

Hot Rolling and Accelerated Cooling Simulations using ABAQUS – a Fertile Basis for Fast Online Algorithms in Heavy Steel Plate Production

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Abstract: Increasing customer requirements concerning product quality (e.g. dimensional tolerances, mechanical properties) in heavy steel plate production, require permanent enhancement of online control systems. Focusing on the main production steps “hot rolling” and “accelerated cooling”, the prediction of the local thermomechanical behavior applying physically based modeling techniques is of vital importance.

An ABAQUS model was developed for calculating the thermomechanical history during hot rolling and accelerated cooling processes. Constitutive laws describing the material behavior were implemented via user subroutines. For both production steps, material parameters of the rolled stock were identified from laboratory experiments. For the hot rolling process, the kinetics of softening (recrystallization and recovery) between consecutive rolling passes and work hardening within the roll gap are essential physical effects. During cooling the dynamics of phase transformation has to be accounted for. Simulation runs for the production cycle of one plate were automated using python scripting, including preprocessing, mapping of results between single pass models and process steps, output reporting and graphical visualization.

In a further step, customized algorithms based on the ABAQUS model were developed. This strategy reduces the computation time for the thermomechanical history to a few milliseconds with an accuracy equivalent to the ABAQUS model. The fast algorithms can be used in online routines for process control systems. In addition to the main process variables such as rolling force, torque and power consumption, the accurate prediction of the evolution of local temperature and deformation provides the opportunity to couple these variables with modeling concepts predicting the mechanical properties of the final product.

1. Introduction

Voestalpine Grobblech GmbH in Linz Austria, produces heavy steel plates and is a supplier in the highest quality segment. Examples for such products are line pipe grades (API X65 to X80 and even X100), especially for sour service applications, construction steels for offshore and deep water applications as well as high strength steels for yellow goods (mobile cranes and earth moving machines).

The main steps of heavy plate production include heating of steel slabs in a reheating furnace, hot rolling at the 4.2m reversing quarto stand (figure 1, top, left side), followed by in-line accelerated cooling (figure 1, top, right side) and a leveling process to obtain flat plates. In the finishing bay there are also machines for edge trimming, cutting to size and of course inspection lines for evaluation of surface and internal quality. Automation, exact guidance and control (“mechatronisation”) of the relevant production processes are standard requirements for achieving high productivity as well as minimum deviations from the desired product properties i.e. narrowest geometrical tolerances (thickness and flatness) and a minimized scatter in the mechanical properties (yield stress and tensile strength as well as toughness).

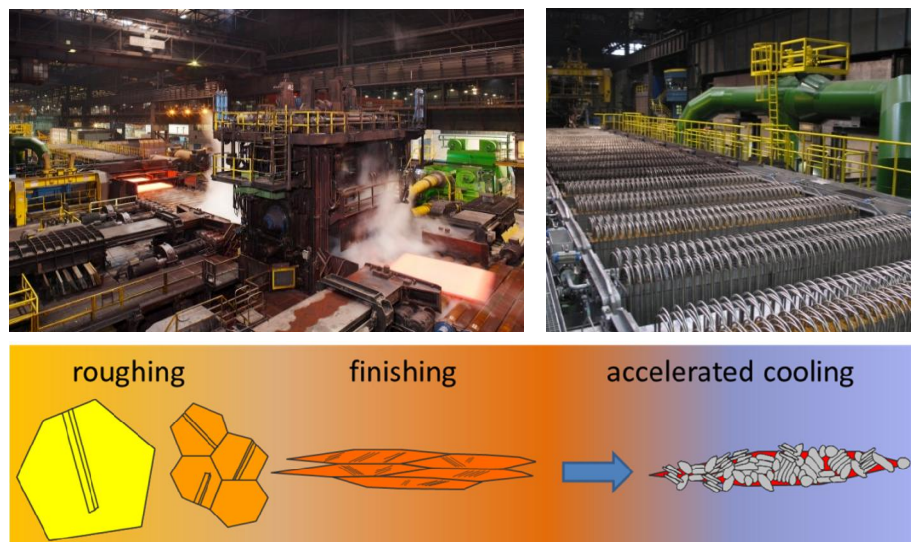


Figure 1. Hot rolling stand and accelerated cooling line with schematic microstructural evolution.

