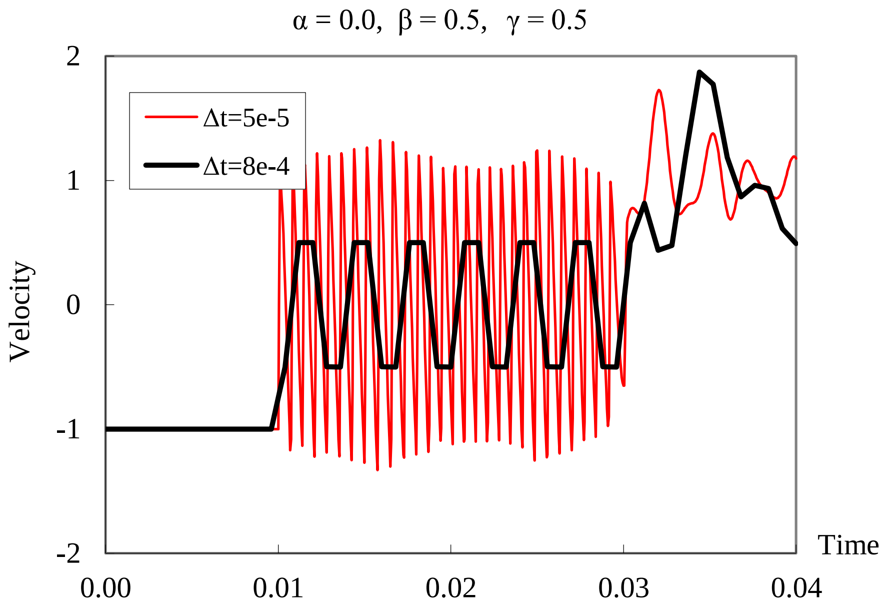


**Figure 4. Newmark for Contact.**

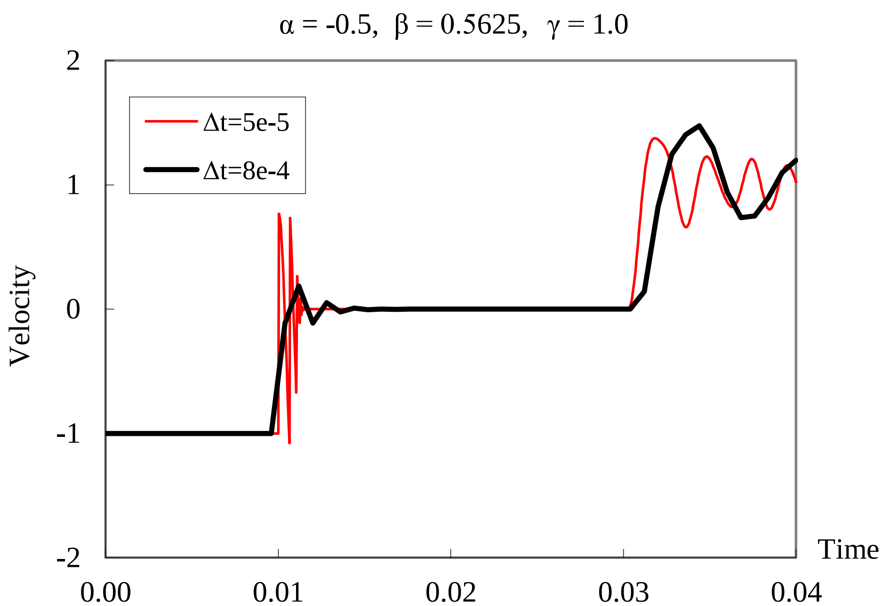


**Figure 5. Contact with weak damping.**

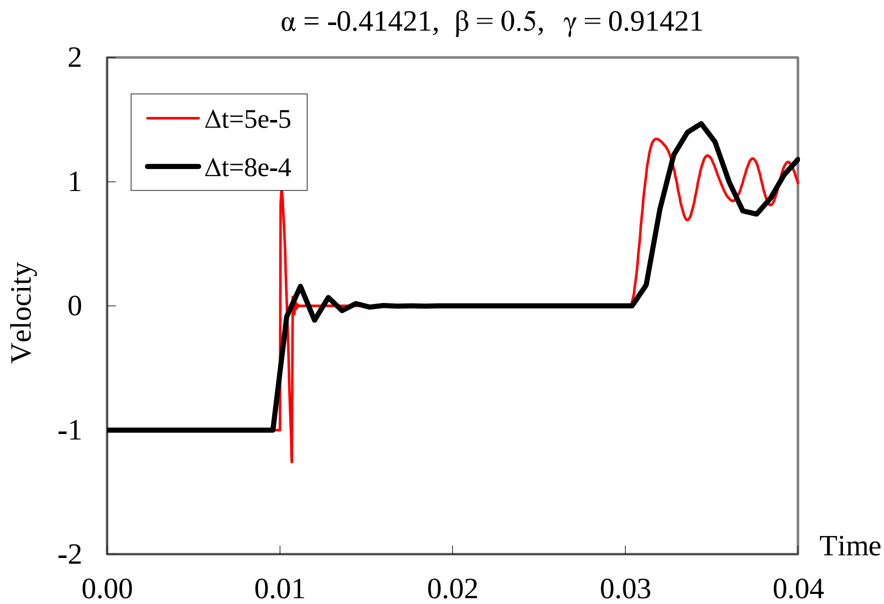


**Figure 6. Moderate Dissipation.**

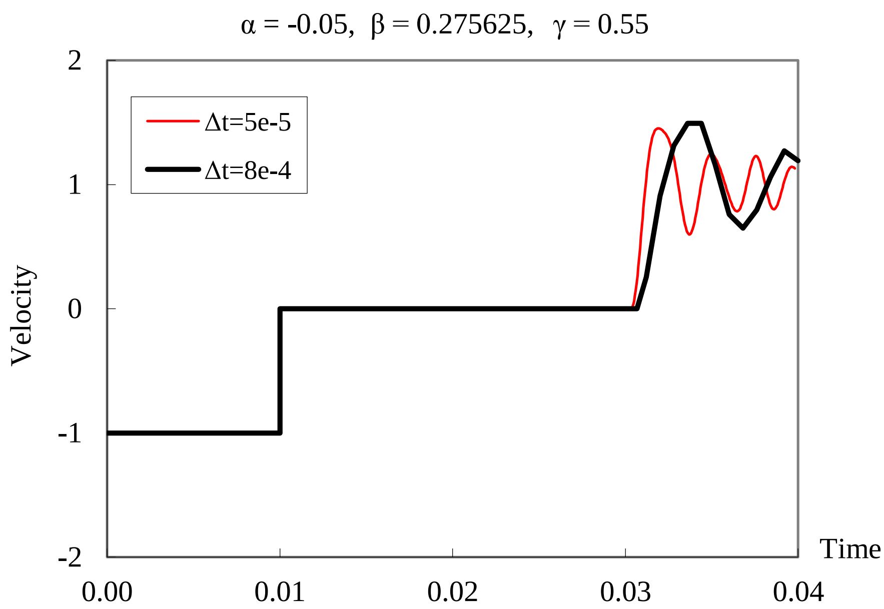


**Figure 7. Transient Fidelity.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 198 | 1 | 0 | 1 | 1 | 0.00990 | 0.00990 | 5.000e-005 |
| 1 | 199 | 1 | 0 | 1 | 1 | 0.00995 | 0.00995 | 5.000e-005 |
| 1 | 200 | 1 | 0 | 1 | 1 | 0.0100 | 0.0100 | 5.000e-005 |
| 1 | 201 | 1U | 0 | 1 | 1 | 0.0100 | 0.0100 | 5.000e-005 |
| 1 | 201 | 2 | 0 | 1 | 1 | 0.0100 | 0.0100 | 4.000e-007 |
| 1 | 202 | 1 | 0 | 0 | 0 | 0.0100 | 0.0100 | 5.000e-011 |
| 1 | 203 | 1 | 0 | 1 | 1 | 0.0100 | 0.0100 | 4.960e-005 |
| 1 | 204 | 1 | 0 | 1 | 1 | 0.0101 | 0.0101 | 5.000e-005 |
| 1 | 205 | 1 | 0 | 1 | 1 | 0.0101 | 0.0101 | 5.000e-005 |
| 1 | 206 | 1 | 0 | 1 | 1 | 0.0102 | 0.0102 | 5.000e-005 |

**Figure 8. Time step cut back in status file.**

### 3.2 Impact of ratchet device

In order to investigate more realistic model, we diverted to the ratchet device model shown in Figure 9 from Abaqus Examples manual (Simulia, 2011, 2.1.17). Since our purpose is to clarify the differences between various applications, all of the analysis conditions were the same as in Abaqus original model. We examined three applications which are Transient Fidelity, Moderate Dissipation and Classical Newmark beta. The angular velocity results are shown in Figure 10.

