

Simulations of Ballistic Impact on Woven Fabric

Ethan M. Parsons and Simona Socrate

Massachusetts Institute of Technology, Cambridge MA

Jonathan Arata

SIMULIA, Providence RI

Tusit Weerasooriya

U.S. Army Research Laboratory, Aberdeen Proving Ground MD

Abstract: Woven fabrics are an integral part of key defense and structural systems, including personal protection (body armor) and vehicles (HMMWV). Textile-based armor, for example, has proven an effective substitute for heavier materials. With the proliferation of such materials, the demand for computationally efficient models is becoming ever more acute. Modeling woven fabrics is challenging, in particular, due to the need to capture the fabric response at both the macroscopic scale of the fabric and the “mesoscale” of the yarns that comprise the weave. Here, we utilize a continuum-level modeling technique for woven fabrics (King et al., 2005) that captures the macroscopic response of the fabric as well as the evolution of its mesostructure with deformation in a single efficient modeling step. A membrane formulation of the model is implemented into Abaqus/Explicit via the VUEL user subroutine. In addition to displacement degrees of freedom, degrees of freedom representing the crimp amplitude of each yarn family are defined in order to solve for the configuration of the fabric unit cell at each step in time. The accuracy of the model is evaluated by comparing the simulation results with the experiments of Weerasooriya et al. (2008) on ballistic impact of Kevlar woven fabric. The simulations successfully predict the experimentally observed deformation of the fabric, including wrinkling.

Keywords: Composites; Contact and Impact Mechanics

1. Introduction

Starting in the 1950's with vests constructed from ballistic nylon, fabrics woven from high strength fibers have been crucial to the safety of military personnel. The most well-known application of woven fabrics is lightweight, flexible body armor for protection against bullets and fragmenting projectiles from sources such as grenades, mortars, mines, and IED's. Woven fabrics are also used to reinforce polymers or ceramics in helmets and armor panels for protection against high-velocity projectiles. Additional common uses include restraint systems such as seat belts and head restraints, deployable fabric structures including air bags and air beams, as well as other forms of

ballistic protection such as pilot seats. Most recently, fabrics are being utilized to increase the portability and durability of personal electronic devices.

Despite decades of research, the design of woven fabric systems is still largely based on empirical methods and experience. Modeling woven fabrics is challenging due to the need to capture the response at both the macroscopic scale of the fabric and the “mesoscale” of the yarns that make up the weave. In fact, there is no widely accepted computational model for fabric deformation which is both computationally efficient and able to represent the behavior and evolving structure of the fabric at the level of the individual yarns. In the literature, either the fabric has been modeled at the level of the mesoscale, with detailed discretization of each yarn in the weave, or the fabric has been modeled at the continuum level, with phenomenological approaches, typically treating the fabric as an anisotropic continuum with two preferred material directions. In the first approach, the discretization length scale is determined by the dimensions of the yarns and weave, and thus real-sized or multi-ply simulations are computationally prohibitive. The second approach cannot be used to design fabrics based on the properties of the yarns and weave, and it is unsuitable for applications where the macroscopic response depends on the fabric's unique underlying mesostructure and its specific deformation mechanisms, such as yarn failure, crimp interchange, shear locking, or relative slip between the two yarn families. As such, without suitable models, the analysis and improvement of existing products and the development of new applications for woven fabrics is currently overly costly and time consuming.

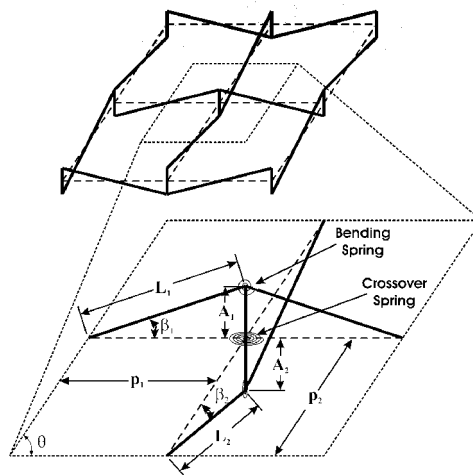


Figure 1. Geometry of fabric unit cell (King et al., 2006).