In combination with the accelerated cooling operation, the hot rolling process is the main process step of heavy plate processing. On the one hand, the plate dimensions (e.g. thickness 20 mm, length 12000 mm and width 2800 mm) will be formed from the slab dimensions (typically e.g. thickness 285 mm, length 1430 mm and width 1650 mm). On the other hand, the mechanical properties, particularly the toughness of the product is dependent on the complex history of

temperature and deformation. The challenge in modeling this process is that a coupling between the thermal history and the mechanical behaviour of the material during each rolling pass occurs. During rolling in the high temperature range above 1100°C, softening of every material point within the plate occurs. The austenite grain will recrystallize but will undergo a grain refinement (see figure 1 – roughing). Dependent on time, temperature and the content of the microalloying elements (especially niobium), the softening between the passes is more and more delayed with decreasing temperature and finally stops at a certain level (see figure 1 – finishing). This process is well known as thermomechanical rolling. Within this rolling regime the rolling force will increase from pass to pass not only as an effect of the decreasing temperature but also as the effect of the delayed softening of the material. During rolling in this lower temperature regime (e.g. 800°C), the austenite grains do not recrystallize, the grains will be elongated (conditioning of austenite), which has an influence on the mechanical properties, particularly the toughness of the final product. Therefore the parameter “accumulated plastic strain” ε_{acc} will be defined, which characterises the state of the conditioned austenite especially during finishing rolling. A second effect is the influence on the physics of phase transformation during the subsequent accelerated cooling process. A highly conditioned austenite grain will accelerate the phase transformation, which influences the strength of the material (see figure 1 – accelerated cooling). The important parameters are the cooling rate and the cooling stop temperature.

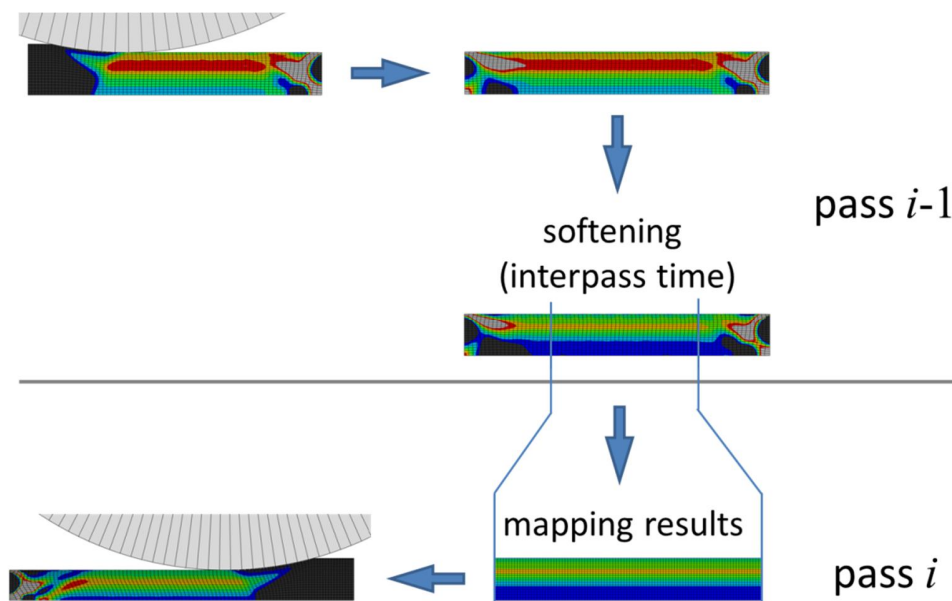


Figure 2. Schematic representation of the hot rolling model, the contour colors are representing the accumulated plastic strain.

2. ABAQUS hot rolling model

The hot rolling process is simulated with a 2D (plane strain) ABAQUS/Explicit thermo-mechanically coupled model using rigid work rolls. The input file for each pass is automatically created applying a python script using all necessary geometry data like plate thickness, reduction per pass, roll diameter, velocity of the roll stock collected from the process database of the hot rolling mill. Optimized meshing parameters are chosen also automatically within given boundaries (e.g. number of elements through thickness as well as along the contact zone, maximum aspect ratio). The point of interest for the simulation is the center point of the plate with respect to the length and width direction, called filet point. For this purpose a small section in length direction is defined. The length of this section is selected in an appropriate way such that a steady state regime is reached. The initial conditions for pass i are gained from the output of pass $i-1$ taking the results from the central region and mapping these results to the initial geometry of path i , see figure 2. For this purpose the results of the central region are averaged in length direction and mapped to the next section.

Simulation of the thermomechanical process means, that also the time between two consecutive passes is of vital importance to cover the physically relevant aspects of softening. The softening equations will be applied to a time step between two rolling passes. All thermal boundary conditions are gained from temperature measurements of numerous plates from the production line. The thermophysical properties like conductivity and specific heat were measured as a function of temperature for a wide spectrum of chemical compositions. To overcome inaccuracies due to the rigid work roll approximation, some Hitchcock-iterations are made, where an analytical formula estimates roll flattening on the basis of the calculated roll force. This estimation leads to a corrected diameter of the rigid work roll for a further iteration run (Hitchcock, 1935).

