On Nonlinear Buckling and Collapse Analysis using Riks Method

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Abstract: *Nonlinear analysis using Riks method is suitable for predicting buckling, post-buckling, or collapse of certain types of structures, materials, or loading conditions, where linear or eigenvalue method will become inadequate or incapable, especially when nonlinear material, such as plasticity, is present, or post-buckling behavior is of interest. These structures usually undergo finite deformations due to complicated loadings or material plasticity before buckling actually occurs, which changes system matrices, and thus, makes the eigenvalue analysis inaccurate, difficult, or even impossible to perform. This study intends to demonstrate the use of Riks method in the nonlinear analysis of buckling and post-buckling behaviors of a flexible structure under bending and compressive loads. The null-point on load-displacement curve is used as criteria for the onsite of instability. The predicted results from finite element analysis compare well with testing data.*

Keywords: *nonlinear, structure, buckle, post-buckle, collapse, Riks, critical load, bifurcation, instability, load-displacement curve, null point*

1. Introduction

Buckling is when a flexible structure loses its stability, which may lead to a sudden and catastrophic failure, such as the complete collapse or breakage of the structure [Ugural, 1987]. When compressive loading is present, buckling may become a concern. Sometimes, it is the limiting factor for structural designs. Some of the examples are found in petrochemical, refining, or nuclear industries, where reactors could be subjected to net external pressures or other types of compressive loads. Understanding the buckling, post-buckling, or collapse behaviors in some cases, is critical for maintaining safe operating conditions.

In linear elastic stress analysis, equilibrium is based on the original undeformed configuration; while for linear elastic instability problem, deformed shape is considered, although the deformation before instability is usually very small compared to structure's original geometry. Typical applications are the long and slender beams under compressive axial loads. The onsite of buckling will lead to an instantaneous increase in lateral deflections. For this type of problems, theory of linear elastic buckling analysis serves well in predicting the onsite of the buckling or critical loads. In other situations, when a structure undergoes finite deformation due to complex load or material plasticity before instability actually occurs, system parameters change along with the deformation, thus, makes the eigenvalue analysis inaccurate, difficult, or even impossible to perform. In this case, a nonlinear analysis becomes necessary in order to simulate this type of highly unstable behavior. To demonstrate the concept, consider a rigid rod of length *L* with its weight being ignored. The rod is pin-supported at its bottom and connected at the top to an axial spring with stiffness *K* as shown in Fig. 1.

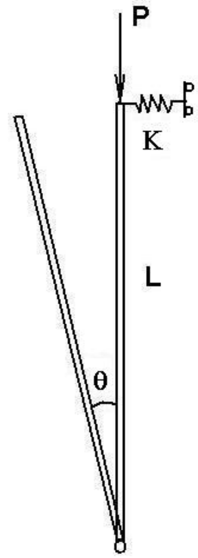

Figure 1 A pin-supported rigid rod.

At equilibrium, the rod is rotated from its vertical position to an unknown angle θ about the pinned-support under an unknown vertical force *P* and the axial spring. The total energy of the system, including external work and spring potential, can be written as

$$U = PL(1-\cos\theta) - \frac{1}{2}KL^2\sin^2\theta \tag{1}$$

By applying a small disturbance $\delta\theta$ to the system at the equilibrium position θ, there must have

$$\frac{\delta U}{\delta\theta} = 0 \tag{2}$$

Carry out the operation, simplify and rearrange, we obtain the following equation

$$(P - KL\cos\theta)\delta\theta = 0 \tag{3}$$

For a nontrivial solution of $\delta\theta$, there must be

$$P - KL\cos\theta = 0 \tag{4}$$

or the system stiffness must reduce to zero. At this critical point, the system loses its stability. In general terms, structural instability or buckling occurs. If there is negligible deformation before the onsite of instability, or $\theta \rightarrow 0$, the critical load or bifurcation load P_{cr} can be calculated easily and accurately from the aforementioned linear or eigenvalue analysis as

$$P_{cr} = KL \tag{5}$$

On the contrary, if the system accumulated deformation is not negligible prior to instability due to the loads that ultimately causes its instability, the critical load becomes system configuration or deformation dependent. The onsite of instability or buckling is determined by its load-displacement history. In this case, linear or eigenvalue analysis becomes inaccurate or even impossible to perform due to lack of the inclusion of large deviation from the original geometry. A nonlinear analysis becomes the essential tool to perform the buckling analysis for this type of systems.

