

Using ABAQUS for reliability analysis by directional simulation

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Abstract: Monte Carlo reliability calculations for high-reliability systems are very computationally expensive. Variance reduction techniques optimize this process greatly and directional simulation is one such technique. Directional simulation is particularly valuable for high reliability systems where the failure surface is highly curved or dislocated. Subsea pipe-in-pipe structures in certain classes represent such a system and Abaqus is ideally equipped to solve this structural problem, which involves contact with friction, buckling, plasticity and fabrication imperfections amongst other phenomena. The pipe-in-pipe structure is non-linear in normal service. The directional simulation algorithms were programmed in VB and Excel. In addition, the VB software generated the Abaqus input files to define a unique model for each combination of parameters to populate the design space/failure surface. The tool also generated the Python scripts required to launch and post-process the Abaqus runs automatically within the directional simulation algorithm. The parameter selection was intelligent to the extent that the algorithm used the available results to cluster runs in the regions of the failure surface that required the best definition. This paper will demonstrate the techniques used and show how the tools were validated on a known-reliability structure. The process of arriving at a probability-of-failure value for a structure (with properties that are random variables) that behaves non-linearly under operating loads will be described.

Symbols:	F	Load, force
	S	Stress
	D	Subscript for demand variable
	R	Subscript for resistance variable, reliability
	t	thickness, with subscripts f for flange and w for web

Keywords: Buckling, Design Optimization, Failure, Pipes, Pipeline, Plasticity, Probabilistic Design, Safety, Python Scripting, Shell Structures, Reliability, Directional Simulation.

1. Introduction

1.1 Subsea pipe-in-pipe structures

In the early days of offshore oil and gas production, fixed platforms were sited directly above the subsurface hydrocarbon reservoirs and the fluids were transported nominally vertically from the

seabed to the platform topside for separation, processing and export. The high costs of such offshore facilities required large hydrocarbon reserves to make them economic. In recent years, technologies have been developed to recover oil and gas from smaller offshore reservoir volumes, which would not have been economic if they required dedicated platforms. In the modern subsea industry, multiple wells are connected to a single host facility through flowlines on the seabed, some many miles long. These flowlines have to be designed for axial stability under fluctuating temperatures and pressures and have to withstand the external pressure associated with water depths up to many thousands of feet. At the same time, they must perform thermally to keep the product warm both during transport over the long distances in cold (4°C) water and in the event of stopped production.

One solution to the demands of subsea flowlines is the pipe-in-pipe bundle concept. An inner pipe provides pressure containment for the hydrocarbons using an acceptable minimum diameter. It is coated in insulation, which in turn is sheathed by a sleeve pipe. The sleeve pipe is then housed along with smaller pipes and umbilicals – for power and control – within an outer pipe. The production line is centralized within the sleeve pipe by means of spacers and the sleeve pipe is likewise held in position alongside the smaller lines within the carrier pipe by spacer assemblies. A schematic cross-section of such a bundle with a view of a spacer assembly is shown in Figure 1.

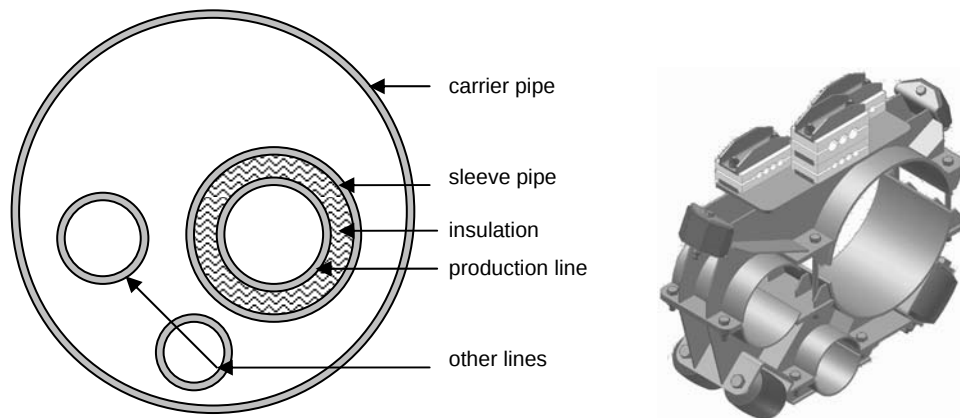


Figure 1. Pipe-in-pipe bundle schematic with typical spacer.

The temperature differentials in the structure during production of hot hydrocarbons require careful design of all the components. Such structures do not fit easily within pipeline design code provisions so alternative methods are required to establish economic fitness-for-purpose. Abaqus can be used to simulate the structural behavior of the system, representing initial imperfections, out-of-straightness due to the seabed profile, lack-of-fit between the spacers and their guide pipes, the thermal effects, contact, friction, buckling and non-linear material properties. This paper shows how the FEA method was used to establish the probability of failure of such designs.