With Moderate Dissipation, we could get reasonable results. The black solid line in Figure 10 revealed that an approximately linear increase in the absolute value of the angular velocity occurs while the pawl slides along the ratchet, and the sign of the angular velocity changed upon impact. In comparison, the option of Transient Fidelity was not effective on this impact analysis. The total time reached only 0.0125 sec even if the analysis wasted on 10000 increments, because there were many time step cutbacks in the analysis. The magnifying plot of Transient Fidelity was shown in Figure 11. Furthermore, the Classical Newmark had the oscillation shown in Figure 12; this analysis also wasted CPU resource.

The run-time (wallclock time in the Abaqus message file) of Moderate Dissipation option was 2194 sec using four parallel processing under Microsoft Windows 7 Professional 64bit operating system and a computer (PC) having Intel Core i7-2640M (quad 2.80GHz processor cores).

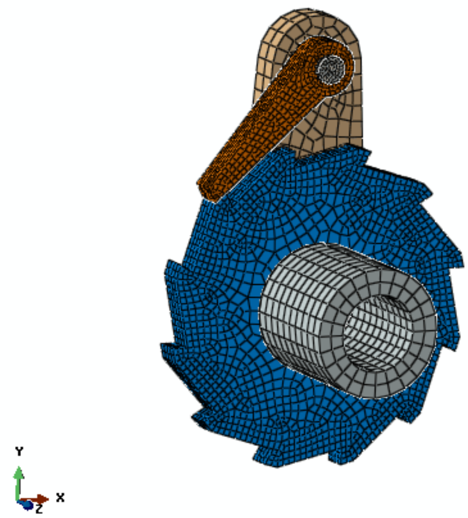


**Figure 9. Impact analysis of a pawl-ratchet device.**

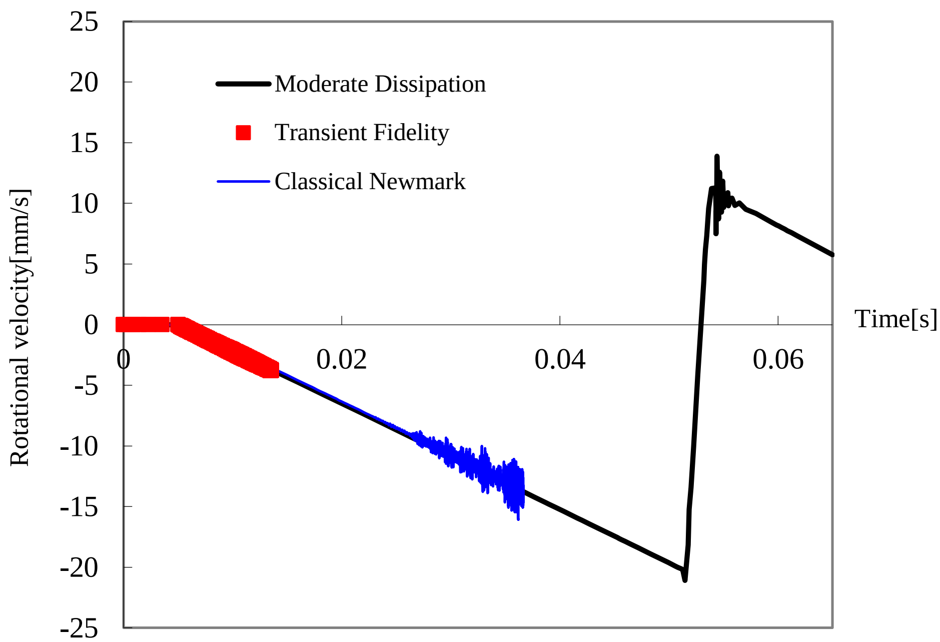


**Figure 10. Rotational velocity plots of three applications.**

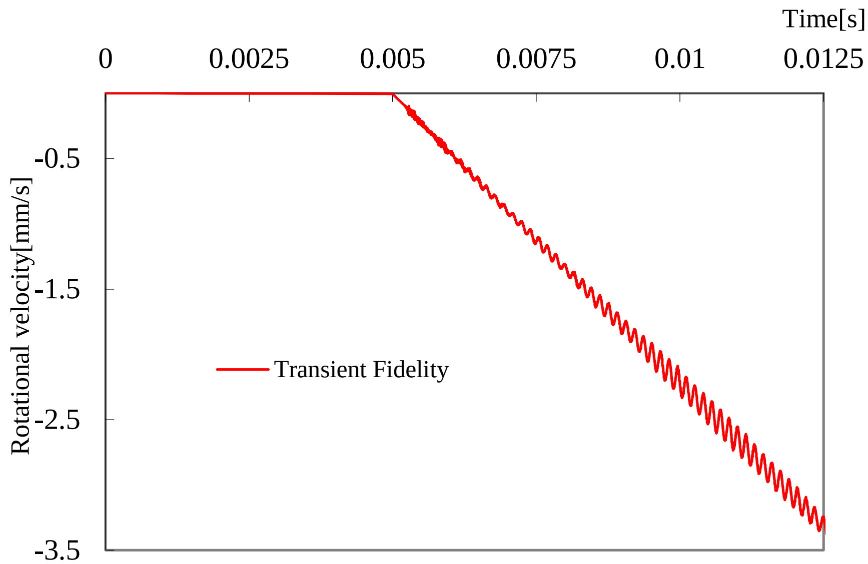


