Transition of Plastic Collapse to Elastic Collapse

Allan Zhong

Halliburton Company

In the oil and gas industry, a typical formula, FEA calculation of the collapse pressure of pipes, is based on the initial yield of the pipes. This type of calculation can produce a rating well above the real collapse pressure when the ratio of pipe OD (D), to the pipe thickness (t), or D/t, is very large. On the other hand, if the API elastic collapse formula for large D/t is utilized across the board, the collapse rating can be artificially lowered. This is because a typically larger D/t section is connected to a smallerD/t section and thus gets support from the latter.

In this paper, two issues are addressed. First, for given material (fixed Young's modulus), what is the transition point (D/t) for plastic (yield strength) collapse to elastic collapse? How does the transition point change with the yield strength? A pipe with a thin section of length L is supported by thicker sections at both ends. It is known that the thin section with a large D/t, the shorter the L, the higher the collapse pressure. The question is how short (L/D) must the pipe be to prevent elastic collapse, or in other words, for a given D/t, what is the transition point (L/D)? The transition curves obtained provide useful guidelines for tubular product design and testing when collapse is a concern.

Keywords: Collapse, Elastic Collapse, Pipes, Plasticity, Transition, Design, FEA, Failure, Yield.

1. Introduction

In the design of completion tool systems, the collapse rating of pipes is of great importance. Commonly available choices of indicators include API formulas, Lame's stress, and von Mises stress. Typical calculation or FEA focus on plastic (yield strength) collapse since elastic collapse pressure is usually much lower than plastic collapse pressure when D/t, where D is pipe OD, t is pipe wall thickness, is very large. Using plastic collapse (Lame's, von Mises stress) can produce a rating well above real collapse pressure – thus significantly increasing the potential of failure in applications. For example, see **Figure 1**.

On the other hand, if API elastic collapse ratings for large D/t are used across the board, it is possible to artificially lower the ratings. This occurs because typically, the finite length (L), larger D/t section, is connected to the smaller D/t section and thus gets support from the latter**.** The essential questions mechanistically become, for a long pipe, what is the transition point (D/t) when the pipe's collapse mode changes from elastic collapse to plastic (yield strength) collapse; for a very thin pipe, i.e. (fixed) large D/t, of finite length (L), what is the transition point (L/D) that elastic to plastic transition occurs? The reason for studying the transition from plastic collapse to elastic collapse is to provide guidelines as to which calculation techniques are the most appropriate for a particular application.

The effect of D/t for long tubing was addressed in API collapse (Economides, 1980). The investigation of the D/t in this work was mainly to check on the "rule of thumb" and additionally to provide a relationship between transition point and steel yield strength.

Figure 1. A collapsed long thin pipe.

The effect of L/D for long tubing was studied in (Pattillo, 2007). The paper provided an analytical solution from a nonlinear shell theory for determination of elastic collapse and power law of a plasticity model for inelastic behavior. It was quite a comprehensive study – the effect of boundary conditions and the effect of work hardening was studied extensively. However, the transition from elastic collapse to plastic collapse was not discussed. The study here is much narrower. The focus is on steel (i.e. fixed Young's modulus), and using initial yield to determine the plastic collapse, thus ignoring the hardening effect. Also addressed is the question when the transition from plastic collapse to elastic collapse occurs.

2. Critical D/t

To determine the critical D/t that the pipe's collapse mode changes to when transitioning from elastic collapse to plastic (yield strength) collapse, two analyses will need to performed. One is to determine the elastic collapse pressure; the other is to determine the pressure that leads to the onset of yield.

2.1 Elastic Collapse

The analytical solution for the elastic collapse of very long pipes was obtained by Clinedinst (1939); see (Rahman, 1995).

$$p_e = \frac{2E}{1-\nu^2} \cdot \frac{1}{(D/t)\left[(D/t)-1\right]^2} \quad (1)$$

The API formula has been specified from equation (1) by introducing a correction factor that decreases the value of the external pressure to 71.25% of the theoretical value. For values of fixed Young's modulus, E = 30 x 10^6 psi, and Poisson's ratio, ν= 0.3, the equation is:

$$p_e = \frac{46.95 \cdot 10^6}{(D/t)\left[(D/t)-1\right]^2} \quad (2)$$

In general, elastic collapse is dependent of (E, ν, D/t). Due to the routine use of API formula (2), many tend to use it for all metals. Equation (2) applies to steel only.