Here, we utilize a continuum-level modeling technique for woven fabrics (King et al., 2005) which captures the macroscopic response of the fabric as well as the evolution of its mesostructure with deformation in a single efficient modeling step. An appropriate unit cell approximation, based on a pin-jointed truss geometry, represents the weave architecture. The configuration of the unit cell is determined by the macroscopic deformation gradient and the satisfaction of local equilibrium. The forces on the unit cell, and in turn the corresponding continuum-level stresses, are calculated from physically motivated constitutive relations.

The fabric model is used to simulate projectile impact on single plies of plain-weave Kevlar. Direct comparisons are made with experimental deformation data obtained with digital image correlation (DIC).

2. Fabric model

The woven fabric model of King et al. (2005) has been modified and extended in order to function efficiently within Abaqus/Explicit.

2.1 Constitutive model

The fundamental basis of the model is the assumed unit cell geometry of the fabric shown in Figure 1 and Figure 2. The unit cell approximates the mesoscale mechanisms of deformation that underlie the macroscopic response of the fabric. The continuum response at each material point along the fabric surface is obtained through homogenization of the unit cell response, which is in turn calculated through the *analytical* model described below. The warp ($k = 1$) and weft ($k = 2$) yarns are approximated as pin-jointed, elastic trusses with the following properties:

- p_k yarn quarter-wavelength
- L_k yarn half-length or simply “length”
- β_k yarn crimp angle
- A_k yarn crimp amplitude
- θ in-plane angle between yarns

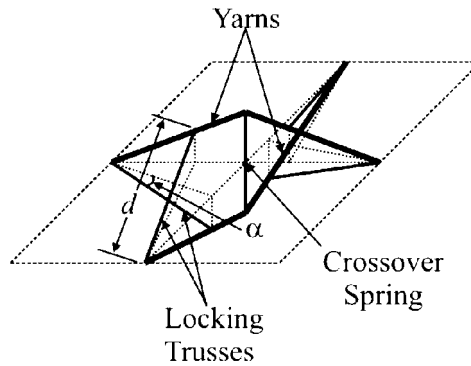


Figure 2. Geometry of fabric unit cell with locking spars (King et al., 2006).

The torsional bending spring provides the resistance to bending of the yarns while the cross-over spring provides, at the cross-over points, torsional resistance to in-plane shear as well as compressive resistance to compaction of the yarns. Figure 2 illustrates the locking trusses of

length d_k and angle α_k from the plane of the fabric that resist both shear and cross locking of the yarns. Constitutive relations describe each mode of deformation of the yarns. They are (with superscript “0” indicating an initial, undeformed value):

1. *extension*: yarn tension, $T_k = k_k(L_k - L_k^0)$
2. *bending*: bending moment, $M_{bk} = k_{bk}(\beta_k - \beta_k^0)$
3. *compaction*: force in cross-over spring, $F_I = K_I(e^{aI} - 1)$, where $I = -\Delta A_1 - \Delta A_2$ is the interference of the yarns at the cross-over points.
4. *yarn locking*: force in locking truss, $F_{Lk} = \begin{cases} 0 & d_k^0 - d_k \leq 0 \\ K_d(d_k^0 - d_k)^c & d_k^0 - d_k > 0 \end{cases}$
5. *in-plane shear*: moment, $M_s = K_s\gamma^e$, where the total relative yarn rotation, $\gamma = \theta^0 - \theta$, is separated into elastic and dissipative components, $\gamma = \gamma^e + \gamma^f$, which evolve according to $\dot{\gamma}_f = \dot{\gamma}_0 \left(\frac{M_s}{M_0} \right)$

k_k , k_{bk} , K_I , a , K_d , c , $\dot{\gamma}_0$, and M_0 are constants determined experimentally from simple experiments on single yarns and strips of fabric.

