

Using Abaqus to understand the orthotropic material interactions that enable zero thermal expansion laminates to be designed.

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Abstract: Poisson's ratio effects, to a large extent, control the thermal expansion characteristics of angle-ply laminates. It is possible to design laminates, that have been manufactured from some highly orthotropic laminae, to exhibit a zero coefficient of expansion along one of its directions, even though none of the expansion coefficients of the laminae themselves are negative. It is often mistakenly believed that at least one of the expansion coefficients has to be negative in order for this effect to be achieved. However, it is the internal coupling of the individual expansion coefficients via the very high laminate Poisson's ratio that actually enables this. The paper illustrates the effect via a simple Abaqus model and in so doing graphically cements the idea in the minds of the FEM analyst.

Keywords: *Abaqus, CTE, Thermal expansion, Orthotropic, Lamina.*

1. Introduction

Because composite materials have properties that are not equal in all directions i.e. they are anisotropic, they may be used very economically as load-bearing material in structures. This economy of usage is achieved by aligning the principal material directions with the anticipated principal loading directions within the structure. Engineers have become quite accustomed to using these directional properties of the material to improve their designs. What is generally not realized, is that the anisotropic nature of composites enable other design possibilities for these materials that are not available with the more traditional materials. Engineers who have many years experience in designing with composite materials still tend to think along traditional ways and consequently are not aware of these other possibilities and the subtleties in effective design with composites. One unique property of composites that is overlooked in this way is the wide variation shown by Poisson's ratio [1].

The ancient Egyptians realized that wood, a naturally occurring composite, could be made more resistant to swelling and thermal expansion by suitably arranging individual laminae to form a laminate [2]. It is possible to use this principle using modern, man-made fibres, to design laminates that will exhibit extremely low coefficients of thermal expansion.

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In this paper, thermal expansion theory of laminates is reviewed and from this, the underlying principle governing the attainment of zero thermal expansion of thin, **balanced symmetric composite laminates** is developed. In so doing, the paper concentrates on extracting the information from theory which is of use to one at the design, rather than the analysis stage; it concentrates on highlighting the **mechanism** at work and to cement the idea, a very simple Abaqus model is used. The purpose of the examples used to illustrate some design cases, is to leave a mental picture of the interactions which need to be understood but which are so easily masked by the theory.

2. Laminate thermal expansion theoretical background

The summary of the theory follows [2,3] – beware of the slightly different notation used here for $[a]$ and that in [2] where it is $[A']$ and also the potential confusion of $[a]$ with $[a^*]$ in [2]. The strain state $\{\varepsilon\}$ within individual laminae is due to mechanical effects and expansion effects; for the latter we consider only **thermal** expansion effects. The **restrained** expansion of such a lamina, subject to a uniform temperature change, ΔT is given by:

$$\{\varepsilon\} = \{\varepsilon\}_m - \{\alpha\}\Delta T$$

Equation 1

where $\{\alpha\}$ is a vector of thermal expansion coefficients. The stress state, $\{\sigma\}$ of such a lamina is given by:

$$\{\sigma\} = [Q]\{\varepsilon\}_m - \{\Gamma\}\Delta T$$

Equation 2

where $[Q]$ is the lamina plane stress stiffness matrix (symmetric) and $\{\Gamma\}$, a vector of thermal influence coefficients.

We have:

$$\{\Gamma\} = \begin{Bmatrix} Q_{11}\alpha_1 + Q_{12}\alpha_2 \\ Q_{12}\alpha_1 + Q_{22}\alpha_2 \\ 0 \end{Bmatrix}$$

Equation 3

Equation 3 represents the thermal influence coefficients in the material axis system; the coefficients are the stress induced per unit temperature increase due to restrained thermal expansion. They transform to the global directions similar to the way that the mechanical stresses do. Because of symmetry, we may replace the terms lying below the diagonal by their mirror terms, lying above the main diagonal – we retain only the upper diagonal terms in the equations.

