Modeling of soils as multiphase-materials with Abaqus

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Abstract: Soils are mixtures of three phases, a porous soil skeleton of solid mineral particles and two fluids water and air filling the pores completely. The soil skeleton and the fluids can move depending on the constitutive behaviour of all constituents and interactions between them. In general a coupled deformation/seepage problem exists which can be uncoupled for certain circumstances. Mechanical models of soils are based on continuum mechanics or particle mechanics. The continuum approach requires a mixture theory, uncoupled problems require the classical continuum mechanics for single-phase materials only. Abaqus has built-in features to carry out uncoupled deformation or coupled deformation/seepage analysis for saturated and unsaturated soil which are described in this paper. Further models and analyses will be shown applying a user subroutine of type UEL. In this model an elastic or hypoplastic stress-strain relation for the soil skeleton is assumed. The user subroutine is applied in some case studies, results will be compared to Abaqus built-in analyses if available.

Keywords: FEM, Abaqus, theory of mixtures, dynamic three-phase model, consolidation

1. Introduction

Soils are heterogeneous mixtures of a porous skeleton of solid mineral particles of different sizes and shapes and two miscible fluids water and air filling the pore space completely (additional phases are disregarded for the sake of simplicity). Under certain circumstances the pores are saturated by a single fluid only: dry or water saturated soil. Under saturated conditions a fully undrained state, a consolidation process or a fully drained state can occur depending on the load velocity, permeability of soil, drainage conditions at boundaries and seepage distance. Due to external actions and seepage the soil can deform. The deformation depends on the material behavior of all three constituents and on the interaction between them (contact, buoyancy, seepage, capillarity). To calculate the deformation of the soil skeleton and the movement of the fluids a coupled boundary value problem has to be solved in general. Under certain circumstances like saturation under drained or undrained conditions the problem can be uncoupled and the deformation and seepage analysis can be calculated consecutively.

Soil models base on continuum mechanics or a discontinuum approach (particle mechanics). The former model corresponds to a hypothetic continuum model where the movement of soil skeleton and fluids is described with field equations, i.e. kinematic, balance and constitutive equations. Corresponding boundary value problems (b.v.p) are completely defined and can be solved only with prescribed initial and boundary conditions.

Abaqus is a program based on the continuum approach solving the corresponding b.v.p. with the Finite Element Method. Abaqus built-in features contain single- and two-phase models enabling uncoupled deformation and seepage analysis for saturated soils under drained conditions and coupled deformation/seepage analysis for saturated soils at consolidation process disregarding mass inertia effects. An additional tool considering the relation between matrix suction and degree of saturation of unsaturated soils is restricted to the case of atmospheric pore air pressure. To overcome these restrictions a user defined element including user defined material models is implemented in Abaqus enabling for static, quasi-static and dynamic fully coupled deformation/two-phase seepage analysis of saturated and unsaturated soils based on the theory of mixtures.

2. Continuum models for soils

Continuum models of soils describe the movement of soil skeleton and fluids based on field equations. In general mixture theories are necessary therefore considering the interaction between phases resulting in models for coupled dynamic deformation/seepage analysis of unsaturated soils. The following continuum models describe selected common combinations of degree of saturation, drainage conditions, boundary conditions and mass inertia effects and can all be regarded as special cases of the coupled dynamic deformation/seepage analysis of unsaturated soils:

- coupled dynamic deformation/seepage analysis for water saturated soil, reducing the model to a two-phase formulation,
- quasi-static consolidation analysis for water saturated soil, reducing the model to a two-phase formulation neglecting inertia effects,
- quasi-static deformation/seepage analysis for unsaturated soil with athmospheric pore air pressure neglecting inertia effects,
- transient and steady-state two-phase flow in unsaturated soil under assumption of rigid soil skeleton, neglecting inertia effects and disregarding soil deformation.

Under certain circumstances, primarily concerning the degree of water saturation and the drainage conditions, the problem can be uncoupled and the deformation and seepage analysis can be carried out consecutively:

- effective stress analyis (static or dynamic) under fully drained conditions for saturated soils (both air and water saturated), where no excess pore fluid pressures occur, i.e. external loads result directly in a deformation of the soil skeleton.
- Total stress analysis (static or dynamic) under fully undrained conditions for saturated soil, where external loads are taken by fluid only, i.e. excess fluid pressures occur corresponding to the external load.
- one-phase seepage analysis in saturated soil under assumption of rigid soil skeleton, neglecting inertia effects and disregarding soil deformation.

In this case the classical continuum mechanics for single-phase materials can be applied considering an effective or total stress analysis, a mixture density and a stress-strain relation for the soil skeleton formulated in effective or total stresses.

3. Abaqus built-in features and enhancements

Abaqus includes uncoupled deformation and seepage analyses as well as consolidation analysis and can be expanded using different user subroutines enabling for more complex continuum models or just more complex stress-strain relations for soil skeleton, see Hügel et al. (2008) for details.