In nonlinear static analysis for buckling, post-buckling, or collapse behavior, the tangent stiffness from the load-displacement response curve could change signs when system changes its stability status as shown in Fig 2. The classical Newton's method will not work in this situation because the corrections for approaching equilibrium solutions during iterations may become difficult to determine when the tangent stiffness is close to null. There are different approaches to solve such problems, such as switching to dynamic analysis, using displacement controlled static analysis, or adding dashpots for stabilization during sudden strain energy release. But those methods are not without limitations in such aspects as high computational cost, non-unique responses due to jump phenomenon, or artificially altered responses. Alternatively, static equilibrium states during the unstable phase of the response can be found by using the "modified Riks method" [Abaqus, 2007]. This method is used for cases where the loading is proportional; that is, where the load magnitudes are governed by a single scalar parameter. The basic Riks algorithm is essentially Newton's method with load magnitude as an additional unknown to solve simultaneously for loads and displacements, thus, can provide solutions even in cases of complex and unstable response such as that shown in Fig. 2. The only requirement is that the system be continuous or reasonably smooth.

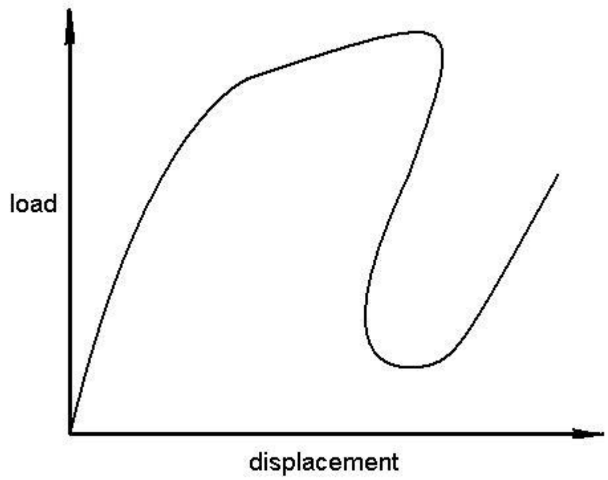

Figure 2 A typical unstable response curve.

2. Buckling of a Louvered Structure

Using finite element and experimental methods, a cylinder with a louvered structure as shown in Fig. 3 is numerically analyzed and tested for the louver buckling and post-buckling behaviors. The whole structure is made of stainless steel *304*. The Elastic Modules is *195,000 MPa*. Fig. 4 shows the stress-strain curve at ambient temperature. The material's average or nominal properties are obtained from ASME standards [ASME, 2004].

Fig. 5 shows the structure on test stand before and after buckling. The louver is welded to the cylinder that is constrained at top. A rigid ring is positioned just under the lower edge of the louver. A piston pushes the ring upward. As the ring gradually moves upward, the louver, which is subjected to bending and compressive loads through the contact with the ring, slowly deforms. At the beginning, the vertical force needed to push the ring upward will increase along with the displacement of the ring or the deformation of the structure. When the force or deformation reach a certain level, the louver starts to loose its stability and buckles. At this point, the force starts to decrease while louver deformation continues to increase, indicating that the structural stiffness has dramatically reduced due to instability or buckling. During testing, the applied forces and corresponding displacements of the ring are recorded. Using nonlinear static analysis procedure with modified Riks method, the buckling and post-buckling behaviors of the structure are numerically simulated, and results including the predicted critical load and post-buckling behavior are compared to test data.