We have:

$$\{\Gamma\}_{xy} = \begin{Bmatrix} m^2(Q_{11}\alpha_1 + Q_{12}\alpha_2) + n^2(Q_{12}\alpha_1 + Q_{22}\alpha_2) \\ n^2(Q_{11}\alpha_1 + Q_{12}\alpha_2) + m^2(Q_{12}\alpha_1 + Q_{22}\alpha_2) \\ mn(Q_{11}\alpha_1 + Q_{12}\alpha_2) - mn(Q_{12}\alpha_1 + Q_{22}\alpha_2) \end{Bmatrix}$$

Equation 4

with $m = \cos \theta$ and $n = \sin \theta$ and the positive direction of rotation is as shown in Figure 1.

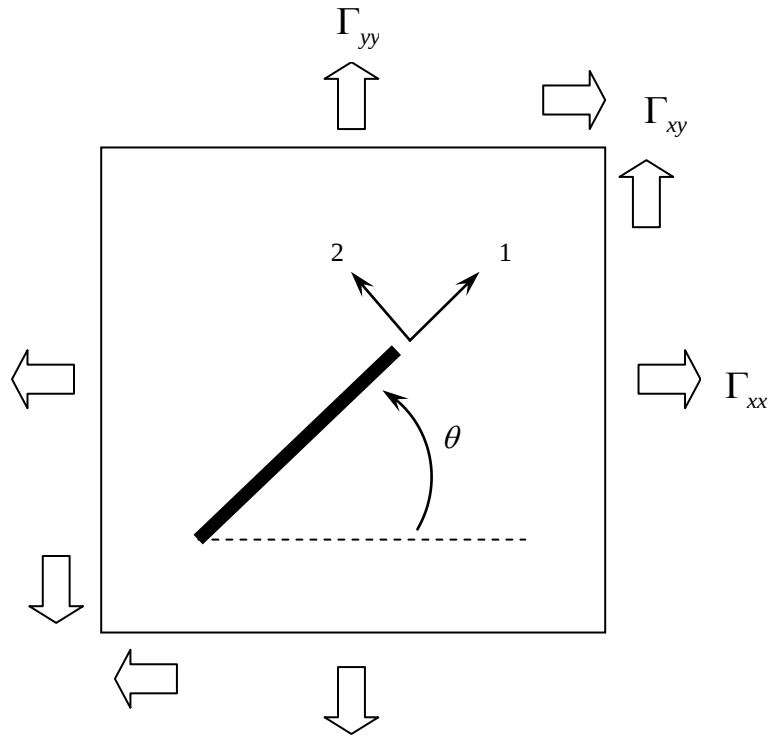


Figure 1 Lamina thermal influence coefficients.

The lamina thermoelastic relations may be extended to laminates by considering a force and moment balance, about a reference surface and applying Kirchoff's assumptions for thin plates as is customarily done in Classical Lamination Theory (CLT):

$$\begin{aligned}\{N\} &= \int \{\sigma\}_{xy} dz \\ \{M\} &= \int \{\sigma\}_{xy} z dz\end{aligned}$$

Equation 5

where the external $\{N\}$ and $\{M\}$ vectors are forces and moments per unit width. Including thermal effects:

$$\begin{aligned}\{N\} &= \int \left([\bar{Q}] \{ \varepsilon^0 \} + z \{ k \} \right)_{xy} - \{ \Gamma \}_{xy} \Delta T dz \\ \{M\} &= \int \left([\bar{Q}] \{ \varepsilon^0 \} + z \{ k \} \right)_{xy} - \{ \Gamma \}_{xy} \Delta T z dz\end{aligned}$$

Equation 6

where $[\bar{Q}]$ denotes the lamina plane stress stiffness in the global (transformed) direction, $\{ \varepsilon^0 \}$ is the strain in the reference surface and $\{ k \}$, the curvature; the xy-subscript denotes that these quantities are to be in the global axis system since the external loading is in this axis system.