3.1 Abaqus built-in analyses for soils

Abaqus offers the following uncoupled deformation and seepage analyses for soils:

- Uncoupled static deformation analysis for saturated soil under fully drained conditions as effective stress analysis. Depending on degree of saturation the self weight of soil have to be calculated by ρ_d (dry soil) or ρ' (water saturated soil). The stress-strain relation for the soil skeleton is formulated in effective stresses.
- Uncoupled dynamic deformation analysis for saturated soils under fully drained conditions based on an effective stress analysis. The scheme to integrate the hyperbolic pde's can be implicit (Abaqus/Standard) or explicit (Abaqus/Explicit). See static deformation analysis for definition of density and stress-strain relation.
- Uncoupled deformation analysis for undrained conditions is actually not provided by Abaqus. A corresponding total stress analysis requires a stress-strain relation for the two-phase mixture and a self weight of soil calculated by density ρ_r. This can be realized as follows:
    1. Execution of a coupled quasi-static deformation/seepage analysis (consolidation analysis) where Abaqus finds out if certain soil layers have undrained or drained conditions or consolidate depending of load velocity, permeability of soil, drainage conditions at boundaries and drainage distances.
    2. The user can categorize soil layers as drained or undrained by assessment of load velocity, permeability of soil, drainage conditions at boundaries and drainage distances. Typically granular soils are declared as drained and cohesive soils are declared as undrained. An uncoupled deformation analysis for undrained conditions can be done by assuming either elastic or linear elastic, perfectly plastic behavior (Mohr Coulomb plasticity) because these models can be formulated in total stresses as well. A corresponding parameter set for undrained conditions ensuring isochoric deformation under elastic or elasto-plastic material response is necessary therefore, for example: Youngs moduls E_u, Poisson's ratio $\nu_u \approx$ 0,495, internal friction angle $\varphi_u \approx 0$ for saturated soils, cohesion c_u and dilatancy angle $\psi_u \approx 0$.

Alternatively the undrained analysis in total stresses can be carried for arbitrary stress-strain relation by using Abqus user subroutine UMAT to define a stress-strain relation for the mixture formulated in total stresses.

- Uncoupled transient and steady state seepage analysis can be realized by setting all displacement degrees of freedom of the soil skeleton to zero.

Abaqus offers the following coupled deformation/seepage analyses for soils:

- Coupled quasi-static deformation/seepage-analysis (consolidation analysis) for water saturated soils where mass inertia effects are neglected. Assuming geometrical and material linearity this corresponds to Terzaghi's theory of consolidation.
- Coupled quasi-static deformation/seepage analysis for unsaturated soils where the pore air pressure p_g corresponds to the atmospheric pressure p_{atm}. The pore water pressure p_w can be negative here considering matrix suction.

3.2 Abaqus enhancements concering analyses for soils

Some uncoupled and coupled deformation/seepage analyses for soil are not built-in but can be implemented via user subroutines for user defined elements (UEL) or user defined stress-strain relations for the soil skeleton (UMAT), for example:

- Uncoupled deformation analysis for water saturated soils under fully undrained conditions for arbitrary stress-strain relations for the soil skeleton.
- Coupled dynamic deformation/seepage analysis (consolidation analysis) for water saturated soils considering mass inertia effects comparable to Biot's theory of poroelasticity.
- Coupled dynamic deformation/seepage analysis for unsaturated soils based on Theory of Mixtures or Theory of Porous Media. See Section 3.3 for an example.
- Uncoupled transient and steady-state two-phase seepage analysis for unsaturated soils assuming rigid soil skeleton comparable to Richards equation.

See Section 3.3 for an example of enhancement of Abaqus by means of UEL subroutine.

3.3 Implementation of user subroutine UEL for coupled dynamic deformation/seepage analysis of unsaturated soils

The coupled deformation/seepage problem was firstly solved by Terzaghi (1943) for one-dimensional consolidation. It was enhanced by Biot (1941, 1957) to three-dimensional problems in Biot's Theory of Poroelasticity considering the compressibility of solid particles and pore water as well as mass inertia effects. Biot's model is still a two-phase model for water saturated soils. More recent soil models are based on the theory of mixtures for the unsaturated soil.

The implemented UEL bases on the equations according to Holler (2006) taking into account the following concepts:

- Volume fraction concept: The soil is devided into the three constituents solid, water and air. n denotes the volume fraction of the pores, filled with water and/or air, also referred to as porosity. $S_w n$ and $S_g n$ are the volume fractions of the water phase and the gas phase respectively and $(1-n)$ is the volume fraction of the solid phase. S_w und S_g are the water and gas saturation respectively ($S_w + S_g = 1$). The volume fractions of the three constituents add up to 1.

- Principle of effective stresses for saturated soils (Terzaghi, 1943) and for unsaturated soils, first discribed by Bishop (1959). Setting the parameter according to Bishop $\chi = S_w$ leads to the following equation:

$$\boldsymbol{\sigma}' - \mathbf{m}^T \left(S_w p_w + S_g p_g \right) - \boldsymbol{\sigma} = \mathbf{0} \tag{4}$$

  Herein $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}'$ denote the total and effective stresses respectively (negative for pressure) and $\mathbf{m}$ is the second order unit tensor.