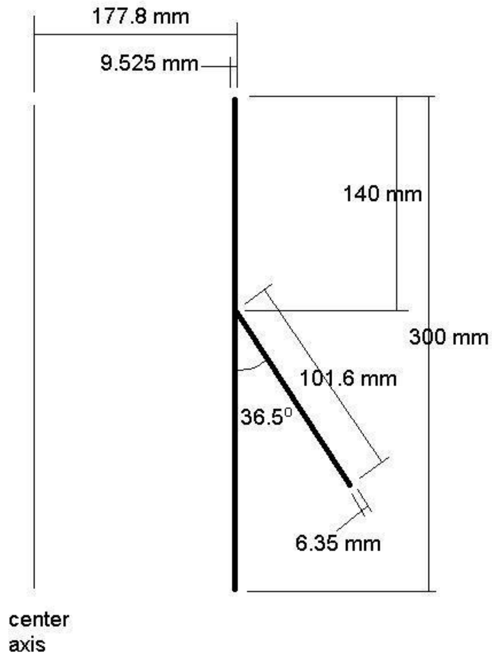

Figure 3 Crosssection of louvered structure.

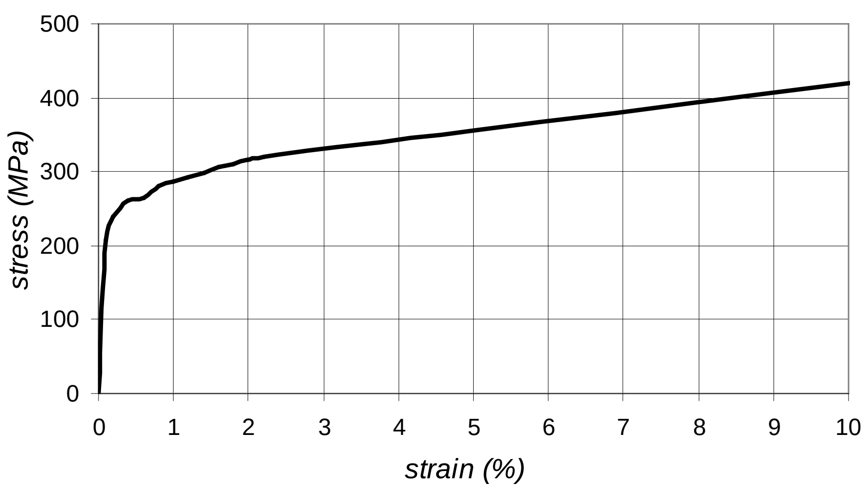

Figure 4 Stress-strain curve for ss304 at ambient temperature.

Fig. 6 shows the finite element model and simulation of the structure and the ring. Shell elements are used for the structural model and the ring is modeled as a rigid body by solid elements. Frictional contact with large sliding is modeled between the louver and the ring. The frictional coefficient is set at *0.3*. Same constraints as those in the experiment are imposed to the top of the cylinder. A variable point force is applied to the center of the ring and tries to push the ring, and thus the lower edge of the louver, upward. For analysis, geometrically nonlinear static analysis procedure using the modified Riks method has been performed to simulate the buckling process. To start the simulation, a small trial force is applied to establish the initial equilibrium state. To let the simulation to progress, an increment of the force, which is treated as an unknown, is added to the previous value and solved simultaneously with deformation for the next equilibrium state along the path using iterative method. The force increments could become negative or change signs along the path as dictated by equilibrium and stability status. This process repeats until preset criteria, such as maximum deformation, is met.