The deformed geometry of the unit cell in three dimensions is fully determined by the macroscopic deformation gradient, $\mathbf{F} = \text{Grad} \mathbf{x}(\mathbf{X}, t)$, and through-thickness equilibrium. $\mathbf{F}$ specifies the orientation and stretch of the yarn vectors, $\mathbf{g}_1$ and $\mathbf{g}_2$, everywhere in the fabric:

$$\mathbf{g}_1 = \mathbf{F}\mathbf{g}_1^0 \quad \text{and} \quad \mathbf{g}_2 = \mathbf{F}\mathbf{g}_2^0. \quad (1)$$

p_1 , p_2 , and θ are calculated from $\mathbf{g}_1$ and $\mathbf{g}_2$. The out-of-plane state of the fabric (crimp amplitude) is derived from a force balance at the cross-over spring. At the end of each cross-over spring, for static equilibrium, the force in the cross-over spring, F_I , must balance the net through-thickness component of the force in the yarn trusses, F_k^N :

$$F_I = F_k^N, k = 1, 2. \quad (2)$$

F_I is given above, and F_k^N depends on the moments in the bending springs and the forces in the yarns and locking spars:

$$F_k^N = 2T_k \sin \beta_k - 2F_{Ll} \sin \alpha_l \left(1 + \frac{p_l |\cos \theta| \cos \beta_k}{L_k} \right) + 2 \left(\frac{k_{bk} (\beta_k - \beta_k^0)}{L_k} \right) \cos \beta_k, \quad (3)$$

where $l = 1, 2$ ($l \neq k$).

Finally, as illustrated schematically in Figure 3, in order to homogenize the mesoscopic, yarn-level forces into a continuum-level stress measure, the stress on the borders of the unit cell must be computed. The first Piola-Kirchoff stress, $\mathbf{P} = \mathbf{P}(\mathbf{F}, \Delta A_1, \Delta A_2)$, is therefore calculated by equilibrating the external stress acting on the loaded unit cell with the in-plane forces that the yarns exert on the unit cell.

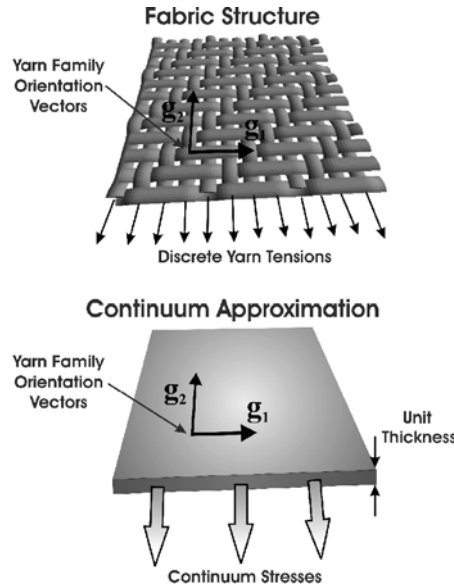


Figure 3. Modeling fabric as a homogeneous anisotropic continuum with current yarn directions g_1 and g_2 (King et al., 2006).

2.2 Finite element model

The fabric model has been implemented into Abaqus/Explicit via the VUEL user subroutine. The degrees of freedom are the three components of macroscopic displacement, u_i , and the changes in yarn crimp amplitude, ΔA_1 and ΔA_2 . The degrees of freedom are integrated forward in time using the explicit central difference rule. The mass matrix, $\mathbf{M}$, is diagonal with the mass apportioned equally throughout the mesh.

The majority of the resistance to ballistic impact loads is generated from in-plane deformation of the fabric and out-of-plane inertial effects. As such, it was deemed sufficient to model the fabric as a membrane. Fully integrated 4-node quadrilateral and 3-node triangular elements were used. An additional full set of “dummy” nodes were created for each element. The u_1 and u_2 “displacement” degrees of freedom of each dummy node represent the changes in yarn crimp amplitude, ΔA_1 and ΔA_2 .

The in-plane position of a material point in the reference configuration is given, as a function of the natural coordinates, ξ_s ($s = 1, 3$ for triangular elements; $s = 1, 2$ for quadrilateral elements) by the shape functions at each node, a :

$$X_J(\xi_s) = X_{Ja} N_a(\xi_s), J = 1, 2. \quad (4)$$

Correspondingly, the three-dimensional position of a material point in the current configuration is given by

$$x_i(\xi_s) = x_{ia} N_a(\xi_s), i = 1, 3. \quad (5)$$

The relevant components of the deformation gradient at a point within the element are therefore defined by

$$F_{iJ} = \frac{\partial x_i}{\partial X_J} = N_{a,J} x_{ia}. \quad (6)$$

Provided that the fabric initially lies in the 1-2 plane, $\mathbf{F}$ then determines the orientation and stretch of the yarn vectors, $\mathbf{g}_1$ and $\mathbf{g}_2$, at each integration point within the element. The internal nodal forces corresponding to the displacement degrees of freedom are then calculated by integrating numerically over the element:

$$(f_{ia}^{\text{int}})^e = - \int_{\Omega_0^e} P_{iJ}(\mathbf{F}) N_{a,J} dV_0. \quad (7)$$