A typical dependence of elastic collapse pressure on D/t is shown in **Figure 2**. It is noted that elastic collapse pressure is independent of material yield strength.

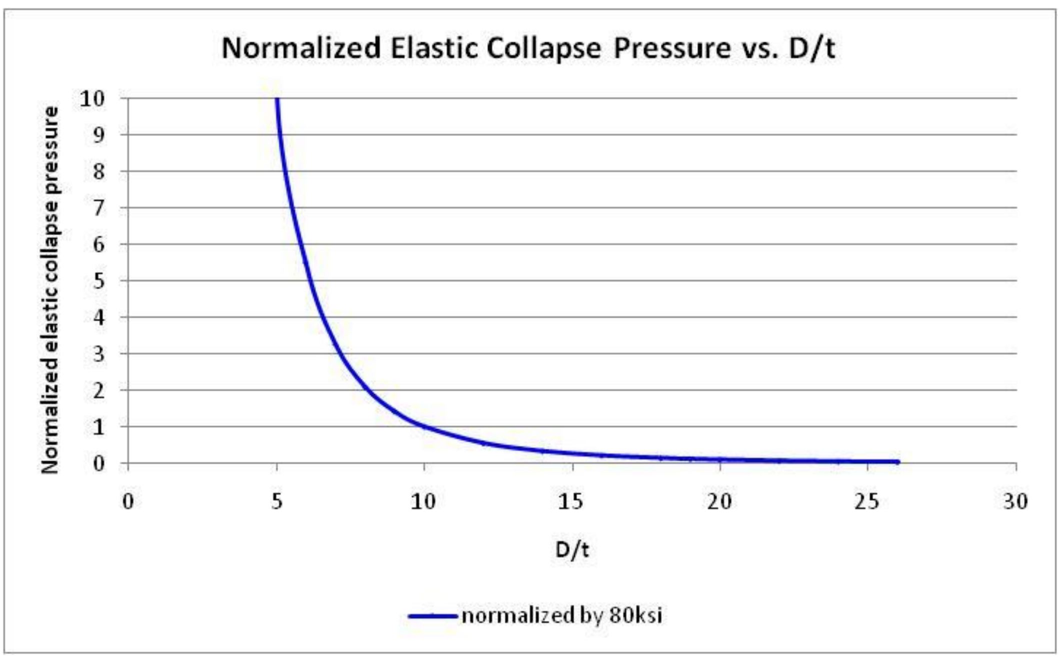

Figure 2. Elastic Collapse Pressure changes with D/t variation.

2.2 Yield strength Plastic Collapse

Based on the analytical elastic solution for long tubing subject to uniform external pressure, the stress in the tubing is dependent of D/t only. Therefore, the initial yield based plastic collapse pressure P_{cr} is dependent of D/t, and yield strength. The following dimensionless relation can exist:

$$\frac{P_{cr}}{\sigma_y} = f\left(\frac{D}{t}\right) \quad (3)$$

The yield strength based collapse pressure for a pipe of finite length is determined via axis symmetric FEA models using Abaqus. Based on relation (3), only one set of analyses is needed for a given yield strength; results for other yield strength can be obtained by scaling per (3). The results are shown in **Figure 3** for 80ksi minimum yield steel.