1.2 Structural failure and reliability

The probabilistic theory in the failure of structures and its complimentary aspect of reliability is now widely covered in the literature (e.g. Thoft-Christensen and Baker, 1982). If the failure of a structure can be described mathematically by demand (D) exceeding resistance (R), as in for example

$$D - R > 0, \quad (1)$$

then there are several methods for determining the probability of failure when the stochastic nature of the demand on the structure and the structure's resistance are known or can be estimated. For structures, the demands are commonly the loads and the resistance is made up of the sizes of the structural elements and their material properties. Clearly, a dimensionally consistent form of the failure function must be established, and a useful unit for structures is stress.

Taking an example where the load-induced stress S_D and the yield stress S_R are random variables (e.g. normally distributed), then the stochastic nature of the failure probability can be visualized as shown in Figure 2. The resistance variate here (s_r), the yield stress for a common grade of structural steel, has a mean of 416 N/mm² and a standard deviation of 25 N/mm². The demand variate (s_d) is more uncertain, as seen by the less peaked PDF. The PDFs overlap and the probability of failure P_f is related to the nature of the overlap. Mathematically,

$$P_f = P[S_R \leq S_D] = \iint_{s_R \leq s_D} f_D(s_D) f_R(s_R) ds_R ds_D. \quad (2)$$

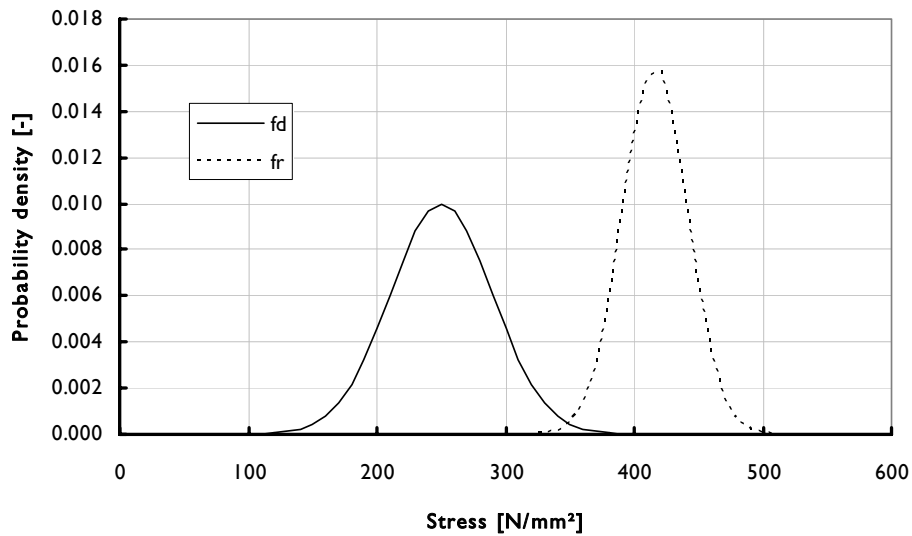


Figure 2. Stochastic failure illustration – interaction of demand and resistance.

In structural systems with many random variables and associated probability distributions, the analytical solution of the failure integral (Equation 2) can become impractical. The development of the First Order Reliability Method (FORM) and its second order variant (SORM) has provided an alternative means for determining the probability of failure (Thoft-Christensen and Baker, 1982). The reliability of the system is the complement to the probability of failure;

$$R = 1 - P_f . \quad (3)$$

However, a limitation on standard FORM/SORM is that the failure function must be available in the form of a mathematical equation. Where an FEA model is used to determine if a structure fails, no such equation will in general be available, so the FORM/SORM tools cannot be used conventionally. One approach that circumvents this obstacle is to link the structural analysis software directly to the reliability software, but this requires access to both source codes which is generally impractical.

2. Monte Carlo methods

2.1 “Crude” Monte Carlo

Alternative approaches for structural reliability analysis are Monte Carlo methods. The basis of the Monte Carlo method is that each random variable is sampled in accordance with its underlying distribution and an FEA is performed using the sampled combination of random variable values (a trial). The ratio of failed outcomes (a set of failure criteria must be defined) to the total number of trials approximates the probability of failure.