By the principle of virtual work, multiplying Equation (2) by virtual changes in amplitude $\widetilde{\Delta A_k}$ and integrating over the element provides the statement of equilibrium (excluding inertial and external forces) in weak form:

$$\int_{\Omega_0^e} \widetilde{\Delta A_k} \frac{F_I(\Delta A_1, \Delta A_2) - F_k^N(\mathbf{F}, \Delta A_1, \Delta A_2)}{4p_1^0 p_2^0} dS_0 = 0. \quad (8)$$

With $\Delta A_k(\xi_s) = \Delta A_{ka} N_a(\xi_s)$, the internal nodal forces corresponding to the amplitude degrees of freedom become

$$(f_{ka}^{\text{int}})^e = \int_{\Omega_0^e} \frac{F_1(\Delta A_1, \Delta A_2) - F_k^N(\mathbf{F}, \Delta A_1, \Delta A_2)}{4p_1^0 p_2^0} N_a dS_0. \quad (9)$$

The total mass of the element is given by

$$M = \rho_f t_f \frac{1}{2} W^q \sum_{q=1}^3 J^q, \quad (10)$$

where ρ_f and t_f are the density and thickness, respectively, of the fabric and J^q and W^q are the Jacobian determinant and weight, respectively, at integration point q . In order to calculate the mass associated with the amplitude degrees of freedom, we take $L_1 \approx L_2 = L$,

$\beta_1 \approx \beta_2 = \beta \equiv 0$ and $m_1 \approx m_2 = m \equiv M / 4$. Moment equilibrium (Figure 4) of a single half-length of yarn about its point of intersection with the unit cell boundary requires

$$Fl \approx I\ddot{\beta} = \frac{mL^2}{3}\ddot{\beta} = \frac{ML^2}{12}\ddot{\beta}. \quad (11)$$

With the through-thickness acceleration of the free end approximated as $\ddot{A}_k \approx \ddot{\beta}L$, the through-thickness equilibrium equation becomes

$$F = \frac{M}{12}\ddot{A}_k. \quad (12)$$

Thus, when the mass of the symmetric other half-length of yarn is included, we find that the lumped masses

of the amplitude degrees of freedom are related to the corresponding masses of the displacement degrees of freedom by a factor of $\frac{1}{6}$.

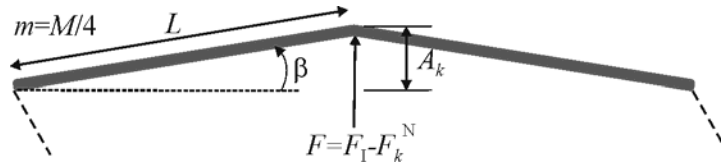


Figure 4: Calculation of lumped mass corresponding to amplitude degrees of freedom.

2.3 Quasi-static verification

In order to verify the accuracy of this new formulation of the fabric model, two simple test cases on fabric strips were run. Figure 5 illustrates that the simulations of warp and weft aligned strips of fabric ($2.54\text{ cm} \times 25.4\text{ cm}$) subject to uniaxial tension agree closely with the experimental results of King et al. (2005). The fabric tested by King et al., commonly known as S706, was 600-denier, plain-weave Kevlar KM2 with 34 threads per inch in both the warp and weft directions. All parameters used in the model were either measured by King et al. or fit to the shear, compaction, or single yarn tension experiments of King et al. 640 4-node planar quadrilateral elements were used to model the strips.