- Kinematic equations with the following primary unknowns: displacement field $\mathbf{u}_\mathbf{s}$ of the soil skeleton and the relative seepage velocities $\mathbf{v}_\mathbf{ws}$ and $\mathbf{v}_\mathbf{gs}$. Concernig the relative seepage velocities the generatlised Darcy's law for the flow of two fluids through a porous media is implemented:

$$nS_i \mathbf{v}_{is} = \frac{\mathbf{k} k_{ri}}{\eta_i} \left(-\nabla p_i + \rho_i \cdot (\mathbf{g} - \ddot{\mathbf{u}}_\mathbf{s} - \ddot{\mathbf{u}}_\mathbf{is}) \right) \tag{5}$$

  $\mathbf{v}_\mathbf{is}$ is the velocity of the fluid phase (water or gas respectively) relating to the solid phase and $\mathbf{k}$ denotes the permeability of the solid phase. k_{ri} is the relative permeability relating to the respective fluid phase, taking into account that the two fluids constrain each other in the case of unsaturated conditions (see e.g. Wyckoff and Botset ,1936). k_{ri} takes values between 0 and 1, e.g. $k_{rw} = 1$ and $k_{rg} = 0$ in the case of water saturated soil or $k_{rw} = 0$ and $k_{rg} = 1$ in the case of dry soil. η_i and ρ_i are the viscosity and the density of the respective fluid phase. ∇p_i is the pressure gradient of the corresponding phase, which is, besides gravity $\mathbf{g}$, the driving force behind the flow. $\ddot{\mathbf{u}}_\mathbf{s}$ is the acceleration of the solid phase while $\ddot{\mathbf{u}}_\mathbf{si}$ denotes the acceleration of the respective fluid relating to the acceleration of the solid phase. Equation 5 enables the description of the fluid phase velocities depending on the solid phase velocity and the respective relative fluid phase velocity.

- Balance equations for each constituent including interactions terms (mass and momentum exchange between constituents) as well as balance equations for the mixture following from the summation of equations for the constituents where the interaction terms must vanish.

- Constitutive equations for each constituent and for certain interactions. In the case of unsaturated soils these are: equation of state for soil particles (incompressible/compressible), pore water (incompressible/compressible) and pore air (compressible), stress-strain relation for soil skeleton, permeabilities of saturated soil, soil-water-characteristic-curve (swcc), relative permeabilities of unsaturated soil.

- In the case of quasi-static and dynamic b.v.p. and in the case of constitutive models depending on state variables initial conditions.
- Dirichlet- or Neumann boundary conditions for all constituents.

The citation of all the corresponding equations goes beyond the scope of this article but can be found in many books, e.g. (Lewis and Schrefler, 2000; de Boer, 2000; Öttl, 2003; Holler, 2006). The developed continuity equations for the water and the gas phase according to Holler (2006) are displayed in Equation 6 and 7 respectively:

Water phase:

$$\left[\frac{nS_w}{K_w}+(\alpha-n)\frac{S_w}{K_s}\left(S_w+p_c\frac{\partial S_w}{\partial p_c}\right)-n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_w}{\partial t}+\left[\frac{S_w}{K_s}(\alpha-n)\left(S_g+p_c\frac{\partial S_w}{\partial p_c}\right)+n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_g}{\partial t}+\alpha S_w\nabla\mathbf{v}_s$$

$$+\frac{1}{\rho_w}\nabla\left[\rho_w\frac{\mathbf{k}k_{rw}}{\eta_w}\left(-\nabla p_w+\rho_w\left(g-\ddot{u}_s\right)\right)\right]=0 \qquad (6)$$

Gas phase:

$$\left[\frac{nS_g}{K_g}+(\alpha-n)\frac{S_g}{K_s}\left(S_g+p_c\frac{\partial S_w}{\partial p_c}\right)-n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_g}{\partial t}+\left[\frac{S_g}{K_s}(\alpha-n)\left(S_w+p_c\frac{\partial S_w}{\partial p_c}\right)+n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_w}{\partial t}+\alpha S_g\nabla\mathbf{v}_s$$

$$+\frac{1}{\rho_g}\nabla\left[\rho_g\frac{\mathbf{k}k_{rg}}{\eta_g}\left(-\nabla p_g+\rho_g\left(g-\ddot{u}_s\right)\right)\right]=0 \qquad (7)$$

K_s, K_w and K_g are the bulk moduli of the solid, water and gas phase respectively, determining the compressibility of the according phase and α denotes the Biot-Parameter. The capillary pressure is defined as difference between the gas and water pressure ($p_c = p_g - p_w$).