The method is easy to understand but in its basic (“Crude”) form, it is extremely inefficient in its use of computer resources and it becomes increasingly inefficient for high reliability problems, and high reliability is the nature of practical components and structures. The predicted probability of failure is not “exact” as in FORM/SORM, but becomes more accurate the more trials are performed. Since the probability of failure is itself a random variable, sufficient sampling has to be performed to give the required confidence level (reduced variance). The number of structural analyses required (millions) for reasonable accuracy would be prohibitively large using “Crude” Monte Carlo, particularly in the type of structure under consideration, where the analysis run time is a number of seconds.

2.2 Monte Carlo with surrogate model

The limitations of the crude Monte Carlo method can be largely removed by determining an analytical surface (strictly a hypersurface) that separates the safe region of the design space from the failed region. For n independent random variables that have an influence on the failure of the structure, the design space is n -dimensional and the failure surface is $n-1$ dimensional.

Sufficient FEAs have to be performed to define the failure surface, but these analyses can be clustered around the part of the design space near the hypersurface. Once an analytical function (the surrogate model) has been established, random trials may be repeatedly performed and tested

as to on which side of the failure surface they lie. Such trials require little more than a robust random number generator and millions of trials can be performed in seconds using current computers and software.

The key to success for this method is therefore the quality of the surrogate model that may be constructed. The more complex the failure surface, the more FEA runs are required and the number of runs is also to a degree related to $n!$ (factorial). While the use of this approach is attractive where the number of random variables is small and the failure surface is well understood and well behaved, the generation of the failure surface may become rapidly very complex and opaque (of uncertain accuracy) as the number of random variables increases and the nature of the surface reveals localized details or is of a separated form (in discrete parts). In such cases it is believed that other methods have the advantage of much greater transparency without any serious disadvantages.

2.3 Reduced variance

The aforementioned weaknesses in the basic Monte Carlo method and its improved variant using the analytical failure surface have led to the development of a number of hybrid computational schemes in which the failure function for the structural reliability problem is mapped into a multi-dimensional unit standard normal space (as in FORM/SORM), but in which the reliability assessment is carried out by advanced simulation methods incorporating variance reduction techniques (Turner and Baker, 1991). The idea of unit standard normal space in reliability theory is central to this work and, while the references cited cover the concept in detail, it is useful to summarize it here. Unit standard normal space is commonly referred to as z -space. To use z -space requires that all the random variables be transformed into normal distributions with zero mean and unit standard deviation. In contrast, x -space is used to denote the physical random variables. The transformation to z -space for a normally distributed parameter X , writing in terms of variates (lower case), is by means of

$$z = \frac{x - \mu_x}{\sigma_x} . \quad (4)$$

and with similar transformations being available for other probability distributions.

The two main variance reduction techniques that are available are importance sampling and directional simulation. The latter is the best option when the failure domain may comprise a number of discrete regions in the data space, as is often the case in buckling problems where the initial imperfection is also a random variable (Chryssanthopoulos et al, 1986 and 1991). Recent work on comparing these advanced simulation methods has been carried out at the University of Aberdeen for the fracture behavior of duplex stainless steel pipelines containing defects in which Abaqus has been used to determine the J-integral values for cracks of different sizes and orientations (Hamid, 2005). This work further supports the suitability of directional simulation for problems of the type to be tackled here.

2.4 Directional simulation

The chi-squared distribution is widely covered in statistical texts and is an important tool in statistical significance testing. Its theory will not be covered here beyond the reminder that it has one parameter: the number of degrees of freedom. Directional simulation uses a central property of the chi-squared distribution: for a unit normally distributed variable Z , the distribution of Z^2 is the chi-squared distribution with a single degree of freedom. More importantly, the sum of the squares of a number of independent random variables with zero mean and unit standard deviation is also distributed according to chi-squared distribution, this time with the number of degrees of freedom equal to the number of the variables. I.e. if

$$X = \sum_{i=1}^n Z_i^2, \quad (5)$$

then X has a chi-squared distribution with n degrees of freedom. The chi-squared distribution can be calculated by a standard function in software such as Excel. Here, we will denote it by χ_n^2 .

Since the n random variables for the pipe-in-pipe structural system can be transformed into unit standard normal variables and the failure surface can then be mapped into standard normal space, it can be shown that the chi-squared distribution with n degrees of freedom (DOFs) can be used to compute the statistical properties of the system, in particular its probability of failure P_f . However, to calculate P_f , it is necessary to know where in z -space the system fails and for this, failure criteria must be defined.

It is no less problematic to construct a failure surface in z -space than in x -space. Directional simulation obviates the need for a complete failure surface by repeatedly finding the distance r from the origin to the failure surface in z -space. It is instructive at this stage to recall that the origin is the location in z -space where all the random parameters are at their mean value. Moving out a unit distance along one of the n axes (such that $r=1$), corresponds to a location where the variate associated with that axis is one standard deviation away from its mean value. On the positive axis, the variate is larger than its mean and on the negative axis it is smaller.

