

# Modeling the Cyclic Response of HY-80 Steel Under Dynamic Loading

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*Abstract: A time-independent constitutive model for HY-80 (High Yield) steels with a yield plateau is outlined. The model couples the non-linear kinematic hardening concept with a memory surface in the plastic strain space, to account for progressive cyclic hardening/softening, and a pseudo memory surface in the deviatoric stress space, to correctly describe the plateau response. The model is incorporated into Abaqus/Standard through a UMAT. The performance of the model is validated against experimental data.*

*Keywords: Constitutive Model, Dynamics, Experimental Verification, Plasticity*

## 1. Introduction

As the engineering community moves towards design concepts based on the performance of a component or structure, finite element methods gain popularity and become an indispensable tool, especially in applications involving cyclic loading in the inelastic range. In finite element applications dealing with large elastic-plastic cyclic deformations, accurate description of the material response is essential in order to arrive at an accurate and reliable prediction of the member or structural response. Inelastic cyclic characteristics of engineering materials are quantified with cyclic plasticity models, that is, mathematical models based continuum mechanics.

Finite element simulations presented by Hodge and Minicucci (1997), Arpin (1999), and Ucak and Tsopelas (2008) show that later time dynamic, and the quasi-static cyclic hysteretic response of structural components made of structural steel with a yield plateau cannot be accurately captured, unless the macroscopic response of the base material is correctly modeled. This can be attributed to the fact that for structural steels with a yield plateau, the stress-strain response

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recorded under monotonic and cyclic load paths are different. Generally an amplitude-dependent cyclic hardening/softening phenomenon is observed, where the amount of hardening/softening depends on the curvature of the load path.

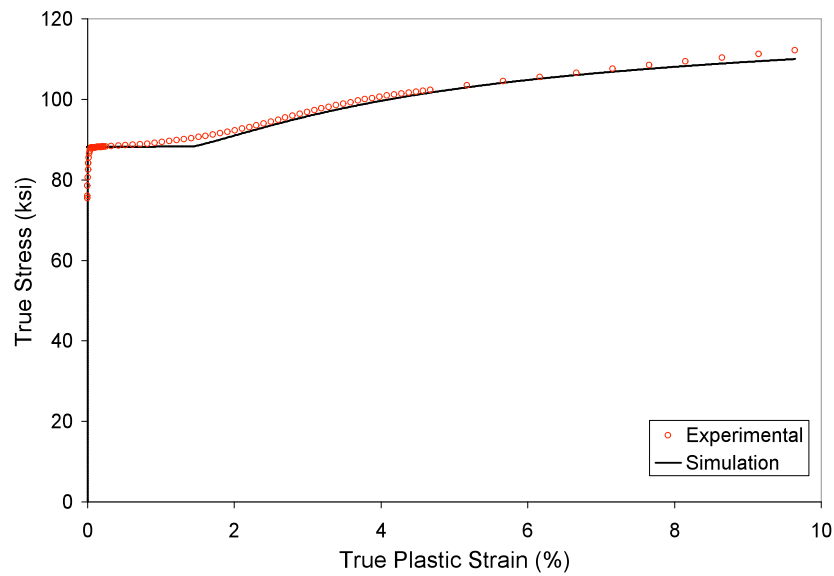
In this paper, a constitutive model for HY-80 (High Yield) steel, used in naval applications, is developed and validated. The material model is coupled with an efficient integration algorithm and integrated into the general-purpose finite element code ABAQUS via user material subroutines (UMAT). A brief description of the model, which is capable of capturing the response of the material for monotonic and cyclic loading conditions, is presented. The performance of the model is validated against experimental data.