Rearranging:

$$\begin{aligned}\{N\} &= [A] \{ \varepsilon^0 \}_{xy} + [B] \{ k \}_{xy} + \{ N \}_t \\ \{M\} &= [B] \{ \varepsilon^0 \}_{xy} + [D] \{ k \}_{xy} + \{ M \}_t\end{aligned}$$

Equation 7

where

$$\begin{aligned}\{N\}_t &= - \int \{ \Gamma \}_{xy} \Delta T dz \\ \{M\}_t &= - \int \{ \Gamma \}_{xy} \Delta T z dz\end{aligned}$$

Equation 8

and

$$[A],[B],[D] = \int [\bar{Q}](1, z, z^2) dz$$

Equation 9

Now, for a uniform temperature change, ΔT is constant and may be taken outside the integration sign and for the case of a laminate composed of a number of discrete lamina, equation 8 becomes:

$$\begin{aligned} \{N\}_t &= -\Delta T \sum_{i=1}^n \{\Gamma\}_{xy}^i (z_i - z_{i-1}) \\ \{M\}_t &= -\frac{1}{2} \Delta T \sum_{i=1}^n \{\Gamma\}_{xy}^i (z_i^2 - z_{i-1}^2) \end{aligned}$$

Equation 10

The laminate **free** thermal expansion and curvature coefficient vectors, respectively, are defined as follows:

$$\begin{aligned} \{\alpha\}_{xy} &= \{\epsilon^0\}_{xy} \frac{1}{\Delta T} \\ \{\delta\}_{xy} &= \{k\}_{xy} \frac{1}{\Delta T} \end{aligned}$$

Equation 11

Equation 11 may be solved, after suitable manipulations, to yield:

$$\begin{aligned} \{\alpha\}_{xy} &= \frac{1}{\Delta T} [L_1]^{-1} ([B][D]^{-1} \{M\}_t - \{N\}_t) \\ \{\delta\}_{xy} &= \frac{1}{\Delta T} [L_2]^{-1} ([B][A]^{-1} \{N\}_t - \{M\}_t) \end{aligned}$$

Equation 12

with $[L_1] = [A] - [B][D]^{-1}[B]$ and $[L_2] = [D] - [B][A]^{-1}[B]$.

Equation 12 represents the thermal expansion and curvature vectors for **general laminates** per degree uniform thermal change i.e. the thermoelastic description of the laminate only, since the ΔT terms cancel – such general laminates will both expand/contract and curl/twist.

3. Design principles of a balanced, symmetric laminate with zero thermal expansion coefficient.

If the laminate is *symmetrically-constructed about the reference plane from identical thickness plies of the same material*, then Equation 12 may be simplified, by virtue of no in-plane/bending coupling and no thermal moment load (since $[B] = 0$, and $\{\Gamma\}_{xy} = \{\Gamma\}_{xy}^i$), to:

$$\{\alpha\}_{xy} = \frac{-1}{\Delta T} [A]^{-1} \{N\}_t$$

Equation 13

So, such special-case balanced laminates undergo only thermal expansion with no curvature changes. The matrix $[A]^{-1}$ is the laminate in-plane compliance matrix, $[a]$.

We want one of the thermal expansion coefficients of the laminate to be zero (say that along the x-axis). Examination of equation 13 shows the factors influencing this are the laminate compliance and the thermal load vector per unit temperature increase. We have:

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} = \frac{-1}{\Delta T} \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}_t$$

Equation 14

Substituting from equation 10 for the thermal load vector and putting $\alpha_x = 0$, one obtains:

$$\sum_{i=1}^n a_{11} (m^2 \Gamma_1^i + n^2 \Gamma_2^i) h_i + \sum_{i=1}^n a_{12} (n^2 \Gamma_1^i + m^2 \Gamma_2^i) h_i + \sum_{i=1}^n a_{16} (mn \Gamma_1^i - nm \Gamma_2^i) h_i = 0$$

Equation 15

One way of easily achieving this, is to make $a_{16} = a_{26} = 0$ by arranging the laminae in alternating positive and negative ply angle pairs i.e. making it balanced. Substituting for the trigonometric identity $n^2 = 1 - m^2$ and noting that the thickness, $h_i = (z_i - z_{i-1})$ of every ply is equal we have $h_i = h$ giving:

$$\sum_{i=1}^n a_{11} \left(m^2 \Gamma_{11}^i + (1-m^2) \Gamma_{22}^i \right) + \sum_{i=1}^n a_{12} \left((1-m^2) \Gamma_{11}^i + m^2 \Gamma_{22}^i \right) = 0$$

