

Representation of Coriolis forces and simulation procedures for moving fluid-conveying pipes

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Abstract: For inclusion of inertia effects of moving fluid columns or strings into structural mechanics simulations, a combined finite-element modelling scheme is proposed in ABAQUS. The concept has been realized in the form of user subroutines (UEL) of beam type. These can be coupled to existing standard (e.g., Euler-Bernoulli or Timoshenko) elements. The proposed Coriolis elements can be applied for, e.g., prediction of loads from high-speed flows and phase shifts of oscillating fluid-conveying pipe systems with arbitrary complex designs. The modelling scheme also allows a flexible coupling to shell or solid finite-element models.

Keywords: finite elements, simulation, vibration, inertial force, Coriolis force, fluid-structure-interaction, multi-physics, mass flow sensors.

1. Introduction

In this short note inclusion of inertia effects for fluid conveying pipes into finite element structural mechanics simulations is discussed.

Fluid structure interaction is a complex and very interesting field. Here we discuss a rather simple system. Consider a (possibly curved) pipe with some lateral deflection $w(x,t)$ conveying a fluid. The fluid motion will influence the pipe deflection in two ways: firstly, if the fluid moves on a curved path, a centrifugal force will occur. Secondly, as the fluid at each position x along the pipe has approximately a transverse velocity corresponding to the velocity of the pipe (neglecting compressibility effects here), the flow along the tube axis will lead to the transport of momentum, if the velocities $w_t(x,t)$ along the tube axis are not constant. Both forces are inertial forces, proportional to the mass density per length of the fluid.

They are also of technical importance: so-called Coriolis flow meters use the latter force for a very precise measurement of mass flow. They have been termed “Coriolis” meters, as the second inertial force described above is the Coriolis force in the case of a curved or straight fluid-

conveying pipe with position-dependent transverse velocities. This is most evident in the case of pipe oscillations which are locally perpendicular to the osculating plane of a curved pipe.

Let us now discuss the effect quantitatively. For a straight tube filled with a fluid which may move with velocity v in axial direction, lateral deflection $w(x,t)$ is described by the differential equation (e.g.: Garnett et al., 1993; Raszillier, 1991)

$$(\rho A + \rho_f A_f) w_{tt} + EI w_{xxxx} - F w_{xx} + \rho_f A_f v^2 w_{xx} + 2 \rho_f A_f v w_{tx} = 0.$$

Here, ρ and ρ_f are the densities of the tube material and fluid, respectively. E is the Young's modulus of the tube material, I the tube cross section's moment of inertia. A and A_f are cross section areas of pipe wall and fluid column, respectively. F is the force defining axial pre-stress.

The equation of motion is easily derived from the Lagrangian L of the tube and L_f of the fluid

$$L = T - V = \frac{\rho A}{2} \cdot \begin{pmatrix} 0 \\ w_t \end{pmatrix}^2 - \frac{EI}{2} w_{xx}^2 - \frac{F}{2} w_x^2,$$

$$L_f = T - V = \frac{\rho_f A_f}{2} \cdot \begin{pmatrix} v \\ w_t + v \cdot w_x \end{pmatrix}^2,$$

which are defined as the difference $T - V$ of kinetic and potential energy. First and second row in the vector notation give axial and transversal velocity component, respectively.

In the following, we show that the term $\rho_f A_f w_{tt}$ can be described realistically with acoustic elements in Abaqus. Furthermore, the centrifugal force term $\rho_f A_f v^2 w_{xx}$ and the Coriolis term $2 \rho_f A_f v w_{tx}$ are modelled by two Abaqus user subroutines of type UEL (Dassault Systèmes, 2008a, ch. 27.16, and Dassault Systèmes, 2008b, ch 1.1.22).

2. Acoustic elements for fluid inertia

The most straightforward method for inclusion of fluid mass inertia in vibration calculations of a pipe is adding nonstructural mass to the walls of the pipe. This method, as it turns out, gives sufficiently accurate results for pure bending modes.

For modes including rotational motion about the pipe's longitudinal axis, however, in most cases the fluid almost does not take part in the rotation. The influence of fluid inertia on pipe motion, if modelled by nonstructural mass attached to the tube walls, is overestimated in such cases. A realistic model of fluid inertia for fluids with moderate viscosity should give the correct result for bending motion without adding inertia to axial rotation. We have tested (Gebhardt, 2008) the suitability of Abaqus acoustic elements for this kind of modelling in the case of standard titanium tubes which are filled with fluids of variable density.