If the failure surface were equidistant from the origin in all directions it would be a hypersphere. Then any direction vector from the origin would have a length r when it crossed the failure surface. Only one vector would therefore be needed to determine the distance to the failure surface. It can be shown that the probability P of a point lying within a distance r of the origin on that vector (i.e. *not* failing) is given by the cumulative chi-squared distribution of r^2 , for n DOFs:

$$P = \chi_n^2(r^2). \quad (6)$$

P_f is then the probability of a point lying on this vector at a distance of more than r from the origin, i.e. the complementary cumulative chi-squared distribution:

$$P_f = 1 - P = 1 - \chi_n^2(r^2). \quad (7)$$

Of course, the failure surface is generally not going to be a hypersphere and any random direction vector will have a different distance to the failure surface. The probability of failure prediction is therefore improved the more such vectors are used and is calculated by taking the average over them all. If q unit vectors are randomly generated (each vector requires the generation of n random numbers in the range -1 to 1), then it can be shown that the probability of failure is given by

$$P_f = \frac{1}{q} \sum_{j=1}^q (1 - \chi_n^2(r_j^2)). \quad (8)$$

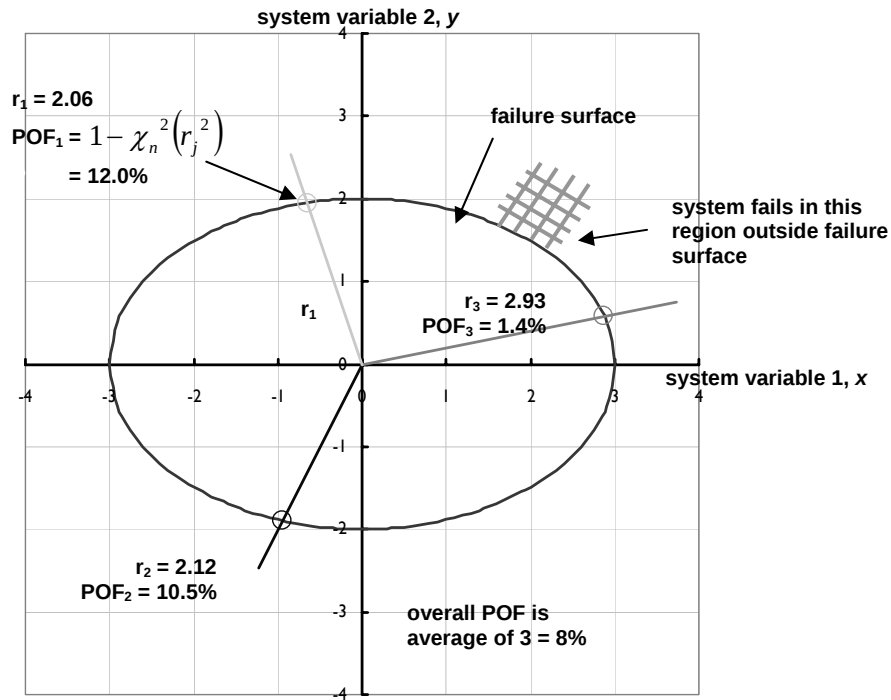


Figure 3. Directional simulation illustration for 2-dof system.

The method can be illustrated as shown in Figure 3, for a simple 2-parameter system. Three random vectors have been generated and the current POF is 8%. Clearly, the value of P_f can be calculated after each random vector has been processed and this gives an indication of the convergence behavior. An example of this behavior is shown in Figure 4 for two load levels.

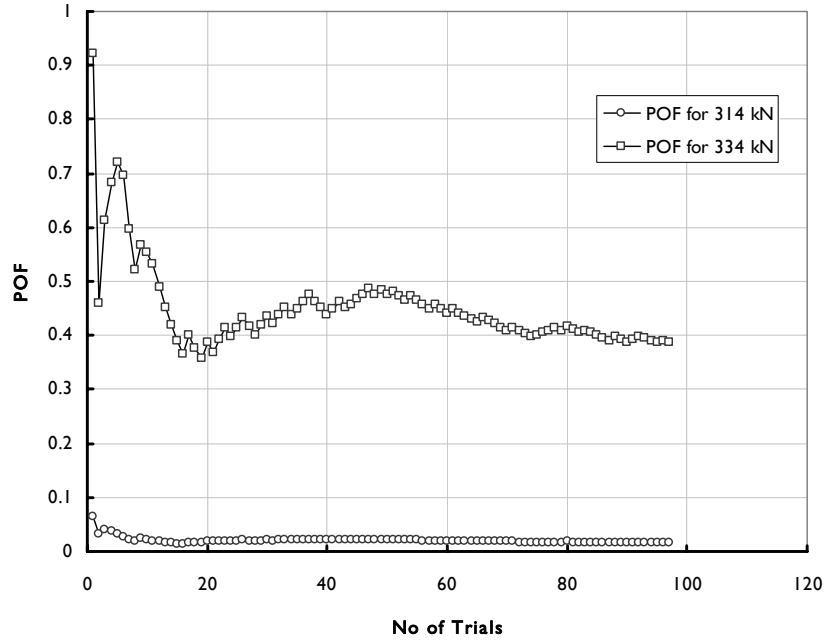


Figure 4. Convergence of directional simulation.