Equation 16

Examination of equation 16 shows that the number of plies required at each orientation in the laminate will depend on the relative magnitudes of Γ_{11}^i and Γ_{22}^i i.e. this determines the weighting of each ply. If $\Gamma_{11}^i = r \Gamma_{22}^i$ and by virtue of all $\{\Gamma\} = \{\Gamma\}^i$ then another simplification gives:

$$\sum_{i=1}^n a_{11} \left(m^2 r + (1-m^2) \right) + \sum_{i=1}^n a_{12} \left((1-m^2) r + m^2 \right) = 0$$

Equation 17

Note that it is not simply the ratio of the terms between the thermal influence coefficients which is important, but this ratio r weighted with the laminate compliance terms. The laminate compliance is in itself a function of the individual lamination angles and total number of laminae. It is not easy to intuitively arrange the lamination scheme so that equation 17 is satisfied, as we have one equation and a multitude of solutions for each m_i . One possible solution to the problem is to constrain all the laminae to have the same lamination angle $\pm \theta$ and then equation 17 reduces to:

$$a_{11} (m^2 r - m^2 + 1) = -a_{12} (-m^2 r + m^2 + r)$$

Equation 18

In general, the compliance terms are given by:

$$a_{11} = (A_{22}A_{66} - A_{26}^2) \frac{1}{|A|}, a_{12} = (A_{16}A_{26} - A_{12}A_{66}) \frac{1}{|A|}$$

Equation 19

Since the laminate consists of many $\pm \theta$ laminae i.e is balanced, the in-plane direct strain to shear strain coupling terms disappear and equation 18 may be rearranged as:

$$\frac{A_{22}}{A_{12}} = \frac{-m^2 r + m^2 + r}{m^2 r - m^2 + 1}$$

Equation 20

This is just the inverse of the laminate major Poisson's ratio and so, the equation to be satisfied is:

$$\mu_{xy} = \frac{m^2 r - m^2 + 1}{-m^2 r + m^2 + r}$$

Equation 21

Equation 21 reinforces the idea that the attainment of a zero coefficient of thermal expansion is dependent on both laminate design (via the lamination angles and thicknesses of the plies) and on the correct choice of lamina materials. A similar expression results for the case where thermal expansion along the y-axis is zero.

4. An Abaqus model to illustrate the principle

We create a simple, single element model of unit planar dimensions with which to illustrate the points made earlier. A linear interpolation function i.e. 4-noded shell element is used, which makes all displacements linear and thus easy to interpret; boundary conditions are such to give only planar response. To compute the effective Poisson's ratio, the model is given a unit displacement on the edge defined by the element's 2- and 3-nodes; the Poisson's ratio is then simply the negative of the displacement in the y-direction. To compute the thermal expansion coefficient, a unit temperature rise is given for all the nodes; the CTE, along the x-axis, is then the displacement in the x-direction; we use a script file, reproduced below, to drive the analysis. In the main input files, T300_Narmco.inp and T300_Narmco_CTE.inp, only the step definitions differ.

```
##
## Python Scripting to combine the results of the Poisson/CTE analyses
##
## Create the parametric study
##
##angle_variable = ParStudy(par='angle', name='T300_Narmco')
angle_variable = ParStudy(par='angle', name='T300_Narmco_CTE')
angle_variable.define(CONTINUOUS, par='angle', domain=(0., 90.))
angle_variable.sample(NUMBER, par='angle', number=31)
angle_variable.combine(MESH)
##
## Generate a number of input files, with a different angle in every case and
execute them
##
##angle_variable.generate(template='T300_Narmco')
angle_variable.generate(template='T300_Narmco_CTE')
angle_variable.execute(ALL)
##
## Where to find the data
##
```

```

angle_variable.output(file=ODB, step=1, inc=LAST)
##
## What to combine i.e to gather and then where to send it for tabular output
##
angle_variable.gather(results='n3_u', variable='U', node=3, step=1)
##angle_variable.report(XYLOT,par='angle', results=('n3_u.2'),file="Poisson.txt")
angle_variable.report(XYLOT, par='angle', results=('n3_u.1'),file="CTE.txt")
##