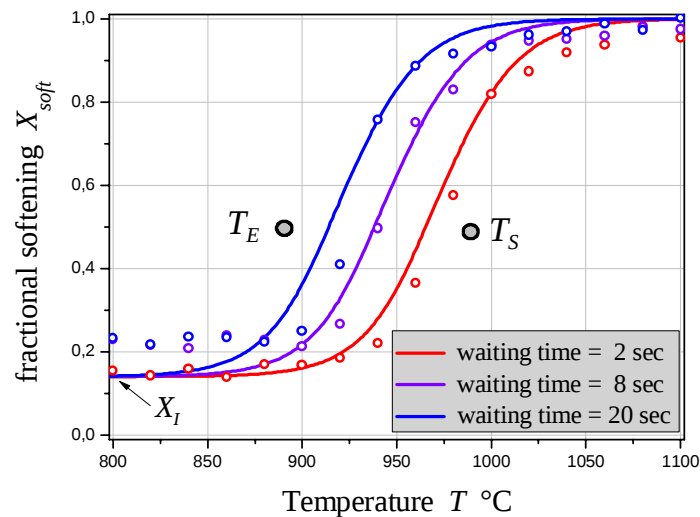


Figure 3. Fractional softening behavior for a medium Nb content of 0.025%.

The flow stress of the material is described by the analytical formula

$$\sigma_y(\varepsilon_{pass}, \dot{\phi}_{pass}, T) = \left[\sigma_{yX}(T) + (\sigma_{yS}(T) - \sigma_{yX}(T)) \left(1 - \left(\exp\left(-\frac{\varepsilon_{pass}}{\varepsilon_C}\right) \right)^n \right) \right] \left(\frac{\dot{\phi}_{pass}}{\dot{\phi}_{ref}} \right)^m,$$

with the initial yield stress σ_{yX} and the saturation flow stress σ_{yS} given by

$$\sigma_{yX}(T) = \sigma_{yS}(T) - X_{soft} (\sigma_{yS}(T) - (\sigma_{y0X} + c_0 \exp(-m_0(T - T_{ref})))),$$

$$\sigma_{yS}(T) = \sigma_{y0S} + c_S \exp(-m_S(T - T_{ref})).$$

ε_{pass} is the equivalent plastic strain at the integration point and coincides with the value **PEEQ** of the ABAQUS simulation. $\dot{\phi}_{pass}$ is an average (or “global”) value typical for the rolling pass and not to be confused with a local quantity. T is the current temperature and ε_C , c_0 , c_S , m_0 and m_S are constants dependent on the chemical composition of the steel grade. For identifying these parameters laboratory compression tests were performed. Additionally a recalculation procedure from roll force measurements of the hot rolling mill was applied (Parteder, 2012).

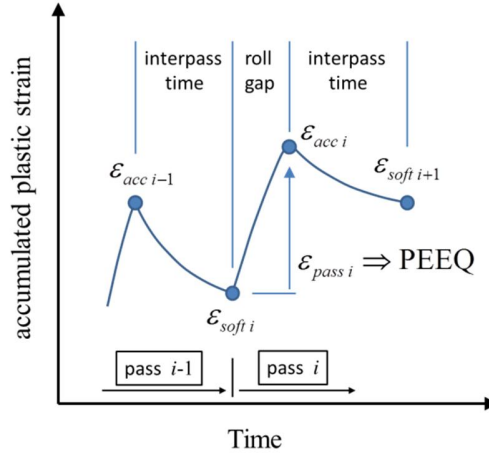


Figure 4. Concept of strain accumulation during hot rolling.

The initial yield stress σ_{yX} is dependent on the softening mechanism during the interpass time. This effect is modeled with the fractional softening X_{soft} of the material point. The evolution of X_{soft} can be described by the phenomenological equation in incremental form

$$dX_{soft} = \frac{c_t p (T_S - T_E) (1 - X_I) \exp(-c_t t + p((T_S - T_E) \exp(-c_t t) + T_E - T))}{(1 + \exp(p((T_S - T_E) \exp(-c_t t) + T_E - T)))^2} dt.$$

T_S indicates the start temperatures (for $X_{soft} = 0.5$ and $t \rightarrow 0$) and T_E ($X_{soft} = 0.5$ and $t \rightarrow \infty$). These parameters as well as the initial value of softening X_I are dependent on the accumulated plastic

strain ε_{acc} and the chemical composition, particularly on the niobium content in solid solution after reheating the slab. c_i and p are constants. All parameters are gained applying multi stage compression tests with four hits for a wide range of temperatures, waiting times and chemical compositions. The waiting time in the multistage compression test is equivalent to the interpass time in hot rolling. The basics of softening behavior in hot rolling were published by (Sellars, 1979). Figure 3 shows an example for the fractional softening behavior for a Nb-microalloyed steel with 0.025% Nb. The points represent the measured values from the compression tests. The fractional softening values are determined following the offset method (Li, 1996). Full lines are indicating the solution of equation dX_{soft}/dt for isothermal softening.