The discretised forms of the continuity equations for the two fluid phases water and air combined with the principle of effective stresses for unsaturated soils are the basis for the three-phase formulation and lead to the following system of equations:

$$\begin{bmatrix}\mathbf{M} & \mathbf{0} & \mathbf{0}\\ \mathbf{M}_w & \mathbf{0} & \mathbf{0}\\ \mathbf{M}_g & \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{u}}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{C} & \mathbf{0} & \mathbf{0}\\ \mathbf{C_{ws}} & \mathbf{P_{ww}} & \mathbf{C_{wg}}\\ \mathbf{C_{gs}} & \mathbf{C_{gw}} & \mathbf{P_{gg}}\end{bmatrix}\begin{bmatrix}\dot{\bar{\mathbf{u}}}\\ \dot{\bar{\mathbf{p}}}_w\\ \dot{\bar{\mathbf{p}}}_g\end{bmatrix}+\begin{bmatrix}\mathbf{0} & \mathbf{-C_{ws}^T} & \mathbf{-C_{gs}^T}\\ \mathbf{0} & \mathbf{H_{ww}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{H_{gg}}\end{bmatrix}\begin{bmatrix}\bar{\mathbf{u}}\\ \bar{\mathbf{p}}_w\\ \bar{\mathbf{p}}_g\end{bmatrix}+\int\boldsymbol{\sigma}'d\Omega\begin{bmatrix}\mathbf{1}\\ \\ \mathbf{0}\end{bmatrix}=\begin{bmatrix}\mathbf{f}_u^{\Omega}\\ \mathbf{f}_{pw}^{\Omega}\\ \mathbf{f}_{pg}^{\Omega}\end{bmatrix}+\begin{bmatrix}\mathbf{f}_u^{\Gamma}\\ \mathbf{f}_{pw}^{\Gamma}\\ \mathbf{f}_{pg}^{\Gamma}\end{bmatrix} \qquad (8)$$

The primary unknowns of the problem are the displacement field of the soil skeleton $\mathbf{u_s}$ and the fluid pressures p_w and p_g (so-called upp-formulation), $\bar{\mathbf{u}}$, $\bar{\mathbf{p}}_w$ and $\bar{\mathbf{p}}_g$ denoting the respective values at the nodes. For the definition of all the matrices and vectors in this system of equations the reader is referred to Holler (2006). It's worth remarking that the usage of both linear and non linear constitutive equations for the solid phase is enabled by the term $\int\boldsymbol{\sigma}'d\Omega[\mathbf{1} \quad \mathbf{0} \quad \mathbf{0}]^{\mathbf{T}}$.

The implemented UEL has the following features:

- Different saturation-capillary pressure relationships according to Liakopoulos (1965), Brooks and Corey (1964) and Van Genuchten (1980).
- Different constitutive laws for the soil skeleton, namely elasticity, Mohr Coulomb plasticity and a hypoplastic stress-strain relation, see e.g. (Gudehus, 1996; Von Wolffersdorff, 1996; Herle, 1997; Niemunis and Herle, 1997; Kolymbas, 2000).
- Different types of elements concerning the number and type of nodes and the associated shape functions (bilinear and/or biquadratic), see Figure 1.
- The element can be used for plane strain as well as axisymmetric.
- The element is suitable for dynamic, quasi-static and static analysis by setting the mass matrix respectively the mass and the damping matrix to zero, see Equation 8. In the first two cases, an implicit dynamic analysis is carried out while in the third case a static analysis is sufficient.
- The element is able to account for geometric nonlinearity.

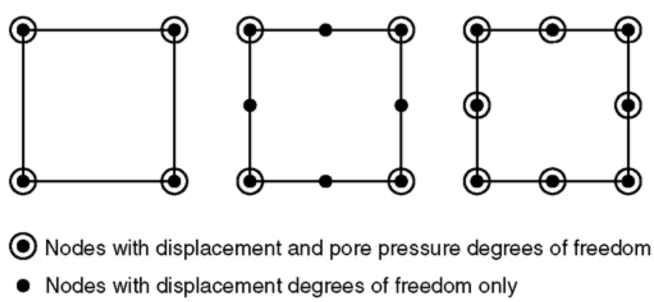

Figure 1. Types of implemented elements.

4. Case studies

The implemented UEL user subroutine will be applied on different boundary value problems. If possible, a comparison to results of Abaqus built-in procedures will be presented.

4.1 One-dimensional consolidation of a soil column

The first benchmark test for the UEL is the one-dimensional consolidation of a water saturated soil column, analysing the time-depent settlement behaviour, which is mainly affected by the permeability of the soil. The geometry, boundary conditions and discretisation of the column is shown in Figure 2, wherein q_w denotes the water flow and u_x and u_y are the displacements in the according directions. The calculation is carried out quasi-static neglecting mass inertia effects and reduced to a two phase simulation by setting the pore gas pressure degrees of freedom (d.o.f.) of all nodes to zero. Elements with 8 nodes are used, of which 4 nodes have additional pore pressure d.o.f. (water and gas pressure). The material behaviour of the solid phase is modelled elastically. The corresponding physical properties of the soil column are listed in Table 1.