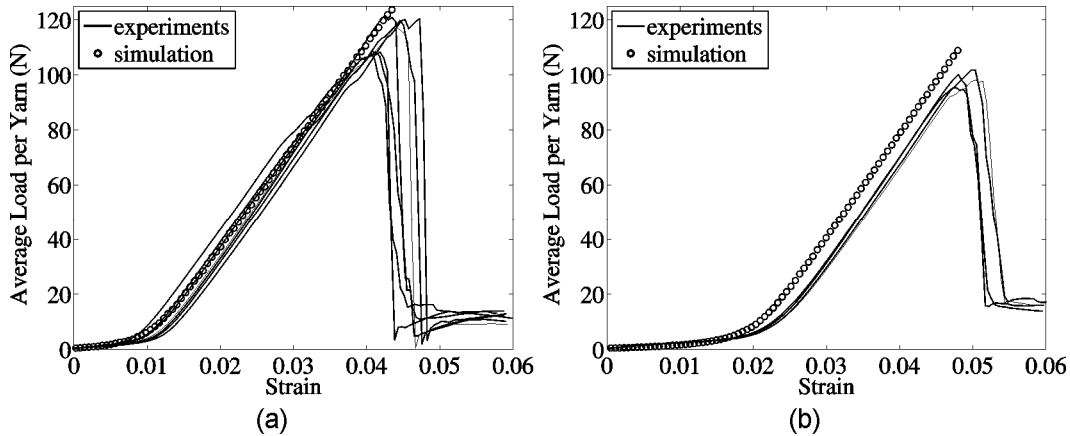


Figure 5. Simulation versus experiments (King et al., 2005) for strips of Kevlar S706 under uniaxial tension in warp (a) and weft (b) directions.

3. Experimental procedure

Weerasooriya et al. (2008) conducted projectile impact experiments on single plies of the Kevlar S706. A gas gun propelled $d = 12.7\text{ mm}$ hemispherically-nosed steel projectiles of mass 104 g at a velocity of 22 m/s. As shown in Figure 6, the approximately 40 cm square fabric specimen was stapled to a 30.5 cm square wooden “picture-frame” with an $R = 12.7\text{ cm}$ round cut-out. The weft and warp yarns were aligned with the vertical (y) and horizontal (x) directions, respectively. The stapled specimen was first painted with white spray paint, and then a speckle pattern was applied with black spray paint. The fabric and wooden frame were then clamped between two square aluminum frames of similar dimensions.



Figure 6. Back view (opposite side of impact) of Kevlar fabric specimen stapled to frame with $R=12.7$ cm round hole.

Figure 7 illustrates the experimental set-up. The initial velocity of the projectile was measured at the outlet of the gun barrel with laser beam detectors. The detectors also triggered the two Photron APX-RS high speed digital cameras to start recording images. 256×256 pixel images of the back surface of the specimen were acquired at a rate of 30 kHz. Commercial digital image correlation software from Correlated Solutions, Inc. was then used to measure the full-field in-plane (u , v) and transverse (w) displacements of the fabric. While great care was taken in the experiments to minimize slip of the fabric from the clamps, as with nearly all experiments of this type, some slip did occur. For example, the DIC results for in-plane v -displacement in Figure 8 show in excess of 4 mm of slippage.

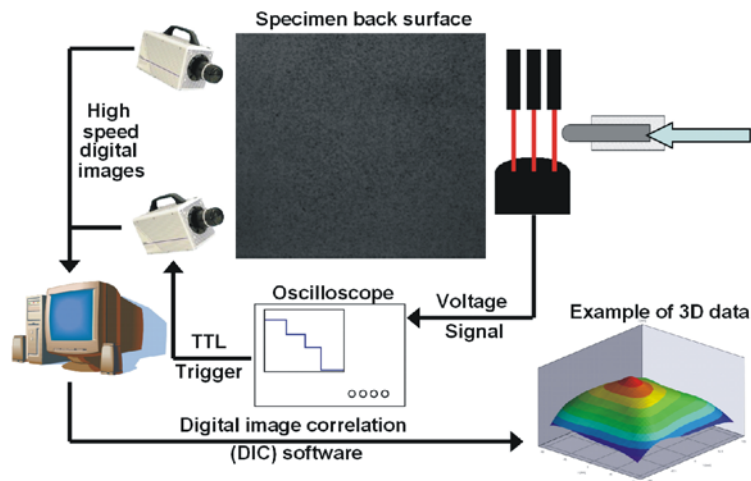


Figure 7. Schematic of experimental set-up for measuring full-field displacements during a ballistic impact test.