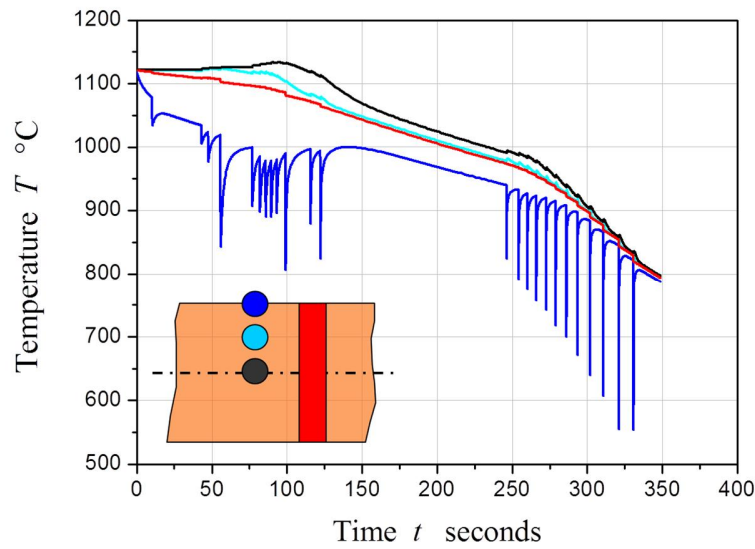


Figure 5. Example of the temperature history of three specific points and the average value through thickness for the hot rolling process.

The accumulated plastic strain ε_{acc} is a variable specifying the plastic strain level accumulated during the rolling passes (ε_{pass}) considering softening phenomena during the interpass times. Please note that the magnitude of ε_{acc} may also decrease due to softening effects. In that respect it must not be confused with what is called ‘equivalent plastic strain’ in ABAQUS. The accumulated plastic strain value for every material point is given by the equations

$$\varepsilon_{acc\ i} = \varepsilon_{soft\ i} + \varepsilon_{pass\ i},$$

$$\varepsilon_{soft\ i} = \varepsilon_{acc\ i-1} \lambda_i(X_{soft}),$$

(Ouchi, 1980) and (Bai, 1996). ε_{soft} is the plastic strain remaining after softening during the interpass time. The function $\lambda_i(X_{soft})$ varies from 0 to 1 and links the fractional softening value

with the strain accumulation. The value 0 indicates that the material undergoes full softening between two passes, whereas the accumulated plastic strain will be 0 and the value 1 means that there is no softening during the interpass time. The flow stress formulation and the softening theory is implemented via the user routines `vuhard` and `vusdfld`. The schematic representation of the evolution of strain accumulation is shown in figure 4.

Figure 5 shows the results of the simulation of the temperature history of a high strength steel (thickness 18 mm, 23 rolling passes) for three different points through thickness and the corresponding volume average value. The rolling model applies symmetry conditions with respect to the plate's mid surface. In figure 6 the evolution of the accumulated plastic strain is depicted. The graphical representation is done automatically using the python script for extracting the results from all odb files.

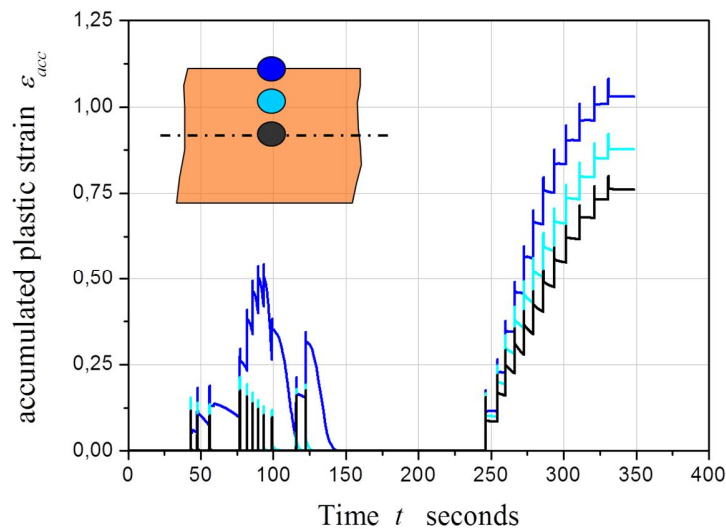


Figure 6. Example of the history of the accumulated plastic strain of three points.