Table 1. Physical properties of the soil column for the one-dimensional consolidation.

E (kN/m²)	ν (-)	n (-)	ρ_s (t/m³)	ρ_w (t/m³)	k (m²)	η_w (kNs/m²)	K_s (kN/m²)	K_w (kN/m²)
5000	0.25	0.5	2.7	1.0	$1.31 \cdot 10^{-10}$	$1,31 \cdot 10^{-6}$	∞	$2.0 \cdot 10^{7}$

E and ν are the Youngs moduls and the Poisson's ratio of the soil skeleton. The parameters listed above result in the following hydraulic conductivity:

$$k_w = \frac{\rho_w \cdot g \cdot k}{\eta_w} = \frac{1 \cdot 10 \cdot 1.31 \cdot 10^{-10}}{1.31 \cdot 10^{-6}} = 10^{-3}\,\text{m/s}$$

Initially the pore water pressure is set to zero for all nodes with pressure degrees of freedom. In a first step the load of $\sigma = 10$ kN/m² is applied on the top of the column within 10^{-6} seconds. The second load step is the consolidation of the soil colunm having a duration of 10 seconds.

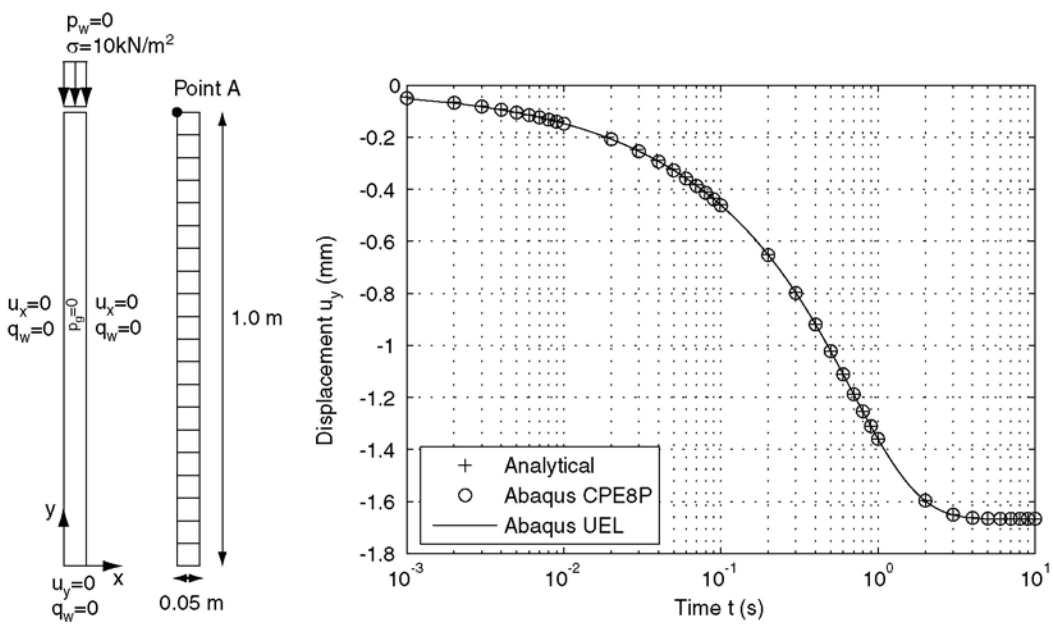

Figure 2. Geometry, boundary conditions, discretisation (left) and calculated time displacement history for point A (right) of the soil column.

The results of the calculation are also shown in Figure 2. A comparison with the results of a calculation with the Abaqus built-in elements of the type CPE8P and the analytical solution according to Verruijt (1995) shows a very good agreement.

4.2 Dewatering of a soil column (Liakopoulos test)

The second benchmark test for the UEL is the dewatering of a soil column, known as the Liakopoulos test (1965), which is also included in the Abaqus 6.9 Benchmarks Manual (2009). Before the start of the test a constant water flow from the top to the bottom of the soil column is generated to avoid a builtup of pore water pressure in the column. The test is started by stopping the inflow of water at the top of the column. During the test the evolution of pore water pressure in the column and the amount of outflowing water at the bottom of the column is measured. This experiment has already been used as a benchmark problem by several other authors, e.g. Holler (2006), Öttl (2003), Lewis and Schrefler (2000). The geometry, boundary conditions and discretisation of the column are nearly the same as shown in Figure 2. The calculation is again performed quasi-static, neglecting inertia effects. The main difference is the consideration of all three phases by removing the boundary conditions for the pore gas pressure of the whole column. The sides of the column are impermeable for gas flow additionally. Besides the column has a width of 0.1 m and there is no pressure applied at the top of the column. For this calculation 20 elements with 8 nodes are chosen, all having additional pore pressure degrees of freedom (water and gas pressure). An elastic stress-strain relation for the soil skeleton is used. The corresponding physical properties of the soil column are listed in Table 2.