Calculations with Abaqus showed that, indeed, a fluid-column model built from acoustic elements can accurately account for inertia in bending modes. Deviations from a Timoshenko beam model, which is used as reference, are less than 0.4 % for the modes considered here. On the other hand, different from the model with nonstructural mass, representation of the fluid as acoustic medium in Abaqus effectively decouples rotations of fluid and tube around the tube axis. In the latter case tube rotation frequency remains independent from fluid filling, whereas an effective-mass formulation strongly affects torsional mode frequencies.

For simulation of a finite boundary layer in cases of higher fluid viscosity, both approaches may be combined.

3. User element for centrifugal forces

For a general bent pipe filled with a moving fluid, centrifugal forces have two consequences. First, they cause static loads on all sections of the tube which have non-vanishing curvature in the base state. Second, they have to be considered as solution-dependent forces on those tube sections which show a curvature after deflection.

An Abaqus user element can account for both of these effects. User subroutine UEL provides, in particular, the variables **coords(N)** and **u(N)** for base state node coordinates and deflections, respectively. Looking at the discretized system of equations for the node deflections **u** in static and dynamic analyses

$$\mathbf{M} \mathbf{u}_{tt} + \mathbf{C} \mathbf{u}_t + \mathbf{K} \mathbf{u} = \mathbf{F}_{\text{ext}},$$

(where **M**, **C** and **K** denote mass, damping and stiffness matrix, respectively) we note that the centrifugal load gives additions to the stiffness matrix and to the right-hand side vector of external forces.

The UEL is written as a three-node element as depicted in Figure 1.

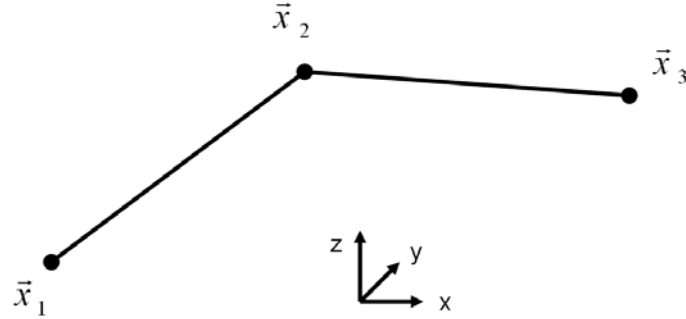


Figure 1. Geometry of the centrifugal force element.

In its simplest form, sufficiently accurate for many applications, the element acts on the corresponding nine translational degrees of freedom, and its addition K_{UEL} to the stiffness matrix reads

$$K_{UEL} = \rho_f A_f v^2 \begin{pmatrix} -1 & 0 & 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 2 & 0 & 0 & -1 \\ \hline -2 & 0 & 0 & 4 & 0 & 0 & -2 & 0 & 0 \\ 0 & -2 & 0 & 0 & 4 & 0 & 0 & -2 & 0 \\ 0 & 0 & -2 & 0 & 0 & 4 & 0 & 0 & -2 \\ \hline -1 & 0 & 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 2 & 0 & 0 & -1 \end{pmatrix}.$$

The contribution to F_{ext} , which in Abaqus is accessible as the variable **rhs(N)**, is defined analogously. The contribution to the right-hand side vector has, e.g., successfully been tested (Gebhardt, 2008) for a rotating ring structure, reproducing the correct tangential principal stress $\sigma = \rho v^2$, where ρ and v denote the ring material's density and tangential velocity, respectively.

Correctness of the stiffness contribution has been checked through the frequency decrease of the lowest eigenmode of a standard stainless steel pipe filled with water. Results have been compared to Euler-Bernoulli theory for the ground mode shift with pinned-pinned boundary conditions, where an analytical formula can be derived and used as reference. Numerical data show good agreement with theory even for rather extreme fluid velocities, where the model is shifted considerably towards the instability of the physical system itself.

4. User elements for Coriolis forces

The user element geometry is given in Figure 2.

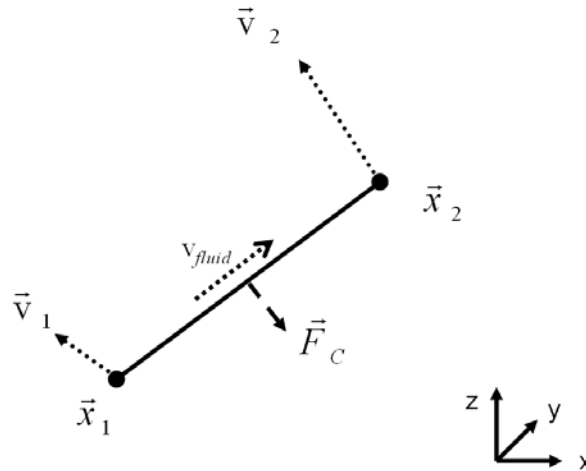


Figure 2. Geometry of the Coriolis force element.