3. ABAQUS accelerated cooling model

After thermomechanical rolling the plates are cooled down within the accelerated cooling line to a predefined cooling stop temperature. ABAQUS/Standard is used for modeling the thermo-mechanical history of the plates during cooling. The initial temperature field is taken from the hot rolling model. The boundary conditions are defined as heat transfer coefficients for the top and the bottom surface respectively. The experimental procedure to attain the parameters for the function $HTC(T_{surf}, \text{water impingement})$ is shown in figure 7 (Horsky, 2009). Due to the different cooling conditions between top and bottom the plates are susceptible to warping. During cooling the material is transformed from the austenitic phase to the product phase (this can be ferrite, pearlite, bainite or martensite). Together with the phase transformation kinetics the mechanical response during phase transformation is of utmost importance to get reliable results concerning the residual

stress state as well as the strain state after cooling. The model is set up as a generalized plane strain model. A comparison with a full 3D model shows that this modeling assumption is a reasonable simplification for the center point of the plate (Eßl, 2013), see figure 8.

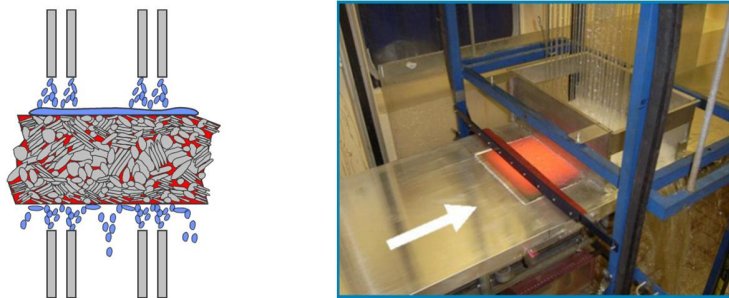


Figure 7. Laboratory equipment for measuring the heat transfer coefficient.

The total strain of a material point is gained by applying decomposition into five components

$$\varepsilon = \varepsilon_{el} + \varepsilon_{pl} + \varepsilon_{th} + \varepsilon_{vol} + \varepsilon_{trip}.$$

ε_{el} is the elastic strain, ε_{pl} , the classic plastic strain, ε_{th} , the thermal expansion strain, ε_{vol} , the transformation dilatational strain and ε_{trip} , the transformation plasticity strain. For the elastic as well as the plastic strain the classical theories are employed. It should be noted that the flow stress σ_y has to be provided as a function of temperature for the austenitic phase as well as the product phase (Eßl, 2011). The first one is defined as a function of the accumulated plastic strain and is equivalent to the flow stress coming from the hot rolling model, the second one is gained from tensile tests of the final product at different temperatures. During phase transformation a linear rule of mixture is applied for all relevant material descriptions, including thermal strain, heat transfer coefficient and yield stress.

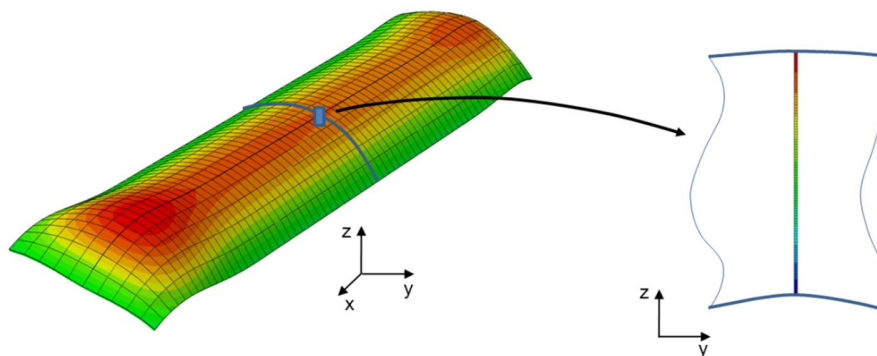


Figure 8. 3D (continuum shell) model for the cooling line and the generalized plain strain model, the contour colors are indicating the z-coordinate (scaled).