Table 2. Physical properties of the soil column.

E (kN/m²)	ν (-)	n (-)	ρ_s (t/m³)	ρ_w (t/m³)	ρ_g (t/m³)	k (m²)	η_w (kNs/m²)	η_g (kNs/m²)	K_s (kN/m²)	K_w (kN/m²)	K_g (kN/m²)
1300	0.4	0.2975	2.0	1.0	0.0012	$4.5\cdot10^{-13}$	$1{,}0\cdot10^{-6}$	$1{,}8\cdot10^{-8}$	$1{,}0\cdot10^{9}$	$2.0\cdot10^{6}$	100

The saturation-capillary pressure relationship and the relative permeability of the water phase are chosen according to Liakopoulos:

$$S_w = 1 - 1.9722 \cdot 10^{11} \cdot p_c^{\,2.4279} \tag{9}$$

$$k_{rw} = 1 - 2.207 \cdot (1 - S_w)^{1.0121} \tag{10}$$

Equations 9 and 10 are valid for saturation $S_w \geq 0.91$. The relative permeability of the gas phase is assumed to be as given by Brooks and Corey and derived according to Burdine (1953):

$$k_{rg} = (1 - S_e) \cdot (1 - S_e^{\,5/3}) \tag{11}$$

$$S_e = \frac{S_w - 0.2}{1.0 - 0.2} \tag{12}$$

In Equation 11 and 12 S_e denotes the effective water saturation. Additionally a minimum value for the relative gas permeability of $k_{rg,\min} = 10^{-4}$ is specified.

Initially the pore water and gas pressure is set to zero for all nodes. In a first step the gravity load with $g = 10\,\text{m/s}^2$ is applied to the column. In a second load step the boundary conditions concerning the pore water and gas pressure are removed and replaced by setting the pore water and gas pressure for the bottom of the column and the gas pressure for the top of the column to zero. The results of the calculation concerning the evolution of the pore water pressure, pore gas pressure, water saturation, effective vertical stress and vertical displacement with time are shown in Figure 3 to Figure 5 and compared with the results of the experiment and the simulations of other authors. A very good agreement with the other authors is achieved concerining the pore water pressure distribution, and the evolution of effective vertical stress and vertical displacement with time. A possible explanation for the slight difference between the results concering the pore gas pressure and consequently the saturation regarding the comparison to Öttl is the non-concideration of the compressibility of the solid phase by Öttl ($K_s = \infty$).

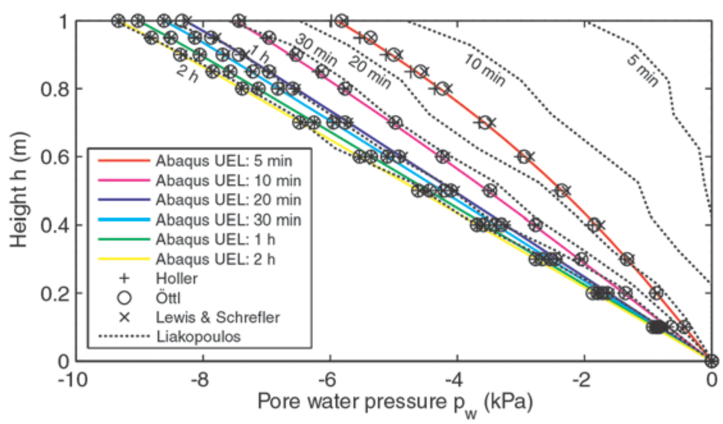

Figure 3. Evolution of pore water pressure with time compared to the results of Holler, Öttl, Lewis & Schrefler and Liakopoulos.