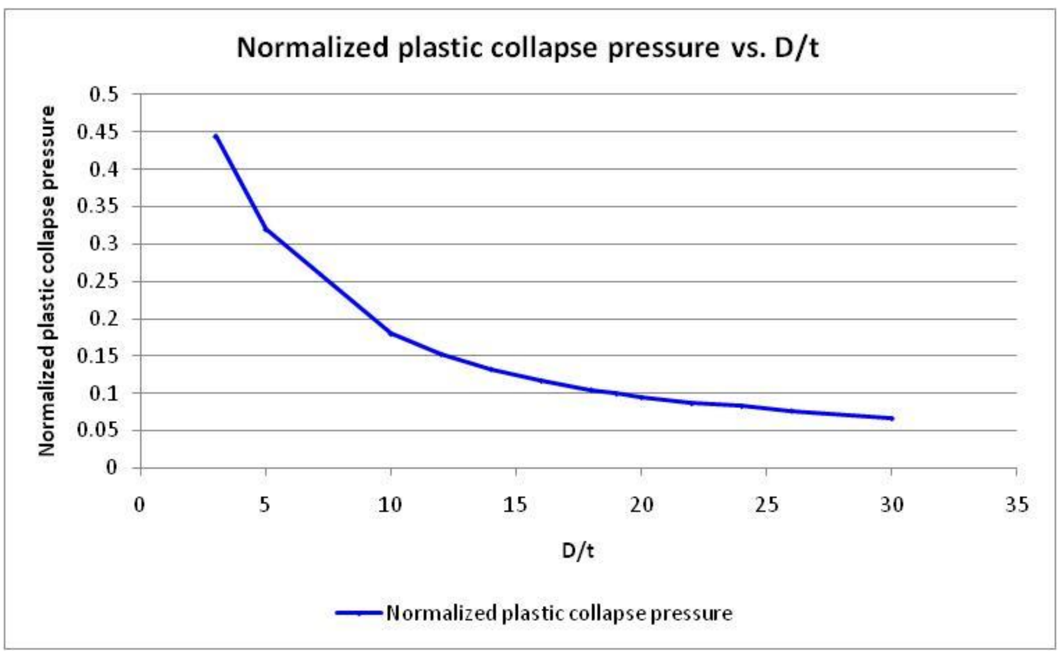

Figure 3. Normalized elastic collapse pressure vs. D/t curve.

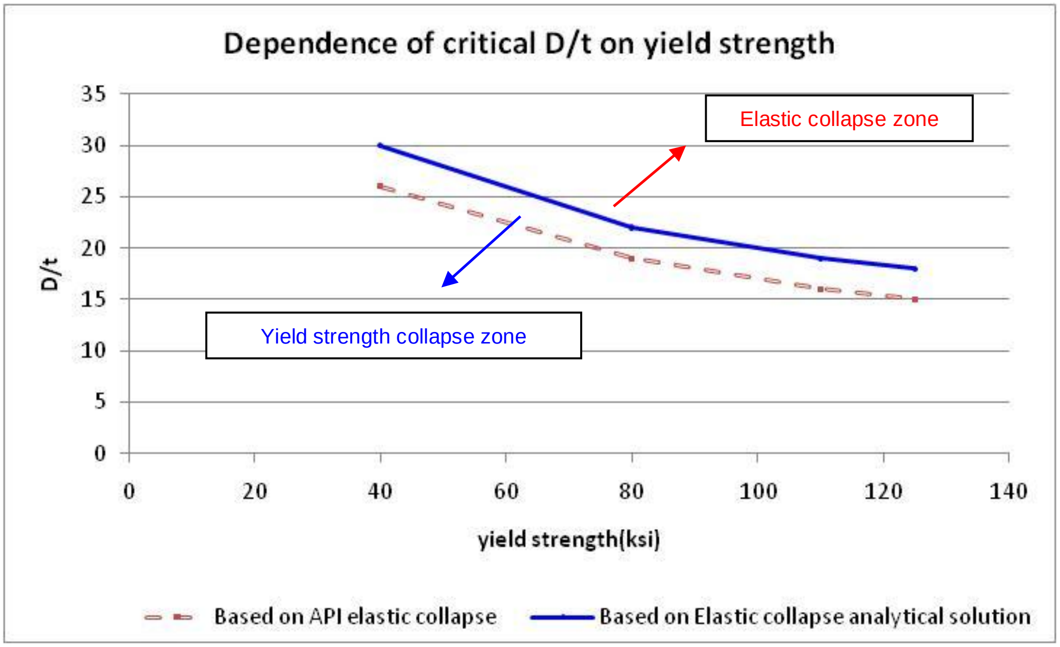

Figure 4. Dependence on yield strength of transition point from plastic (yield strength) collapse to elastic collapse.

2.3 Critical D/t

The critical D/t is determined by the intersection of curves P_e (D/t) and P_{cr} (D/t). For given D/t, if $P_e < P_{cr}$, then elastic collapse will occur; otherwise plastic (yield strength) collapse will occur. The transition curve in plane (D/t, σ_y) is shown in **Figure 4**. Apparently, the transition point (D/t) is dependent of the yield strength of the material, the higher the yield strength, the lower the transition point D/t. It also shows that for yield strength between 80ksi to 110ksi, the critical D/t is ~ 20. The common rule of thumb that elastic collapse occurs when D/t > 20; plastic yield strength collapse otherwise, is approximately correct for most commonly used steels.