## **2. Observed Material Behavior of HY-80 Steel**

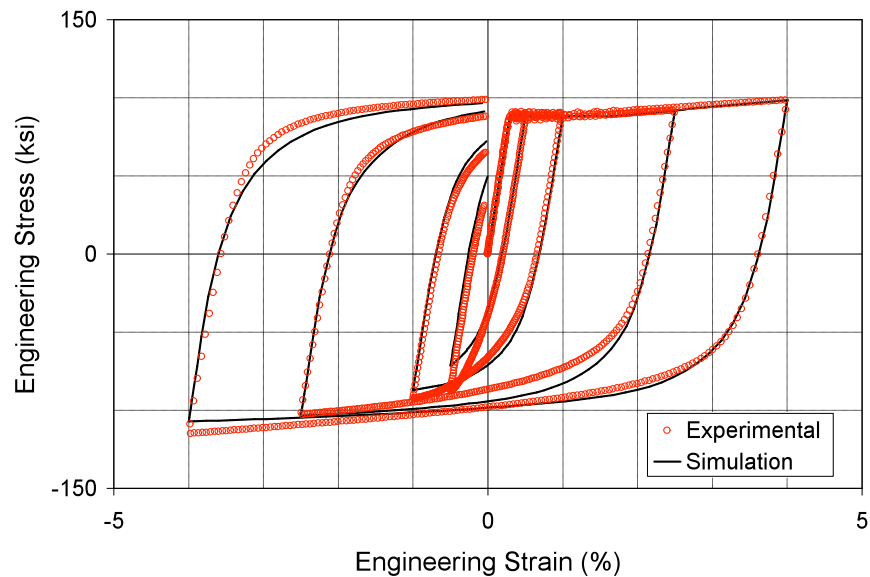
All experimental coupon data presented in this paper for HY-80 steel were reported by Arpin and Trimble (1997), and Hodge et al. (2003). The experiments were conducted on round specimens under displacement controlled loading protocol.

The monotonic stress versus plastic strain curve of a HY-80 specimen is depicted in Figure 1. During monotonic loading, after the homogeneous elastic deformation, HY-80 steel shows a rather sharp yield point, followed by a yield plateau. The plastic deformation along the yield plateau is caused by Luders band propagation. During this process the plastic deformation along the gage is inhomogeneous. Once the Luders bands cover the whole gage, the plastic deformation becomes homogeneous, and the specimen starts hardening. The hardening curve of the material is non-linear with respect to loading amplitude. From a macroscopic point of view, the yield plateau region is treated as plastic material without hardening.

Material response under cyclic loading is much more complex as compared with monotonic loading. Generally the monotonic hardening curve will not be representative of the cyclic characteristics of the material. Figure 2 depicts the hysteresis curves of HY-80 steel under constant amplitude cyclic loading, for different loading amplitudes. As evident from Figure 2, for fully reversed loading amplitudes smaller than approximately 1%, the observed material response is cyclic softening, while for larger amplitudes the material will undergo cyclic hardening.



**Figure 1. Monotonic true plastic strain vs. true stress curve of HY-80 steel (experimental data after Hodge et al. 2003).**



**Figure 2. Hysteretic curves of HY-80 steel under constant amplitude cyclic loading (experimental data after Hodge et al. 2003).**

### 3. Description of the Constitutive Model

Detailed description of the constitutive model used to describe the cyclic behavior of structural steels with a yield plateau was outlined in Ucak and Tsopelas (2008). In this section only the constitutive relations that apply to HY-80 steel is presented.

The constitute model used to describe cyclic response of HY-80 steel is based on the basic principles of time-independent theory of plasticity with a von Mises yield surface and an associative flow rule

$$f = \left[ \frac{3}{2} (\mathbf{s} - \mathbf{x}) : (\mathbf{s} - \mathbf{x}) \right]^{1/2} - \kappa \leq 0, \quad \kappa = R + k, \quad (1)$$

$$d\boldsymbol{\epsilon}^p = \lambda \frac{df}{d\boldsymbol{\sigma}} = \frac{3}{2} \lambda \frac{\mathbf{s} - \mathbf{x}}{\kappa}, \quad dp = \left[ \frac{2}{3} d\boldsymbol{\epsilon}^p : d\boldsymbol{\epsilon}^p \right]^{1/2} = \lambda, \quad (2)$$

where, bold letters indicate second order tensors,  $\mathbf{s}$  is the stress deviator,  $\mathbf{x}$  is the back stress,  $\kappa$  is the size of the yield surface,  $k$  is the initial size of the yield surface,  $R$  is the isotropic hardening/softening variable,  $\lambda$  is the plastic multiplier, and  $p$  is the accumulated (or equivalent) plastic strain.

A memory surface is incorporated into the plastic strain space, to measure the loading amplitude

$$F = \left[ \frac{2}{3} (\boldsymbol{\epsilon}^p - \boldsymbol{\xi}) : (\boldsymbol{\epsilon}^p - \boldsymbol{\xi}) \right]^{1/2} - q \leq 0, \quad (3)$$

$$dq = c^F H(F) \langle \mathbf{n}^f : \mathbf{n}^F \rangle dp, \quad (4)$$

$$d\boldsymbol{\xi} = (1 - c^F) H(F) \langle \mathbf{n}^F : d\boldsymbol{\epsilon}^p \rangle \mathbf{n}^F, \quad (5)$$

where,  $\boldsymbol{\xi}$  and  $q$  are the center and radius of the memory surface respectively,  $H(F)$  is the Heaviside function, i.e.  $H(F) = 1$  if  $F = 0$  and  $H(F) = 0$  if  $F < 0$ ,  $\langle \rangle$  denotes the Macauley brackets, i.e.  $\langle a \rangle = a$  if  $a > 0$  and  $\langle a \rangle = 0$  if  $a < 0$ ,  $\mathbf{n}^f$  and  $\mathbf{n}^F$  are the unit normal vectors to the yield surface and memory surface respectively, and  $c^F$  is a material dependent parameter.