```

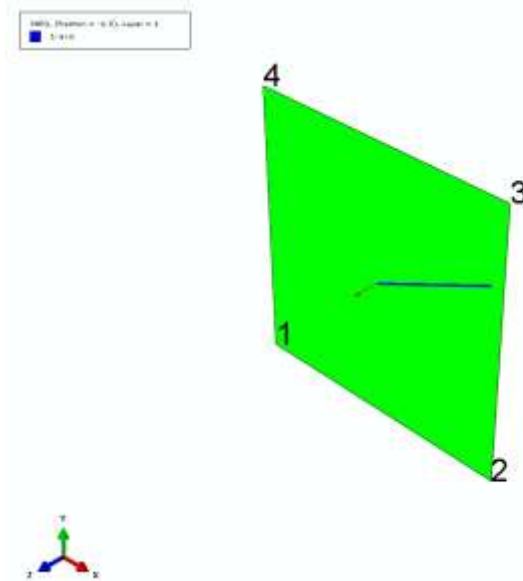


Figure 2 Single element, showing fibre direction aligned at 24.9° to x-direction, for maximum Poisson-effect in T300/5208 laminate.

4.1 Abaqus model-definition keywords

```

*heading
Simple problem to check orthotropic behaviour
*parameter
**angle = 24.9
angle = 1.91
minus_angle = -1.0*angle
*node,nset = all_nodes
1,0.0,0.0
2,1.0,0.0
3,1.0,1.0

```

```

4,0.0,1.0
*nset,nset = node_3
3
*material, name = T300_Narmco_5208
*elastic,type = lamina
181.0E9,10.3E9,0.28,7.17E9,5.0E9,5.0E9
*expansion,type = ortho
0.02E-06,22.5E-06,22.5E-06
*element,type = s4r,elset = shell
1,1,2,3,4
*shell section,elset = shell,composite
0.5E-03, ,T300_Narmco_5208,angle
0.5E-03, ,T300_Narmco_5208,minus_angle
*orientation,name = angle,definition = offset to nodes
2,4,1
3,<angle>
*orientation,name = minus_angle,definition = offset to nodes
2,4,1
3,<minus_angle>
*boundary
all_nodes,3,6,0.0
**
1,1,2,0.0
4,1,1,0.0

```

4.2 Abaqus step-definition for computing Poisson's ratio

```

*step
Composite laminate analysis - single element, defined displacement
*static
0.1,1.0
*boundary
2,1,1,1.0
3,1,1,1.0
*output,field
*node output,variable = preselect
*end step

```

4.3 Abaqus step-definition for computing expansion

```

*step
Composite laminate analysis - single element, unit temperature change
*static
0.1,1.0
*temperature
all_nodes,1.0
*output,field
*node output,variable = preselect
*end step

```

4.4 Results

To illustrate the dependence of the thermal expansion coefficient on elastic moduli, four balanced-symmetric $\pm \theta$ laminates designed from carbon fibre, aramid fibre, glass fibre and boron fibre embedded in an epoxy matrix are considered. The thermoelastic constants for the four materials appearing in Table 1 are taken from Tsai [4], with r being computed from these tabulated properties.

The left and right sides of equation 21 are plotted as separate curves for each of these laminates in turn in figures 3 to 6 [5]. Any points where these curves intersect represent solutions to the zero coefficient of thermal expansion laminate problem (along the x-axis).

The T300/5208 laminate has two solutions viz. at $\pm 1.91^\circ$ at $\pm 41.71^\circ$. The Kevlar/epoxy laminate has only one solution at $\pm 43.93^\circ$. These laminates have very similar Poisson's ratios around $\pm 40^\circ$ but widely differing values of r due to the negative coefficient of thermal expansion of the Kevlar/epoxy along the fibre direction. So, in spite of this large difference in driving thermal load, both of these laminates are able to generate no strain along the x-axis. This is because *their thermoelastic coefficients enable their respective Poisson's ratios to have a large enough range to achieve a balance with the thermal load according to equation 21.*

Examination of Figure 4 shows that for the case of Kevlar/epoxy, the ratio of the driving thermal load in the global direction asymptotes at $\pm 58.14^\circ$. At this point, the thermal influence coefficients in the material axis system resolve to only the x-direction.