The desired result for the pipe-in-pipe structure was P_f as a function of axial load in the production line. For any given distance r , the load that corresponds to failure may be computed by FEA and recorded. Starting with a suitably small value of r , this involves first calculating the values of the n random variables in physical x -space (by reverse transforming according to the normal FORM/SORM techniques) corresponding to the current z -space sample values. These current values of the x -space random variables are used for running an FEA and recording the load at the first encountered limit state. This is repeated for increasing values of r until a sufficiently small load is required for failure.

Each vector gives the failure load F_f as a function h of r :

$$F_f = h(r). \quad (9)$$

Once q random vectors have been generated and processed in this fashion by FEA runs, the values of r for a given load level are computed for each vector by linear interpolation or other curve fitting, i.e.:

$$r = h^{-1}(F_f) \quad (10)$$

and Equation 8 is used to obtain P_f .

It will be seen from the above that the process is highly efficient in terms of limiting the number of FEA runs to a minimum. For vector directions where the resistance variables are increasing, very few FEA runs will be required since the vector is only going to intersect failure surfaces associated with unrealistic large loads. Where the failure load is reasonably linear with r , fewer FEA runs are required, as illustrated in Figure 5. Here initial increments for r of 0.5 are used and additional FEA runs are added where the failure load varies more rapidly with r . The relationship between the failure load and r defines the function h , and thereby h^{-1} , for this direction vector. In this example, h^{-1} is completely defined for loads in the range of 50-200 kN and examples of extracting P_f at two load levels are given.

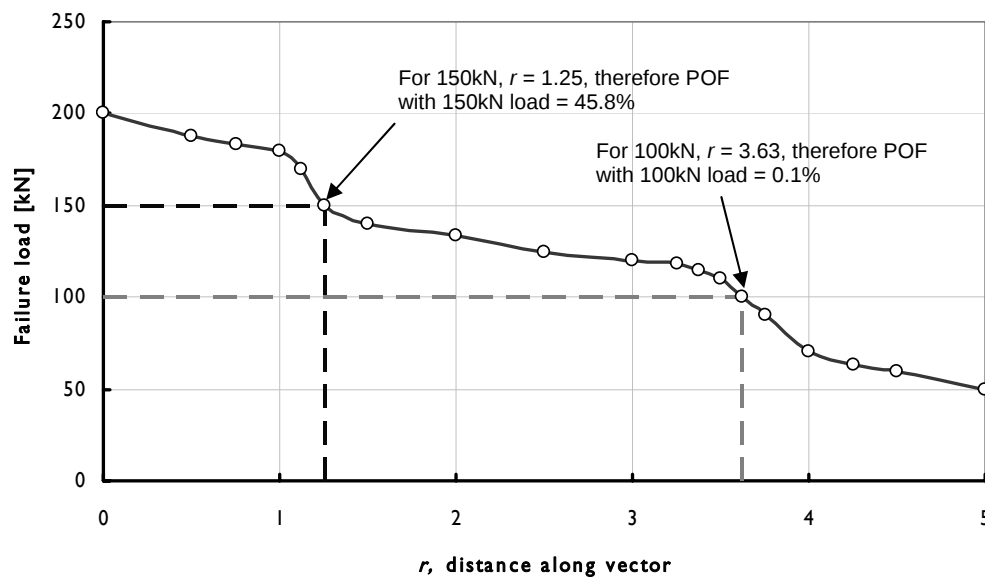


Figure 5. Example of failure load variation along a random direction vector.

3. Test case

The pipe-in-pipe system is highly complex structurally, so it was desirable to validate the approach using a system for which the reliability could be established by other means. A plain I-beam end-loaded cantilever was selected (Figure 6) and although FEA was not required to solve this structural problem, it was clearly desirable to use Abaqus so that the interaction between the automated FEA runs and the reliability calculations could be tested.