2.4 Critical L/D

In a pipe string, if there is a section of pipe which has D/t > $(D/t)_{cr}$, should the elastic collapse rating be used? The usual scenario is that thicker pipes (smaller D/t) usually support a thin section of pipe (large D/t). Whether the elastic collapse or plastic (yield strength) collapse pressure should be used is dependent on the length of the section (L) and the support it receives.

In this work, the following assumptions were made to determine the critical L/D. The tubing is supported by a section of pipe with D/t=10 at both ends – as illustrated in **Figure 5**.

The critical L/D is determined in this way:

1. Determine pressure P_{cr} (L/D, D/t) to initial yield based on the deformation analysis – axis symmetric models.
2. Determine elastic collapse pressure P_e (L/D, D/t) based on eigenvalue analysis – full 3D models – note: analytical solution cannot be used due to finite length of the pipe.
3. The collapse pressure P = min (P_{cr}, P_e).
4. The intersection of curves P_{cr} (L/D, D/t=fixed) and P_e (L/D, D/t=fixed) corresponds to the critical L/D.

The real variables here are D/t and L/D, and yield strength for one.

Figure 5. Illustration of the FEA model geometry for L/D study.

The transition curve for 80MY of steel is shown in **Figure 6**. When L/D is above the curve, the pipe will have elastic collapse. When L/D is below this curve, the pipe will have a yield strength collapse. Again, the transition curve is dependent of steel yield strength. The higher the yield strength, the smaller the L/D needs to be in order to have a yield strength collapse. (See **Figure 7**) Conservatively, when L/D is smaller than 2.5, a thin pipe section will have a plastic collapse instead of an elastic collapse for all yield strengths and D/t. (i.e. Lame's or von Mises stress can be used to determine the part collapse rating.)

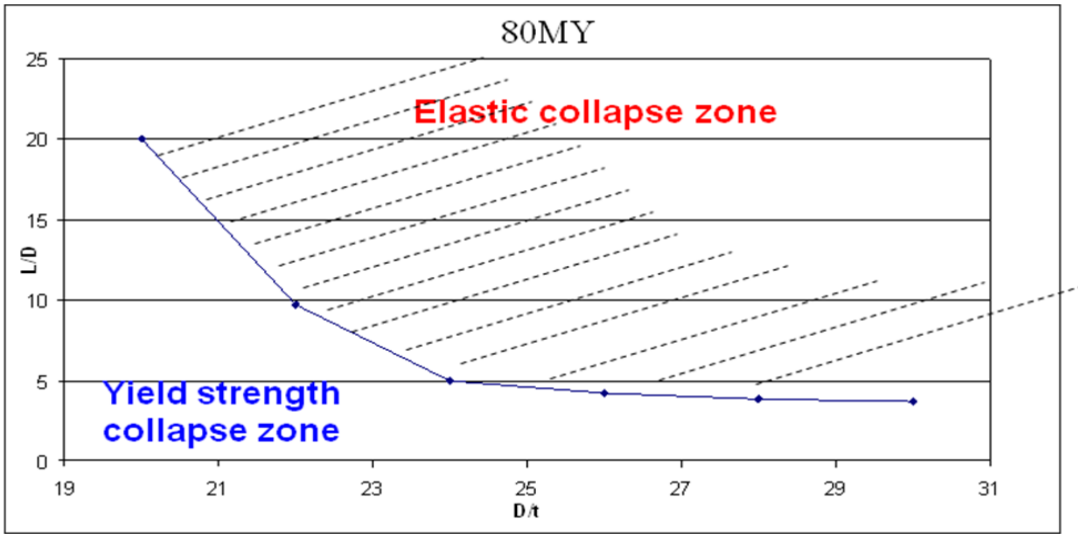

Figure 6. The plastic to elastic collapse transition boundary for 80ksi MY steel.