The glass/epoxy laminate does not have any solution at all because the lamina is not orthotropic enough to generate an appreciable Poisson effect when an off-axis laminate is constructed as shown in Fig 5. The B/5505 laminate is also not capable of producing a solution, although it comes very close as can be seen in Fig 6. A small reduction in the transverse stiffness of the latter material will indeed enable a zero coefficient of thermal expansion to be achieved.

Table 1 Representative materials' thermoelastic constants.

	T300/5208	K49/epoxy	Glass/epoxy	B/5505
E_{11} , GPa	181	76	38.6	204
E_{22} , GPa	10.3	5.5	8.27	18.5
μ_{12}	0.28	0.34	0.26	0.23
G_{12} , GPa	7.17	2.3	4.14	5.59
α_1 , /°C	2.00E-08	-4.00E-06	8.60E-06	6.10E-06
α_2 , /°C	2.25E-05	7.90E-05	2.21E-05	3.03E-05
r	0.2958	-0.3651	1.886	2.341

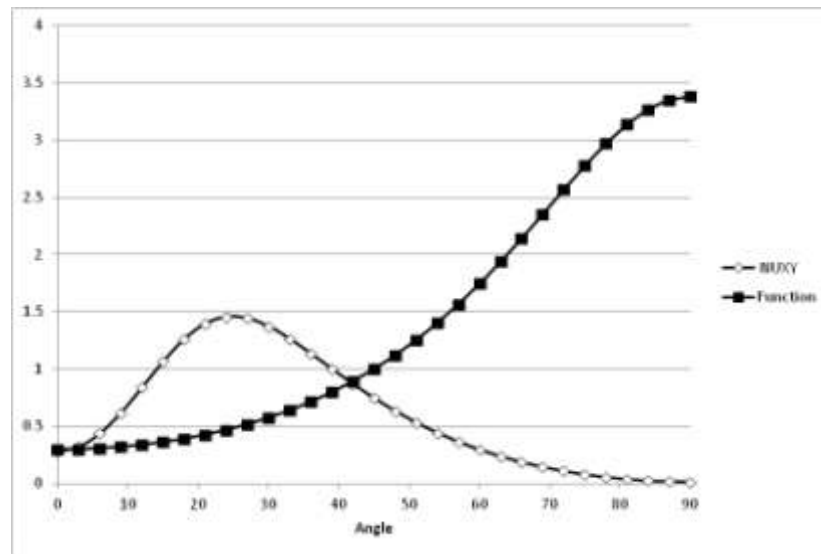


Figure 3 Laminate thermoelastic interaction diagram for T300/5208.

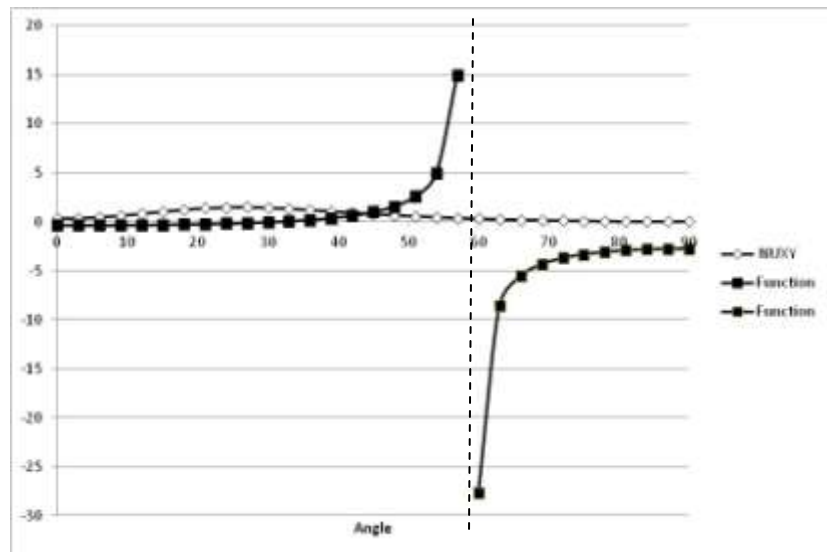


Figure 4 Laminate thermoelastic interaction diagram for Kevlar/epoxy, asymptote at ± 58.14 .