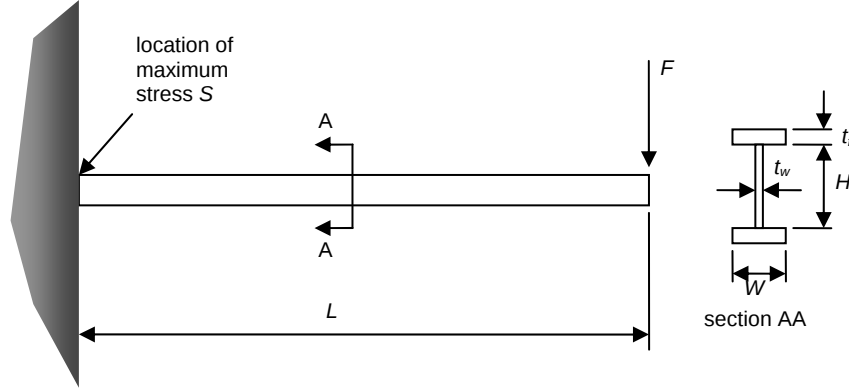


Figure 6. Test case structure.

The random variables for the test were selected as H and W so that the problem would be non-linear. The applied load F implies a stress s at a location in the structure. Here the outer fiber at the root of the cantilever is an obvious choice, since it is the location of the highest stress. s is linearly related to the load F through the engineer's beam theory by

$$s = \frac{FL(H + 2t_f)}{2I} = \frac{FL(H + 2t_f)}{\frac{1}{6}(t_w H^3 + 2Wt_f^3) + t_f W(H + t_f)^2}, \quad (11)$$

where I is the section modulus, which is a function of the random variables H and W . A definition of failure is needed and the bending stress exceeding 60% of the yield stress, i.e. $0.6s_y$, was selected. The problem can then be formulated in terms of stress, writing the margin M between resistance and demand as

$$M = 0.6s_y - s. \quad (12)$$

Since the margin function is non-linear in the random variables, the linear expression for the reliability index β , which in terms of the mean μ and the standard deviation σ of the margin is

$$\beta = \frac{\mu_M}{\sigma_M}, \quad (13)$$

cannot be used. Instead an iterative procedure may be adopted to find β as described in the cited literature. The P_f can then be shown to be given by the cumulative normal distribution function:

$P_f = F_z(-\beta)$, often denoted $\Phi(-\beta)$. The results of this are illustrated in Figure 7, which is a depiction of z-space. The failure boundaries are plotted for different values of load and it is instructive to note that the failure boundary corresponding to 50% P_f goes through the origin, which is fundamental to z-space. (At this scale the failure boundaries look like straight lines, but they are in fact slightly curved.) The alpha-vector shown links the origin to the “design” point – the point on each failure boundary closest to the origin – for the 1% P_f load level.

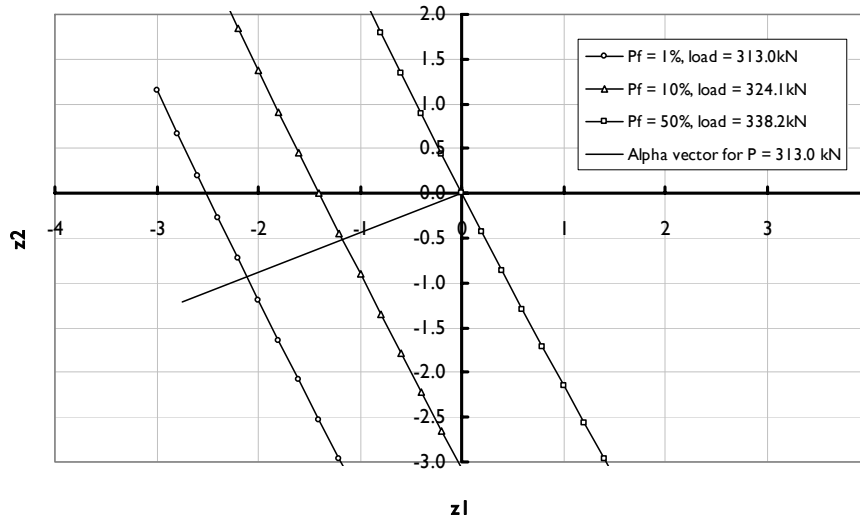


Figure 7. Failure boundaries in z-space for cantilever.

The relationship between load and P_f is clearly available from the FORM calculation – simply by changing the deterministic load – and it is plotted in Figure 8.

The directional simulation approach was applied to the cantilever modeled in Abaqus and the resulting probability of failure curve is shown in Figure 8 for comparison. At a load of 330 kN, the difference in the calculated P_f is 3%, which may be partly attributed to FORM errors due to the non-linearity in the margin function. A SORM calculation would resolve this issue. The correspondence is however good enough to give confidence in the approach and the automated tools.