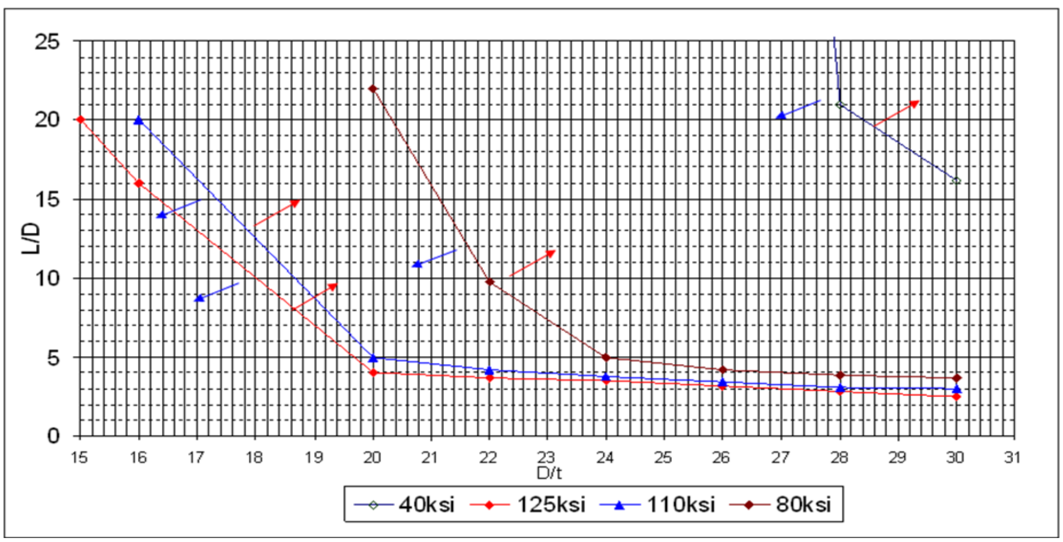

Figure 7. Dependence of the plastic to elastic collapse transition boundary on yield strength.

3. Application in a Design

The initial design of a steel sleeve (thin pipe) for a flow device has a minimum yield of 110ksi, D/t=55 and L/D=7.5. The required collapse rating for this sleeve is 2000psi. Based on **Figure 7**, it is determined that the sleeve will have an elastic collapse, and FEA predicted the elastic collapse pressure is 540psi (see **Figure 8**), which matched the test results (Figure 1), and were well below the target pressure rating. (Note: API elastic collapse formula yielded a collapse rating of 292psi; the collapse pressure should be ~ 4000psi per thin wall solution for this sleeve).

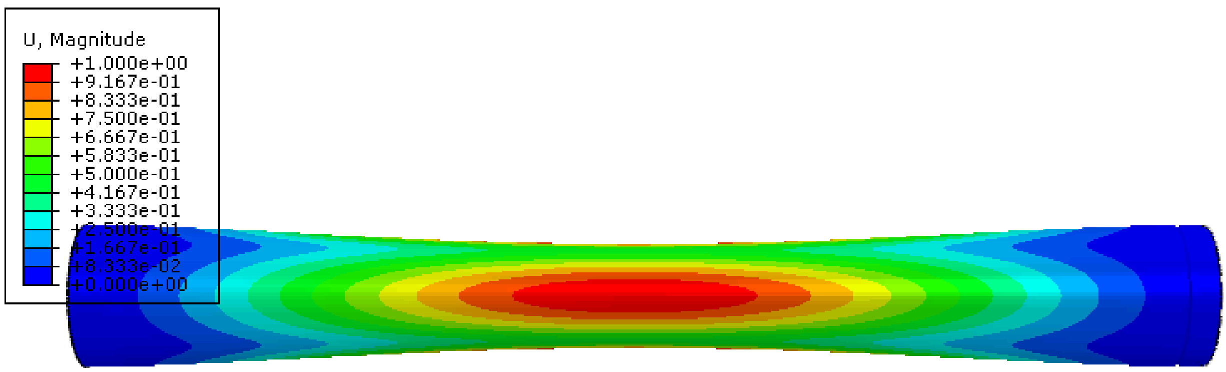

Figure 8. First collapse mode of the D/t=55, L/D=7.5 thin pipe.

Since L/D is fixed per system level requirements, it was determined that the only way to improve the sleeve's collapse resistance was through a decreased D/t.