The fraction of the product phase is given as an extended Avrami formulation in incremental form which can be integrated in an explicit way (Antretter, 2010 and Hunkel, 1999)

$$dX_{trans} = n(\dot{T}) k^{1/n(\dot{T})} \frac{1 - X_{trans}}{t_s(T, \varepsilon_{acc})} \left[\ln \left(\frac{1}{1 - X_{trans}} \right) \right]^{\frac{n(\dot{T})-1}{n(\dot{T})}} g(\dot{T}) dt.$$

All parameters and functions $n(\dot{T})$, k and $g(\dot{T})$ and $t_s(T, \varepsilon_{acc})$ are defined in a way to achieve a good agreement with the results from the dilatometer experiments. The release of heat energy q_{trans} during phase transformation can be implemented with an enthalpy formulation

$$dq_{trans} = \Delta H_{aust \rightarrow prod}(T) dX_{trans}.$$

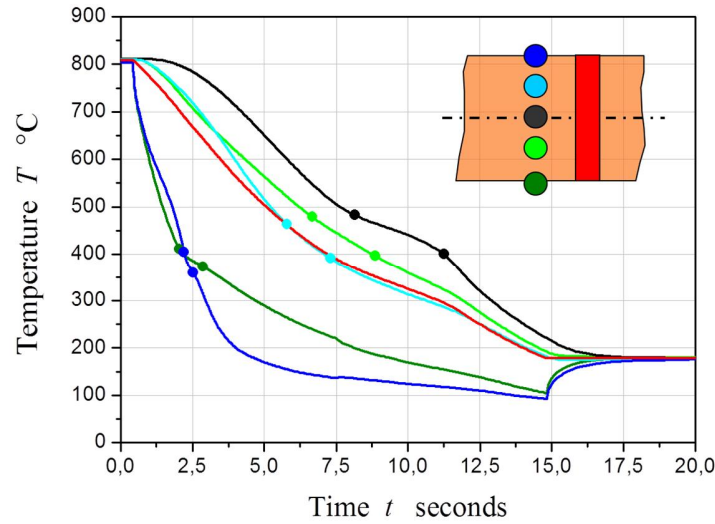


Figure 9. Temperature history for five specific points of the plate and the average value through thickness for the accelerated cooling process.

The trip strain increment is given by (Leblond, 1989)

$$d\varepsilon_{trip} = \frac{3}{2} K S_D f'(X_{trans}) dX_{trans},$$

$$f(X_{trans}) = (2 - X_{trans}) X_{trans},$$

$$f'(X_{trans}) = 2(1 - X_{trans}),$$

see (Denis, 1983). The constant K is the Greenwood Johnson coefficient and S_d is the deviatoric part of the stress tensor. The theory is implemented via the subroutines **usdfld** for the phase transformation kinetics, **hetval** for the heat volume flux during phase transformation, **creep**

for definition of the transformation induced plasticity term, **uexpan** for the thermal expansion for the different phases as well the volume strain due to phase transformation. The material parameters for the transformation kinetic law are the results of a huge number of dilatometer experiments. On the one hand the chemical compositions as well as the cooling rates were varied to identify the transformation start and stop temperatures. On the other hand different levels of predeformation before cooling (e.g. at a temperature of 800°C) were applied to identify the influence of the accumulated plastic strain level from hot rolling on the transformation behavior. The Greenwood Johnson coefficient was verified with the dilatometer experiments at different load levels during phase transformation.

Figure 9 shows the result of the temperature calculation of the cooling model for five selected points in thickness direction. The phase transformation start ($X_{trans} = 0.05$) and end ($X_{trans} = 0.95$) are indicated as dots. The release of heat during transformation can be seen as a result of a decrease in the cooling rate in the center of the plate.

4. A customized fast algorithm for calculating the thermomechanical history for heavy plate production

In modern automation and control systems physically based modeling becomes more important. An accurate description of the thermomechanical behavior of a material point allows process guiding in narrow ranges, and thus the minimization of the variance of the mechanical properties of the final products. For this purpose the algorithms of the ABAQUS models were implemented to a customized algorithm applying some simplifications but with almost no loss of accuracy.

4.1 Customized hot rolling model

The customized fast algorithm solves the heat equation on a rectangular grid using the Finite Element Method (FEM). The rolling process itself enters the simulation by applying the corresponding boundary conditions. For a given velocity profile, heat transfer due to radiation and convection to environment is alternating with heat transfer due to roll contact. This massive change in the boundary condition (and in the respective heat transfer coefficient) leads to a square root pattern of the top temperature as a function of time. At the temperature minimum the roll loses the contact to the point and the reheating (change in boundary condition) starts. Figure 10 shows an example for the surface temperature history during roll contact and reheating for one pass. Please note that the temperature history of the surface node strongly depends on the discretization. The result shows that after a short reheating time the temperature of the ABAQUS model and the customized model are almost equal. A perfect accordance can be detected for all points within the plate as well as the volume average temperature throughout the whole pass schedule.