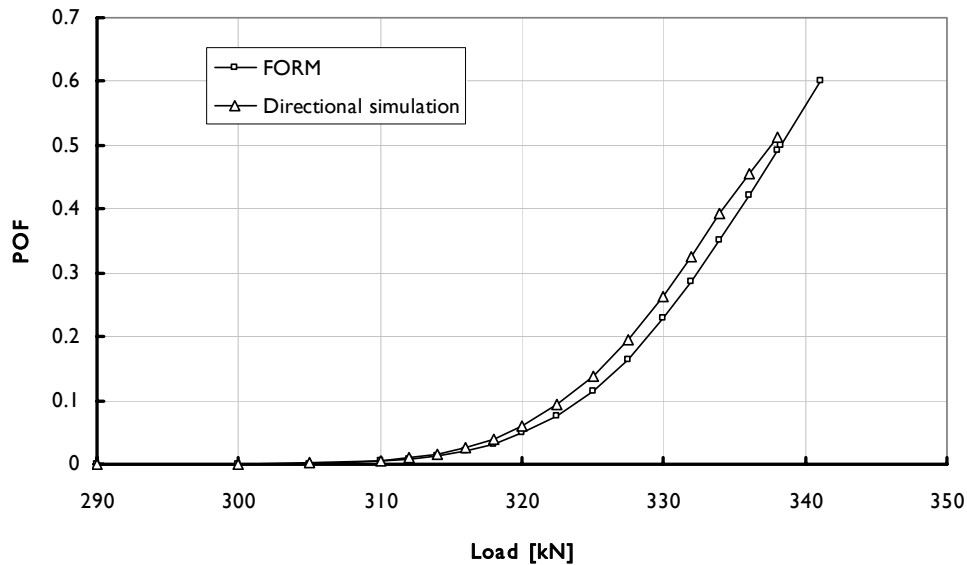


Figure 8. Probability of failure of cantilever.

4. Pipe-in-pipe analysis

The task was to find P_f for the production pipe as a function of axial load, since axial load can be related both to internal pressure and temperature. The random (normally distributed unless stated otherwise) variables to be considered were:

- Wall thickness
- Out-of-straightness magnitude
- Out-of-straightness direction, uniformly distributed
- Yield stress
- Bore eccentricity (wall thickness variation)

The method employed was the same as that for the cantilever. An Abaqus model (template `inp` file) was set up with the random variables defined as parameters. Shell elements were used for the pipe and therefore the bore eccentricity could not be incorporated in this model. To allow for the bore eccentricity, a separate Abaqus model was set up using brick elements for the wall. The sensitivity of the failure load to eccentricity alone was determined using this model (e.g. an eccentricity of one standard deviation gave a 4% reduction in failure load). In the directional

simulation, the effect for eccentricity was included implicitly by reducing the failure load on the basis of a look-up table developed from the eccentric bore model. This approach did not account for the interaction of eccentricity with the other parameters, but it was a pragmatic solution to the problem. A random number between -1 and 1 was generated for each of the other four parameters and the resulting z-space vector was scaled to give it a magnitude $r = 0.5$. The scaled vector components were then transformed into x-space, which became the values of the four parameters for the Abaqus analysis. These values were then inserted (automatically by a Visual Basic (VB) macro with Excel) at the appropriate locations in the template `inp` file and a script was generated to launch Abaqus and run the simulation until the first failure criterion was reached. The failure criteria were

- The maximum von Mises stress in the production pipe reached 96% of the specified minimum yield stress
- The slope of stress-strain curve reached twice the elastic slope
- The maximum pipe lateral displacement reached 26 mm