**Figure 11. Magnifying plot of Transient Fidelity.**

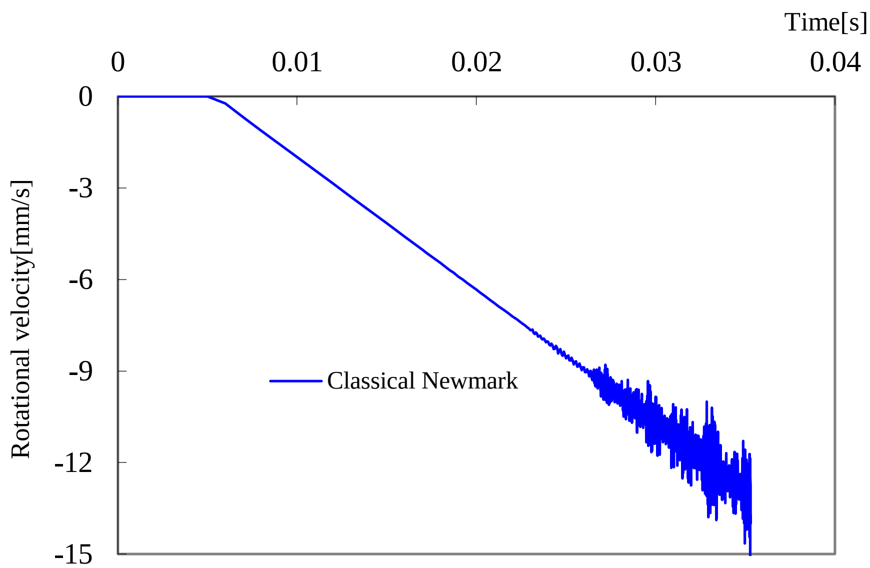


**Figure 12. Magnifying plot of Classical Newmark beta.**

## 4. Conclusions

The Generalized alpha time integration method has been presented for solving structural dynamic analysis. For clarifying numerical damping, the spectral radius involving Abaqus's HHT methods has been revealed. Also we have investigated the various time integration parameters on the FE impact analysis.

- The Classical Newmark method could not deliver proper result in contact problem. There were strong oscillations in the transition result.
- Although the Transient Fidelity method showed good results in the simple contact problem, it could not produce appropriate results in the realistic model. The analysis wasted huge CPU time, because the hidden "IMPACT=AVERAGE" option caused many time step cutbacks.
- The Moderate Dissipation method showed appropriate results and excellent performance for the case of contact. This method can be used in dynamical analyses of industrial products.

## 5. References


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