The Coriolis force on an element is given by

$$\vec{F}_C = -2\rho_{fluid}A_{fluid}v_{fluid}(\vec{v}_2 - \vec{v}_1)_{\perp},$$

where the bracket with subscript denotes the components perpendicular to the element's longitudinal axis. The force has been coded as **rhs** – vector in an Abaqus user subroutine of type UEL.

The UEL has been designed for use in the ***Dynamic, direct** – procedure. This procedure has been chosen because it offers, besides the ability to call user elements, the opportunity to specify heuristic damping for the complete model via ***Damping, alpha= ..., beta= ...**.

For verification of the element formulation, we chose a running steel wire (with Young's modulus $E = 200$ GPa, and density $\rho = 7940$ kg/m³) of length 2m and cross-section radius 1 mm. It has been defined between the end points $x_{1/2} = \pm 1$ m with pinned-pinned boundary conditions.

Damping has been specified by **alpha=0** and **beta=0.01**. This produces quality factors Q_i for the systems i^{th} eigenmode with angle frequency ω_i as follows:

$$Q_i = \frac{1}{\beta \cdot \omega_i}.$$

So, since the system's lowest eigenfrequency ω_0 is about 6.2 Hz, the quality factor Q_0 assumes a value of about 17.

In order to get stable steady state solutions for a situation where mainly the ground state is driven by a periodic force, therefore, we have to integrate the system from arbitrary starting conditions for a time

$$T_{convergence} \approx Q_0 \cdot \frac{2\pi}{\omega_0}.$$

The system is driven periodically at the resonance frequency ω_0 by a lateral force ($F = 10^{-4}$ N) in the middle of the tube.

The steady state under influence of Coriolis forces can be expected to be seen after 20 – 30 s integration time. Testing of the element, now, has been done by the study of induced relative phase shifts between two given points on the wire (here chosen as $x_{L/R} = \pm 0.56$ m).

Analytical investigation of the system (Kassubek, 2004) leads to closed expressions for the expected phase function along the wire in first order perturbation theory. Reducing the dynamics, additionally, to a two-level-system of the ground mode and first harmonic, we get:

$$\phi(x) = \arctan\left(\frac{32}{3} \frac{v_{fluid}}{L} \frac{\omega_0}{\omega_1^2 - \omega_{01}^2} \sin\left(\frac{\pi}{L} x\right)\right)$$

The phase shifts extracted from Abaqus calculations are compared to analytical results in Figure 3. The analytical exact results have been obtained by imposing the boundary conditions on the general solution of the Euler-Bernoulli equation at given frequency (with flow), leading to nontrivial solutions only for a discrete spectrum. The finite element result is compared to the lowest eigenmode.