During simulation, the actual applied forces and the resulting displacements of the ring at each and all equilibrium states are sampled and plotted against the test data as load-displacement curves, which are shown in Fig. 7. From the load-displacement curves, the force and displacement pairs increase nearly proportionally at the first stage, showing that the structure deforms close to linear and is in stable range. As the force and displacement become larger, the slope of the curve gradually reduces, indicating that certain nonlinear effects such as large deformation or material plasticity become prominent. When reaching the null point, where slope of the curve becomes zero because structure is losing its stiffness, instability or buckling has occurred. For critical load, the louvered structure will buckle at about *170 KN*. The numerical predictions match very well with the measured data. For post-buckling stage, the slope of the curve initially becomes negative in responding to the stiffness reduction due to instability, and then, returns to positive when stable equilibrium states are re-established. In post-buckling stage, trend of the load-displacement curve from numerical simulation matches that from experiment, but somewhat off in absolute values. The reason might be the lack of accurate material properties such as stress-strain data at large strains in simulations. The deformed or buckled shapes from simulation and experiment are very similar as shown in Figs. 5 and 6.

Fig. 7 shows how various energy terms, including external work and plastic strain energy for the whole structure, change during the process from numerical simulations. At the two points when system stability status changes at near *0.6 mm* and *1.8 mm*, the curve for external work changes its slopes slightly. Also, the plastic strain energy starts to accumulate well before buckling occurs. Only a nonlinear analysis can capture such behavior.

3. Summary

The nonlinear static analysis procedure using the modified Riks method works very well to simulate the buckling behavior of structures exhibiting large pre-buckling deformations or plasticity, or subjected to bending-compressive loads. Post-buckling or collapse behavior can also be captured given better material data at large strains. The numerical results compare well with experimental data.

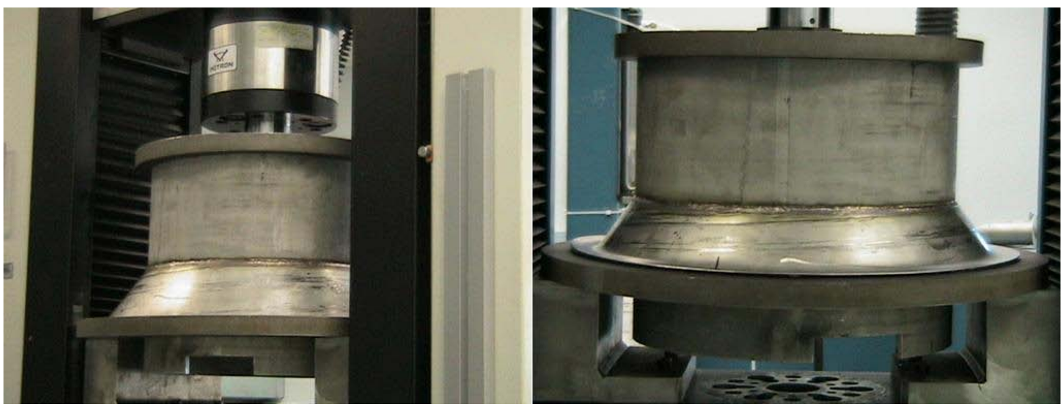

Figure 5 Experimental setup – left for pre-buckling and right for post-buckling.

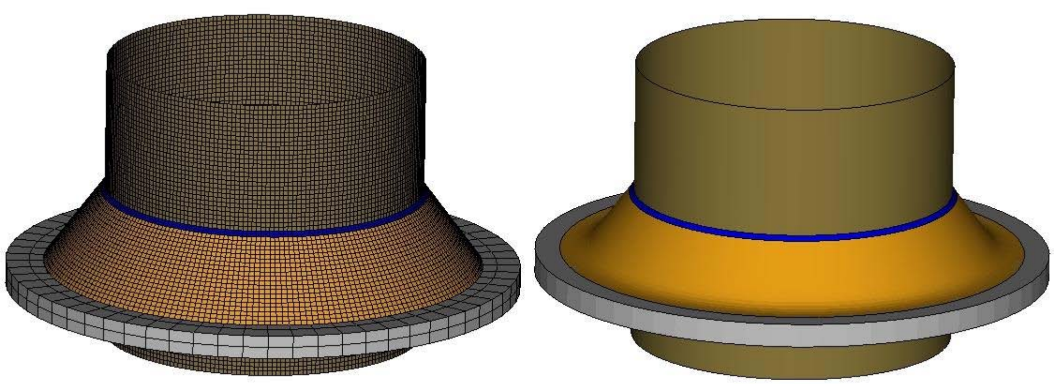

Figure 6 Finite element model – left for original shape and right for deformed shape.