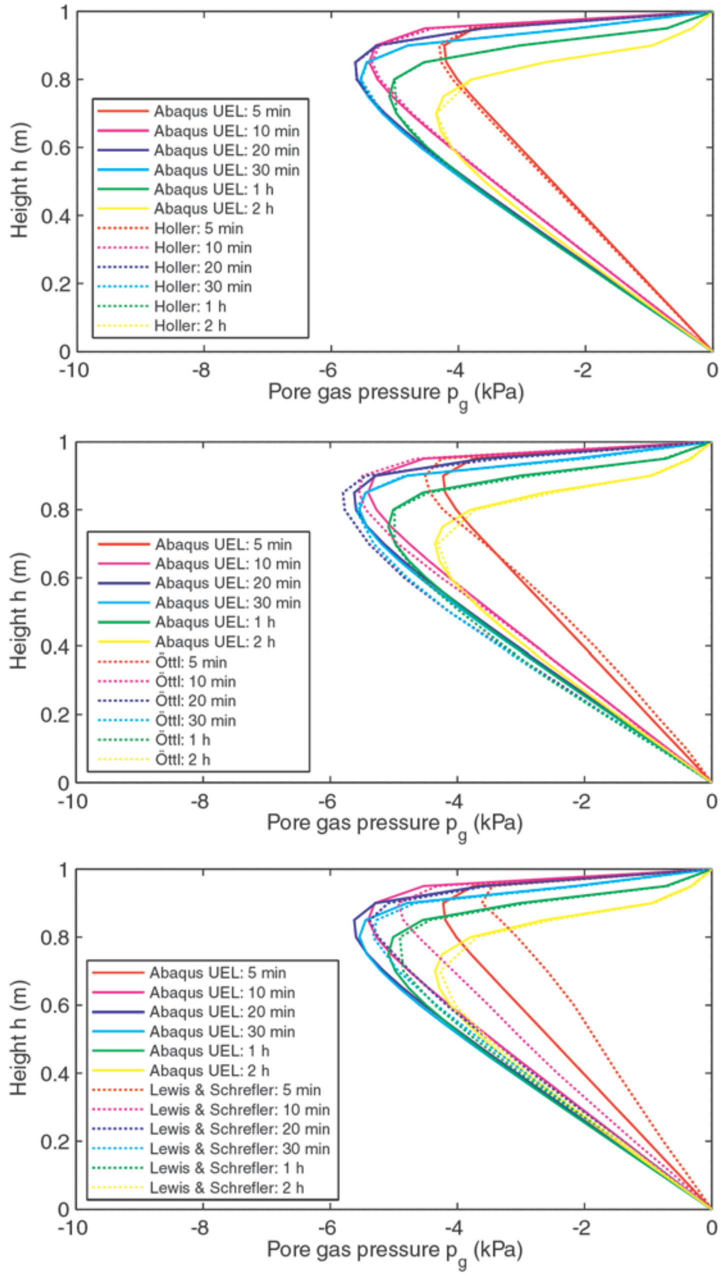

Figure 4. Evolution of pore gas pressure with time compared to the results of Holler (top), Öttl (middle) and Lewis & Schrefler (bottom).

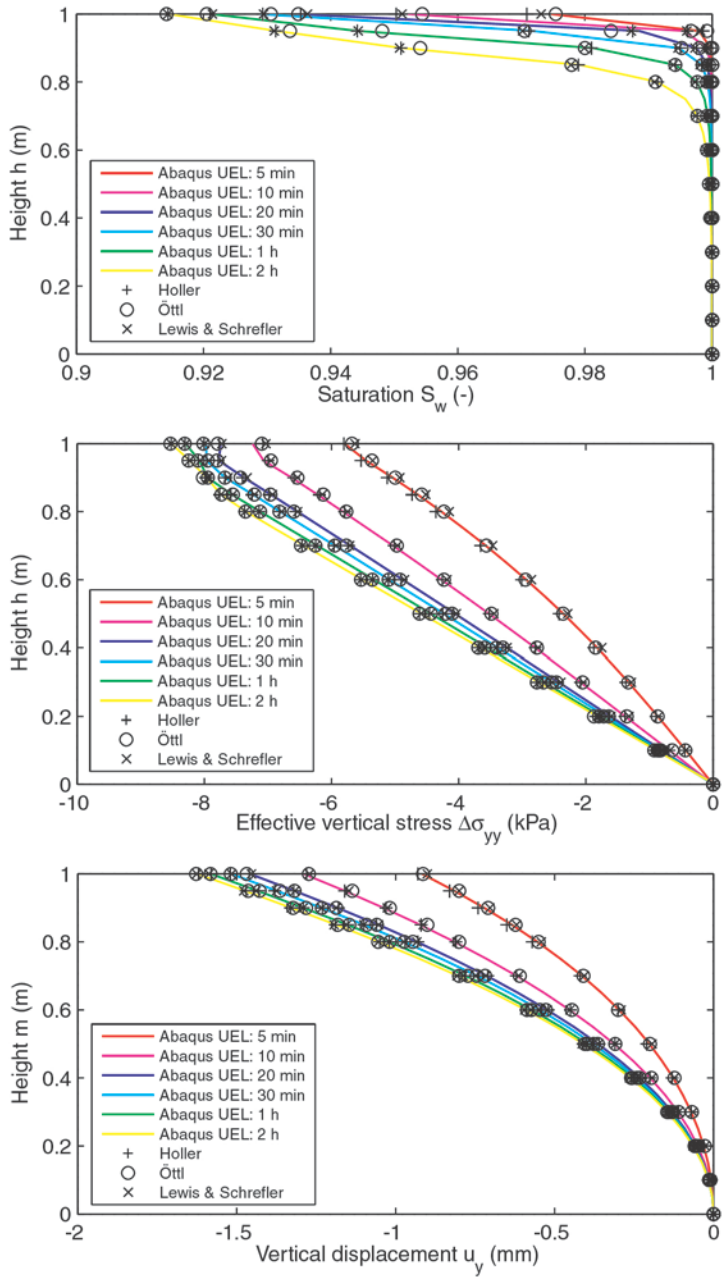

Figure 5. Evolution of saturation (top), effective vertical stress (middle) and vertical displacement with time compared to the results of Holler, Öttl and Lewis & Schrefler.