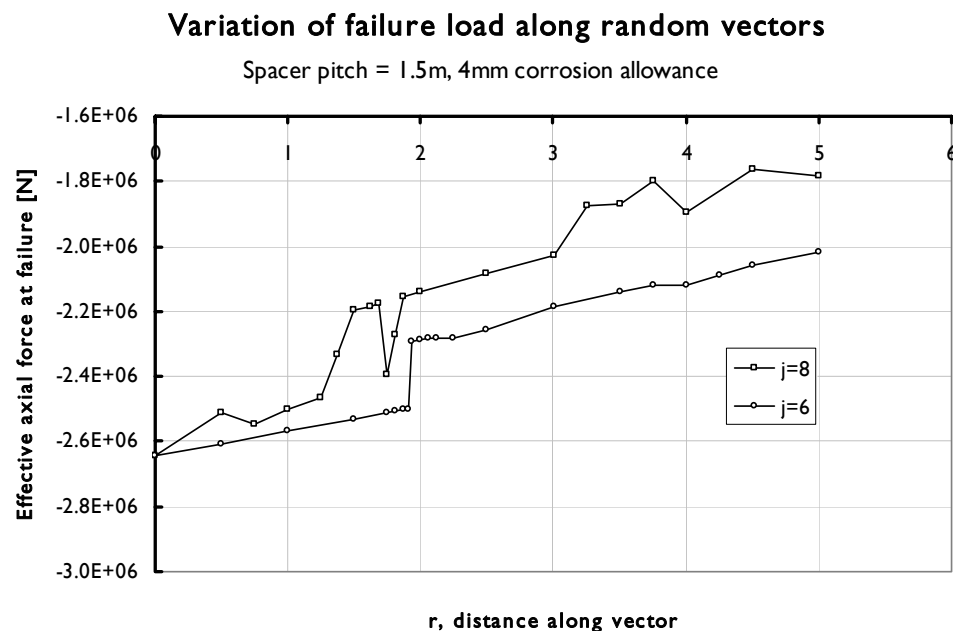


Figure 9. Examples of failure load variation along random vectors.

A Python script and VB utilities were developed to extract the failure load and reduce it as appropriate for the bore eccentricity. The direction vector was then scaled to a magnitude of 1.0, 1.5 and so on and the process repeated until the failure load was sufficiently different from the

mean value (in some directions the failure load will increase with r and in others it will decrease) or the direction vector reached a length of 5.0. This process gives the function h (see Figure 5) for a single direction. A new set of random z -values is generated and the process is repeated as many times as necessary to arrive at a converged value for P_f .

The initial step size used for the direction vectors (0.5) was arbitrarily chosen and additional r -values were used as needed. The criterion for additional points on the h functions was a curvature limit. The infill of additional points can be seen in Figure 9. Some of the dislocations in the h functions are related to the FEA solution (i.e. not physical) but others, such as the step in h_6 are representative of the highly non-linear behavior of the structural system near failure.

Around 20 Abaqus runs were required per vector and at least 60 random directions were required to achieve converge on P_f to within one percentage point. Thus for each case, around 1200 runs were required to generate a POF curve; typical results are shown in Figure 10.

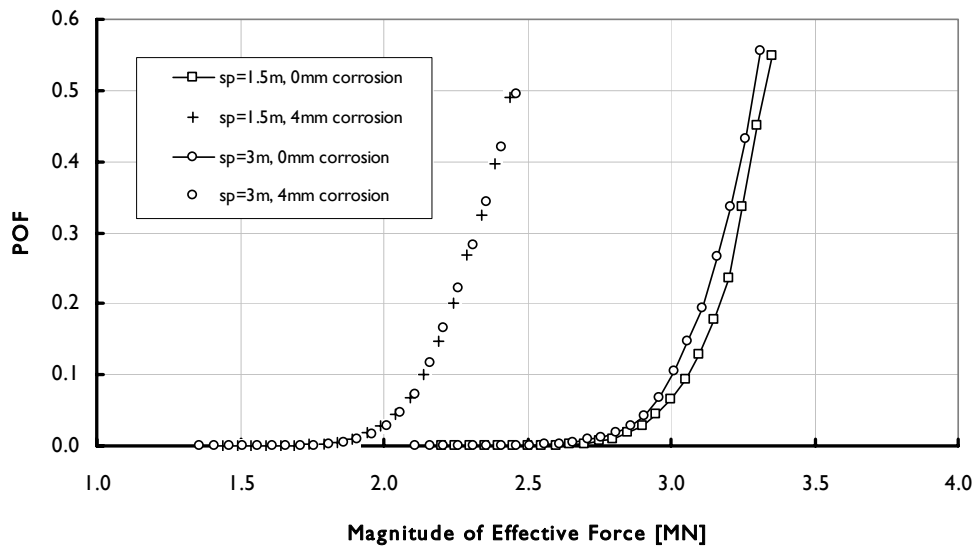


Figure 10. Typical probability of failure curves.

The method was tested for robustness by repeating a base case using different increments in r , different Abaqus step sizes and increasing the number of directions per case and the same POF curve was obtained.

5. Conclusions

An efficient and robust recipe for reliability calculations for complex (non-linear) high reliability structures has been provided. The theoretical background is well reported in the existing literature

in a form that is accessible to the non-specialist. Each step in the process is relatively straightforward and through the use of spreadsheets and simple programs (here Visual Basic has been used) that interact with Abaqus through the input file and Python scripts, the process may be made as transparent as necessary.

Rather than perform the validation test for the cantilever example, which proved to be nearly linear, it would be more rigorous to use a highly non-linear system, albeit one for which the reliability could be accurately derived using FORM/SORM.

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