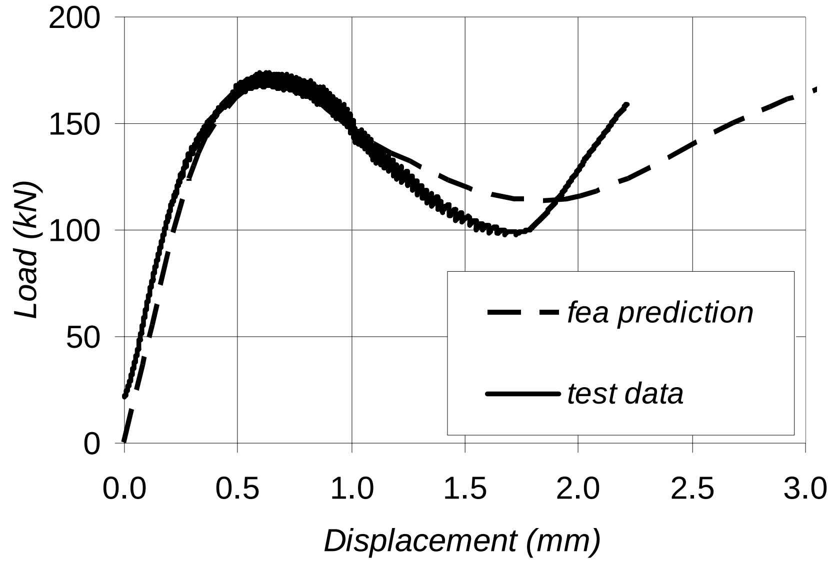

Figure 7 Load-displacement curves of the louvered structure.

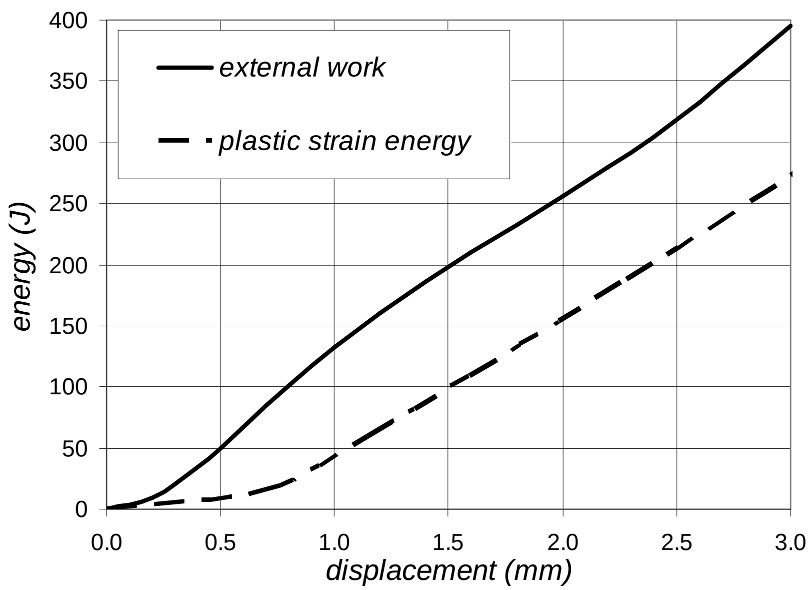

Figure 8 Energy plot of the louvered structure.

4. Acknowledgement

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5. Reference

1. "Abaqus Analysis User's Manual," Dassault Systems, 2007.
2. "ASME Boiler and Pressure Vessel Code, 2004, Section III, Division 1, subsection NH, T-1800," American Society of Mechanical Engineers.
3. Ugural, A. C., and Fenster, S. K., "Advanced Strength and Applied Elasticity," Elsevier Science Publishing Co., Inc., New York, 1987.