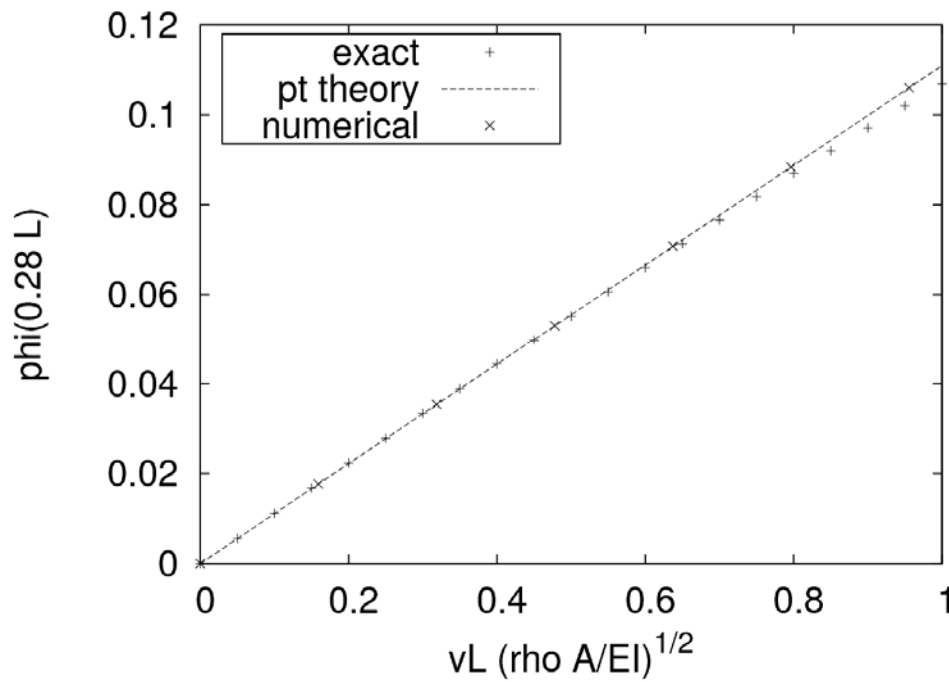


Figure 3. Verification of the Coriolis force element. The phase difference between the points $x_{L/R} = \pm 0.56$ m is given, as a function of dimensionless-scaled flow velocity, for the Abaqus calculation (“numerical”), for first order perturbation theory (“pt theory”) and for the analytical solution (“exact”).

As can be seen, agreement is exceptionally good. Remaining small deviations may be caused mainly by damping influences and the limitations of first order perturbation theory with truncated spectrum. It should be mentioned that the exact solution takes also in account the centrifugal force, which leads to the slight non-linearity for large flows. We have verified that it perfectly agrees with the finite element solutions if the centrifugal terms in the exact solution are neglected. A unit dimensionless flow, as it is defined here, means $1/\pi$ of the critical flow where system instability sets in.

The “tumbling” wave function caused by Coriolis forces can be viewed as real part of a complex solution

$$w(t, x) = \Re \left[(a(x) + i \cdot b(x)) \cdot e^{i\omega_0 t} \right]$$

of the equation of motion. For illustration, in Figure 4 we have plotted for our system analytical solutions for $a(x)$ and $b(x)$ in the special case of zero drive and zero damping. In this case, $a(x)$ is a symmetrical function in x , whereas $b(x)$ is anti-symmetrical.

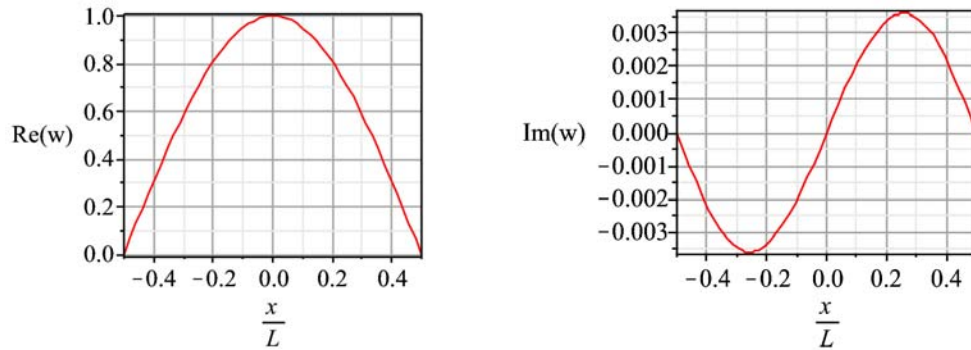


Figure 4: Eigenfunction vs position (in x/L). Left: Real part of the basic function (dimensionless velocity $vL/(\rho A/EI)^{1/2}=0.1$). Top right: imaginary part.

5. Conclusions

We have seen that Abaqus may serve as a very flexible tool for modelling inertia of systems like pipes with fluid flow, conveyor belts or chain drives. It has been shown that Abaqus UELs can be applied for modelling Coriolis forces, which is slightly more intricate than modelling other inertia forces because it contributes to the damping structure rather than to the stiffness or mass matrices.

A drawback in currently available Abaqus versions: It is at least inconvenient, if not impossible, to model additions to the damping matrix directly by user subroutines, since the latter are currently not yet supported by the relevant procedures ***Steady dynamics,direct**, ***Steady state dynamics,subspace** and ***Complex frequency**. A request for enhancement on this topic has been issued.

In this note, a practical workaround has been presented: Using ***Dynamic, direct**, the vibrating system can be driven and integrated in time until the steady state is reached. Of course, this procedure is computationally expensive for larger systems and in cases where damping is small. For this reason, and for convenient postprocessing of phase and damping data in vibrational systems, it would still be very desirable to include the Coriolis effect into procedures for steady-state calculation or complex eigensolvers.

Nevertheless, user elements as proposed in this note, together with advanced strategies for coupling and contact to general structures, give remarkable opportunities to study dynamical effects e.g. of pipe boundary conditions (Kassubek, 2004) or of time-dependent flow conditions (see, e.g. Gebhardt, 2004; Gebhardt, 2006).

6. References

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