The elastic collapse for L/D=7.5, D/t = 28 to 37 can be represented approximately by the following formula via fitting to FEA predictions.

$$P_e = 5 \cdot 10^7 (\frac{D}{t})^{-2.8861} \text{psi} \quad (4)$$

Decreasing D/t from 55 to 37, the elastic collapse pressure will be 1489psi; to D/t=32, the elastic collapse pressure is 2264psi. This FEA based collapse calculator was used by the designer to tune the design of the sleeve to fit into the overall system. The final design of this sleeve was physically tested successfully.

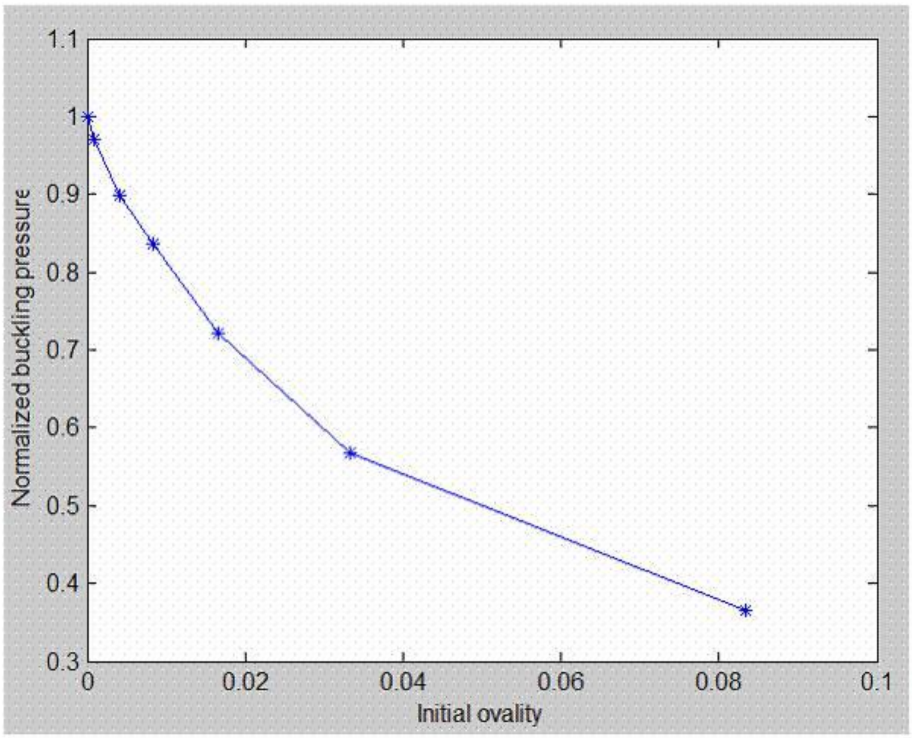

Figure 9. Empirical curve for collapse pressure sensitivity to initial ovality.

4. Discussions

There are several *limitations* to the results obtained in this work. Strictly speaking, the results are only valid for steel since elastic modulus and Poisson's ratio are fixed to be those for steel in all FEA calculations, though results for other materials can be easily obtained by proper scaling. Secondly, the results obtained are for steel pipes under collapse pressure only; there is no axial load or bending load on the pipes.

In addition to D/t, and L/D, the effect of egging (out of round cylinder) has a significant impact on collapse. Even with standard machining practice, there can be enough egging to significantly reduce collapse pressure. Four jaw and three jaw egging caused by the lathe chuck clamping on the heavy wall ends can easily propagate early failure into the thin section. An empirical relation between ovality and the loss of collapse rating is shown in **Figure 9**. It shows that a 0.01 initial ovality will lead to the reduction of buckling pressure to 78% of perfect pipe.

In real world applications, consideration should be given to potential causes for significant reduction in collapse. For example, non-uniform wall thickness, external loading (techtronic forces are never uniform), marks and strain from assembly as well as impact damage (strain or deformation) to thin wall during assembly or operation.

5. Summary

This comprehensive study established the relation between (L/D, D/t) and the nature of tubing collapse. It provided a general guideline when determining the appropriate method for a collapse assessment. As illustrated in the application example, many potential mishaps have been avoided in the early design stages due to the awareness of this relation.

6. References

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7. Acknowledgements

The author is grateful to Halliburton Company Management for permission to publish this work.