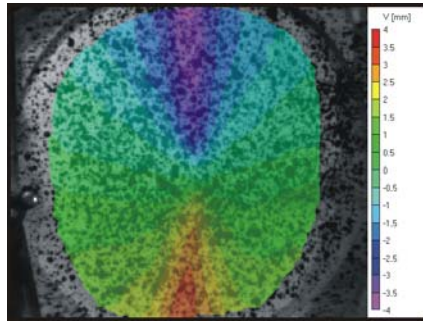


Figure 8. Contours of in-plane v -displacement for $R=12.7$ cm experiment at $t=2.0$ ms.

4. Results

One-quarter of the fabric specimen was modeled with 2308 3-node triangular membrane elements. The rectangular sections of the fabric outside of the cut-out were clamped flat with a force of 29 kN between two surfaces constructed from rigid elements. The projectile was modeled with an analytical rigid surface. The interactions between the clamps and the fabric, the frame and the fabric, and the projectile and the fabric were modeled with the kinematic predictor/corrector (hard) contact algorithm and the Coulomb friction model with $\mu = 0.3$.

In Figure 9, the transverse displacements along lines in the warp and weft directions through the point of impact are plotted for the experiment at four instances in time, with the final time

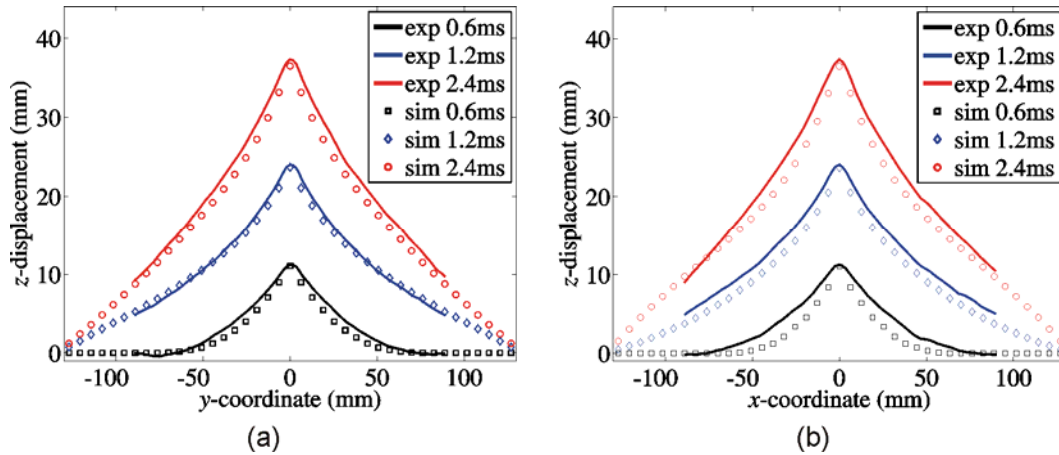


Figure 9. Simulation versus experiment for $R=12.7$ cm fabric with $v_i=22$ m/s: (a) warp yarn transverse displacement through point of impact, (b) weft yarn transverse displacement through point of impact.

corresponding to the maximum displacement of the fabric. During the experiment, the fabric deforms almost identically in the warp and weft directions. The isotropy of the experimental results is an indication that the specimen is subject to a preload. In fact, it is nearly impossible to clamp flat a fabric specimen without applying at least a small preload. In the simulation, the effects of preload are accounted for by specifying an initial deformation gradient, $\mathbf{F}^{\text{pre}}$, and initial changes in yarn amplitude, ΔA_1^0 and ΔA_2^0 . These parameters were determined by subjecting a single 4-node, reduced integration finite element to increasing levels of biaxial tension. Single element simulations were conducted with both equal force and equal stretch in the warp and weft directions. A parametric study with varying levels of $\mathbf{F}^{\text{pre}}$ and ΔA_i^0 was conducted for the $R=5$ inch simulations. It was found that a preload with equal force in the warp and weft directions provided a good fit to the experimental data. The preload was determined to be approximately 1.0 N per yarn, and the initial changes in amplitude are $\Delta A_1^0 = -0.0093\text{mm}$ and $\Delta A_2^0 = -0.0238\text{mm}$. The corresponding initial deformation gradient is

$$[\mathbf{F}^{\text{pre}}] = \begin{pmatrix} 1.00396 & 0 & 0 \\ 0 & 1.0135 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (13)$$