During contact of the plate with the work roll, the geometry of the plate changes, which would require a continuous remeshing of the plate if the problem is formulated in spatial (Eulerian) coordinates. However, if a reference (Lagrangian) coordinate system of the initial (undeformed) geometry is used, the same grid can, in principle, be used for the complete rolling process. The heat equation, of course, has to be formulated in the somehow unnatural Lagrangian system, leading to anisotropic heat conductivity with high(er) values in the thickness and low(er) values in

the length and width direction. Figure 11 shows the concept of the customized model. The vertical arrow denotes the position at time t (contact with work roll). The horizontal arrow shows the direction the plate is going through the rolling mill. The red area is the already rolled part of the plate. The heat conduction is scaled and becomes larger in z -direction (due to the thickness reduction of the plate) and smaller in the lateral dimension (due to lengthening of the plate). This modeling concept allows the simulation of the whole plate including the thermal situation of the head and tail end of the plate within appropriate calculation times (approximately 20 sec) and is applicable for offline purposes. For online application only the filet point of the plate has to be calculated which reduces the time needed for the whole pass schedule to approximately 20 msec. The time consumption for the ABAQUS model is approximately 2 hours, please note that the most time consuming procedure of the ABAQUS calculation is not the equation solving but extracting and mapping the results from one pass to the next one.

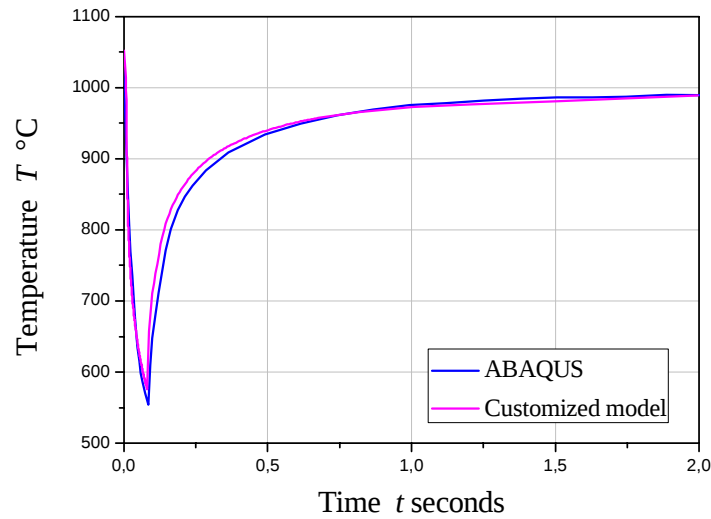


Figure 10. Top temperature of a point of the plate for one pass.

One essential effect on the temperature profile through the plate thickness is the conversion of deformation energy to thermal energy. In ABAQUS this effect is “automatically” done by the coupled simulation. In the present work the fully coupled model is replaced by an efficient strategy to calculate the correct heat source in dependence on the thickness coordinate. This is done by the determination of the distribution of the plastic strain through thickness in advance applying the ABAQUS model. The calculations were done for the whole range of possible rolling parameters. In flat rolling two independent quantities are characterizing the geometry of the roll gap, the pass strain φ_{pass} and the shape factor S

$$\varphi_{pass} = \ln \frac{h_0}{h_1},$$

with h_0 and h_1 the entry and the exit thickness resp., and

$$S = l_d / h_m ,$$

$$l_d = \sqrt{R \Delta h} .$$

R is the radius of the work roll and Δh is the difference of the entry thickness and the exit thickness and h_m the average value of entry and exit thickness of the roll stock.

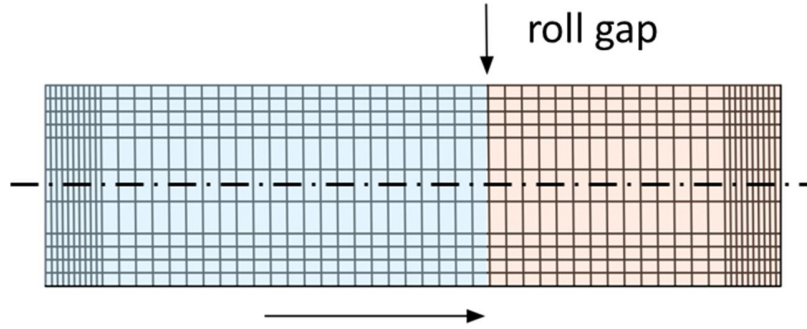


Figure 11. Schematic display of the customized model for the rolling process.