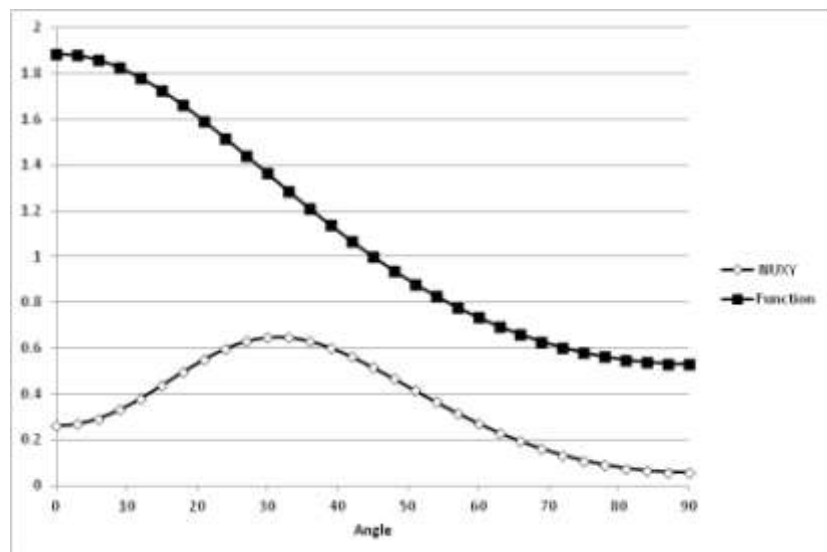


Figure 5 Laminate thermoelastic interaction diagram for Glass/epoxy.

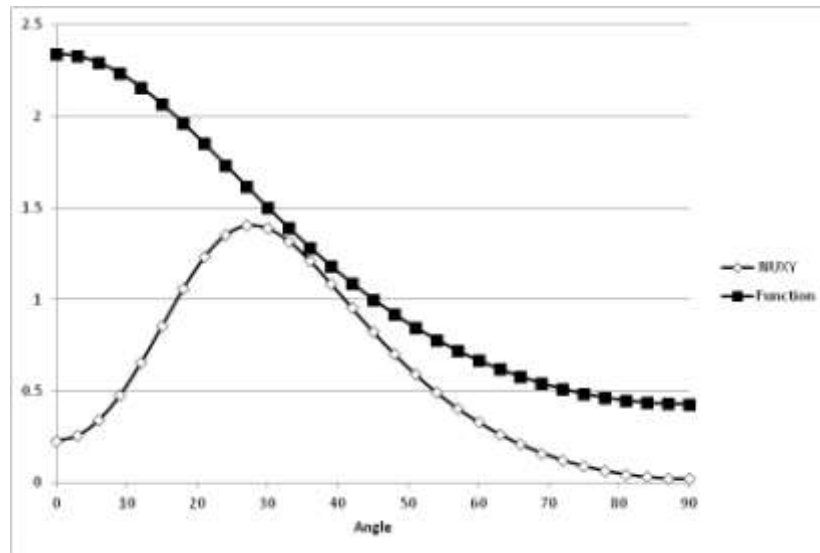


Figure 6 Laminate thermoelastic interaction diagram for B/5505.

5. CONCLUSIONS

The large differences in stiffness along the principal material directions of typically available composite laminae are primarily responsible for allowing one to tailor the thermal expansion characteristics of balanced, symmetric angle-ply laminates. Materials such as these, enable one to design laminates that have their Poisson's ratio matched to the driving thermal load.

It is possible to specify a lamina material with elastic constants that do indeed allow a large range of Poisson's ratio but whose thermal expansion characteristics are such that the match between Poisson's ratio and driving thermal load cannot be achieved, as in the case of B/5505. So, the thermal expansion characteristics do play a role, but for lamina materials commonly available, it is a secondary one as can be verified from the numerical example results of Kevlar/epoxy and T300/5208.

The simple examples in the text have attempted to cement the basic ideas of the interactions that occur. Of course, in 3-dimensional cases that cannot be regarded analytically as a plane stress laminate, the full 3-dimensional elasticity equations are to be used. The ideas are generally-applicable, but in the general case using the full equations, the central lesson is lost and the designer/analyst does not gain an understanding of the internal *mechanisms* at work.

6. References

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