(Note: alternatively, to the same end, $\mathbf{F}^{\text{pre}}$ could be applied macroscopically to the fabric.) The results of the simulation exhibit exceptional agreement with those of the experiment (Figure 9).

Figure 10 displays the history of displacement of the fabric at the point of impact for both the experiment and simulation. Overall, the simulation compares very well with the experiment, even during the rebound phase of the deformation. Figure 11 shows that the deformed shape and

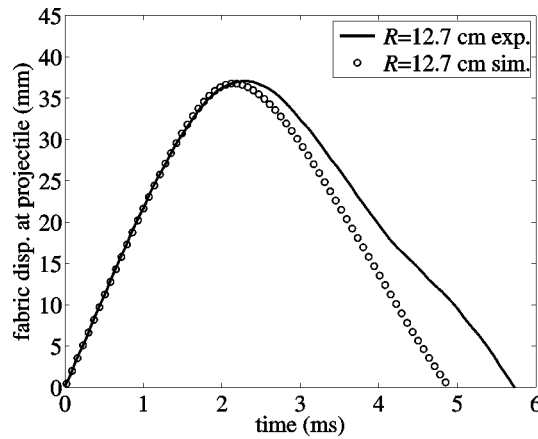


Figure 10. Simulation versus experiment of $R=12.7$ cm fabric with $v_i=22$ m/s: transverse displacement at point of impact.