Isothermal ABAQUS simulations were done for one pass within the whole range of possible parameters φ_{pass} and S including different friction conditions (friction coefficient μ with τ_{max} dependent on $\sigma_{yS}(T_{en})$, T_{en} is the surface temperature at roll gap entry). The results for **PEEQ** were normalized with respect to the thickness direction ($z_{norm} = 0$ is the plate center, and $z_{norm} = 1$ is the plate surface) using the following formulation

$$SH_{FE}(z_{norm}) = \frac{PEEQ(z_{norm})}{1.15 \varphi_{pass}} - 1 .$$

$SH_{FE}(z_{norm})$ is called the shearing part of deformation. The ABAQUS result $SH_{FE}(z_{norm})$ can be approximated by a polynomial equation of the form

$$SH_{Fit}(z_{norm}) = a z_{norm}^2 + b z_{norm}^3 + c z_{norm}^4 .$$

The coefficients a , b and c can be described as functions of the pass strain, the shape factor and the friction condition. Applying this procedure it is possible to calculate the distribution of plastic strain through thickness in a very fast and easy way,

$$\varepsilon_{pass Fit}(z_{norm}) = 1.15 \varphi_{pass} (1 + SH_{Fit}(z_{norm}, \varphi_{pass}, S, \mu)) .$$

Figure 12 shows an example of $SH_{Fit}(z_{norm})$ for different pass strains and shape factors for the friction coefficient 0.3 in comparison to $SH_{FE}(z_{norm})$. The function is used for calculation of the accurate heat source due to deformation and is also the basis for the local evolution of the accumulated plastic strain. Prediction of the rolling force is done with a similar approach. In this case the influence of the roll gap geometry is characterized by a geometry factor depending on φ_{pass} , S and μ (Parteder 2012).

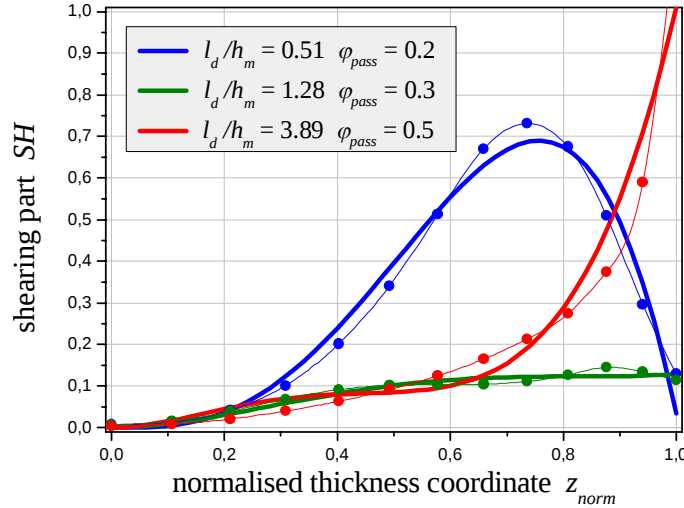


Figure 12. Approximation of the shearing part of deformation (SH_{FE} – dotted line, SH_{Fit} – full line) for different shape factors and pass strains and a friction coefficient of 0.3.

4.2 Customized accelerated cooling model

In case of the cooling model the thermal mechanical coupling effects are negligible. The algorithm of the customized model can be applied for the cooling model in a similar way. The transformation kinetics described above has been integrated into the thermal code. The computing time for the filet point of the plate is reduced to a few msec. The code was already implemented as an online routine to the control system of the accelerated cooling device. The thermal model is used to calculate the set up values (water impingement on top and bottom side, velocity of the plate during cooling) to reach the predefined values like cooling stop temperature and cooling rate.

The mechanical calculation predicting the warping of the plate during and after cooling is done by a special code replacing the generalized plane strain model described above. This strategy is based on a single one dimensional element with only three global degrees of freedom that performs equally well as the ABAQUS model (Eßl 2013).

5. Summary and Outlook

Process models for hot rolling and accelerated cooling simulations were set up with ABAQUS. Python scripting was used for creating input files automatically, mapping results between rolling passes and report history data for specific points of interest. With this procedure the thermo-mechanical history for individual heavy plates can be calculated in an appropriate time. An efficient customized modeling strategy was developed decreasing the computation time significantly. Hence the physically based modeling technique is applicable to online routines for control systems of the heavy plate production. A future step will be the coupling of the

thermomechanical model with numerical tools for predicting the mechanical properties of the plates after processing (strength and toughness). This opens the opportunity to set up a CAQC (computer aided quality control) system adapting specific target values during processing. With this method a further decrease of the variance of mechanical properties will be achieved.

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