4.3 One-dimensional wave propagation in a soil column

The third benchmark test for the UEL is the one-dimensional propagation of waves in a water saturated soil column. The propagation of elastic waves in fluid-saturated porous solid for the low and higher frequency range was first described by Biot (1956 a, 1956 b). The one-dimensional case is a common benchmark problem to investigate the dynamic behaviour of fluid saturated porous media and has been solved by several other authors, e.g. Soares (2008) and Schanz & Cheng (2000). The geometry, boundary conditions and discretisation of the column nearly correspond with those shown in Figure 2. The main difference is the sudden application of the load on the top of the column, according to a so called heavyside type load function. Besides the column has a height of 10 m and a width of 0.5 m. In this case a fully dynamic analysis is carried out taking inertia effects into account. The simulation is reduced to a two phase simulation by setting the pore gas pressure of all nodes to zero. The column is discretised by 20 elements with 8 nodes, all having additional pore pressure degrees of freedom (water and gas pressure). The material behaviour of the solid phase is described by elasticity. The corresponding physical properties of the soil column are listed in Table 3.

Table 3. Physical properties of the soil column.

E (kN/m²)	ν (-)	n (-)	ρ_s (t/m³)	ρ_w (t/m³)	k (m²)	η_w (kNs/m²)	K_s (kN/m²)	K_w (kN/m²)
254423.077	0.298	0.48	2.7	1.0	$3.55 \cdot 10^{-12}$	$1{,}0 \cdot 10^{-6}$	$1{,}0 \cdot 10^{7}$	$3.3 \cdot 10^{6}$

Initially the pore water and gas pressure is set to zero for all nodes. In a first step the gravity load with $g = 10$ m/s² is applied to the column. The second load step has a duration of 0.3 s and starts with the sudden application of a surface load with a magnitude of $\sigma = 1$ N/m². This step is calculated with a constant time increment of 10^{-5} s.

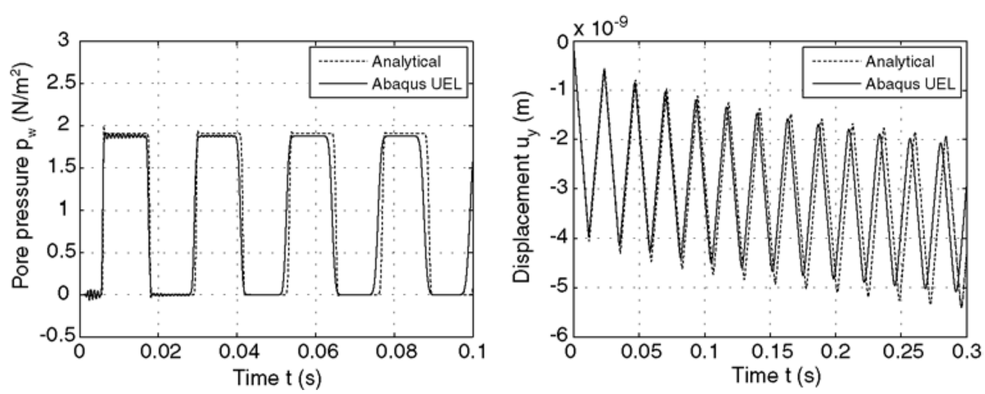

Figure 6. Evolution of pore water pressure for the bottom of the column (left) and the vertical displacement of the top of the column (right) with time compared to the analytical solutions given by Schanz and Cheng (2000).

The comparison of the results concerning the evolution of the pore water pressure at the bottom of the column and the vertical displacement of the top of the column with the analytical solution given by Schanz and Cheng (2000) shown in Figure 6 yields a satisfying agreement.

5. Summary

It could be shown that Abaqus built-in procedures modelling soils with one-phase or two-phase models can be extended by implementing a user subroutine UEL enabling static, quasi-static and dynamic coupled deformation/seepage analyses for unsaturated soils. Different benchmark tests have shown that this extension works quite well compared to Abaqus built-in procedures as well as FE-simulations published by different authors.

6. Outlook and potential applications

The user subroutine UEL implementing a dynamic three-phase model for unsaturated soils already offers different constitutive equations and will be enhanced further. The transfer of this UEL subroutine to a corresponding VUEL subroutine for Abaqus/Explicit will be checked and realized if possible. A potential application for the presented user defined element is the simulation of laboratory and in-situ testing of saturated and unsaturated soils, e.g. the propagation of waves and vibrations, cone penetration testing with additional measurement of pore pressures or high strain dynamic pile testing. Further, the element can be used to simulate construction processes like pile jacking, pile driving or vibratory pile driving, as described by Hügel et al. (2008) for dry soils, or tunnelling in aquifers by means of compressed air, see e.g. Öttl (2003). With regard to the field of coastal and offshore engineering the presented user defined element can be applied to investigate the stability of embankments and underwater slopes under seismic loading or the wave driven seepage within the seabed taking into account the phenomenon of soil liquefication.

7. References

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