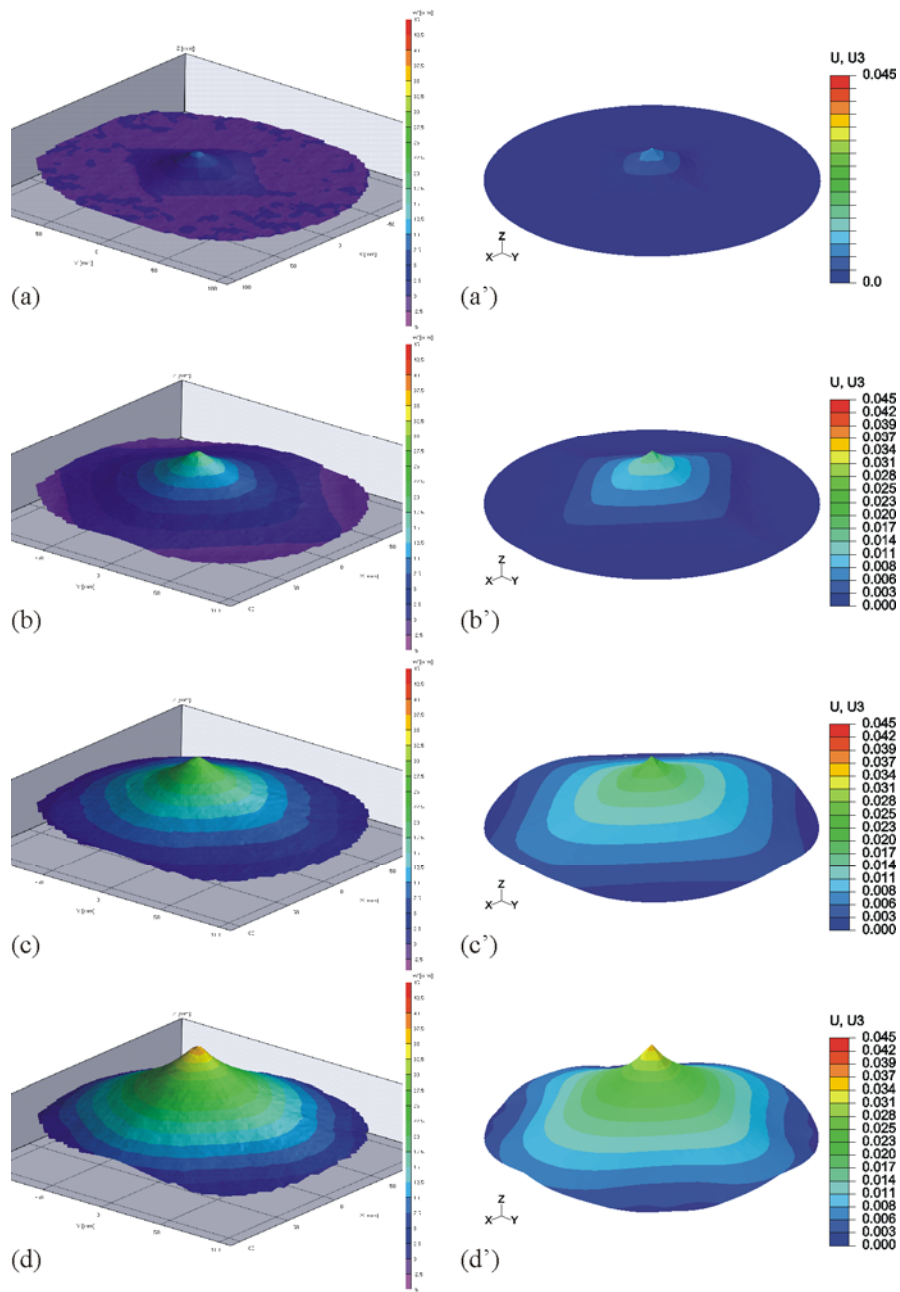


Figure 11. Contours of transverse displacement, as a function of time, for experiment (left, in mm) and simulation (right, in m) of $R=12.7$ cm fabric with $v_i=22$ m/s: (a), (a') $t=0.4$ ms; (b), (b') $t=0.9$ ms; (c), (c') $t=1.3$ ms; (d), (d') $t=2.0$ ms.

contours of transverse displacement of the experiment and simulation also correlate impressively. We note the “tent-like” deformation patterns of both the experiment and simulation. Observed by numerous experimentalists, this pattern of deformation is due to the highly anisotropic properties of the fabric.

5. Conclusion

The extension of the fabric model to an explicit finite element formulation with the deformed configuration of the unit cell determined solely from degrees of freedom driven by inertia has been a success. The model has proven to simulate projectile impact events accurately without excessive computational cost.

The model has many potential uses. Its computational efficiency enables the simulation of large and complex (including multi-ply) structures. Although a continuum-level model, it also simultaneously monitors the deformation of the fabric on the scale of individual yarns. Knowledge of local quantities such as yarn tension and yarn wavelength enables the prediction of phenomena such as penetration due to yarn breakage (e.g. bullet, knife threat) or yarn separation (puncture threat).

The framework of the model is versatile and may be applied to many existing or proposed materials. Other plain-weave Kevlar fabrics may be modeled by simply adjusting the purely physical material parameters. Fabrics of a different weave pattern require only a new unit cell geometry. Viscoelastic yarn deformation may be included in the model by modifying the tensile constitutive response of the yarns. Furthermore, its continuum nature will allow the model to be integrated with other material models for the simulation of fabric composites such as those used in helmets and armor panels.

6. References

1. King M. J., “A Continuum Constitutive Model for the Mechanical Behavior of Woven Fabrics Including Slip and Failure,” PhD Thesis, Massachusetts Institute of Technology, Cambridge MA, 2006.
2. King M. J., Jearanaisilawong P., and Socrate S., “A Continuum Constitutive Model for the Mechanical Behavior of Woven Fabrics,” *International Journal of Solids and Structures*, vol. 42, pp. 3867-3896, 2005.
3. Weerasooriya T., Gunnarsson C.A., and Moy P., “Measurement of Full-Field Transient Deformation of the Back Surface of a Kevlar KM2 Fabric During Impact for Material Model Validation,” *Proceedings of the 2008 International Congress and Exposition on Experimental and Applied Mechanics*, Orlando FL, June 2008.

7. Acknowledgments

This research was supported by the U.S. Army Research Office through the MIT Institute for Soldier Nanotechnologies under contract number W911NF-07-D-0004.