Consistent with the observed material behavior, existence of a plateau region and hardening region is assumed. In the plateau region, the (equivalent) stress cannot exceed the initial yield stress of the virgin material. For ideal von Mises behavior, this implies that,  $\sqrt{\mathbf{s}:\mathbf{s}} < \sqrt{2/3}k$ . The transition from the plateau region to the hardening region depends on the loading amplitude and the accumulated plastic strain, such that

$$\text{if } q \leq \bar{\varepsilon}_{sh}^p \rightarrow \text{plateau region,} \quad (6)$$

$$\text{if } q > \bar{\varepsilon}_{sh}^p \text{ and } p > \varepsilon_{sh}^p \rightarrow \text{hardening region.} \quad (7)$$

In the plateau region the material dependent parameter  $c^F = 1/2$ , whereas in the hardening region  $0 < c^F \leq 1/2$ .

### 3.1 Initial Plateau Response

A bounding surface is incorporated into the deviatoric stress-space, to simulate the plateau behavior during monotonic loading. The bounding surface has the same shape and size as the yield surface of the virgin material, and is not allowed to translate nor change size. During initial plastic loading, the yield surface softens and translates at the same time, and is always in contact with the bounding surface at the loading point. The bounding surface is expressed as

$$\bar{f} = \left[ \frac{3}{2} \mathbf{s} : \mathbf{s} \right]^{1/2} - k = 0. \quad (8)$$

Softening of the yield surface is explicitly defined with the following differential equation

$$dR = b(R_\infty - R)dp, \quad (9)$$

where,  $R_\infty$  is the asymptotic (fully saturated) value of the isotropic softening function ( $-k < R_\infty < 0$ ), and  $b$  is a sufficiently large softening coefficient to cause almost instantaneous softening.

During initial monotonic loading, on the yield plateau, the center of the yield surface is evaluated using the consistency condition that the outward normal of the yield surface and the bounding surface are identical

$$\mathbf{x} = \mathbf{s} - \frac{\kappa}{k} \mathbf{s}. \quad (10)$$

The bounding surface, which is used to simulate the plateau response during initial loading will vanish: (a) when (7) holds during monotonic loading, or (b) if an unloading-plastic reloading occurs while (6) is holding. At this instant, the bounding surface is deactivated and the kinematic hardening variable  $\mathbf{x}$  is decomposed into 2 short-term ( $\mathbf{x}_i^{pl}$ ) and 2 long-term ( $\mathbf{x}_j^h$ ) components

$$\mathbf{x} = \sum_{i=1}^2 \mathbf{x}_i^{pl} + \sum_{j=1}^2 \mathbf{x}_j^h. \quad (11)$$

### 3.2 Unloading From the Plateau

Upon first unloading in the plateau region, while (6) is holding, the long-range and short-range kinematic hardening variables are defined respectively from

$$\mathbf{x}_j^h = 0, \quad (12)$$

$$d\mathbf{x}_i^{pl} = \frac{2}{3} C_i^{pl} d\epsilon_p - \bar{\gamma}_i \mathbf{x}_i^{pl} dp \quad (\text{for } i = 1, 2), \quad (13)$$

where,  $C_i^{pl}$  are material parameters, and  $\bar{\gamma}_i$  are material dependent functions. In the plateau region, the material dependent functions  $\bar{\gamma}_i$  are assumed in the form

$$\bar{\gamma}_i = \gamma_i^{pl}, \quad (\text{for } i = 1, 2), \quad (14)$$

where,  $\gamma_i^{pl}$  are material parameters. For the two short-range kinematic hardening functions the following relation has to hold

$$\sum_{i=1}^2 \frac{C_i^{pl}}{\gamma_i^{pl}} = -R_\infty. \quad (15)$$

### 3.3 Hardening Behavior

At the instant, when (7) is satisfied, the existing memory of the material is erased and re-set with the following initial conditions

$$\boldsymbol{\xi} = \bar{\boldsymbol{\epsilon}}^{p*}, \quad q = 0, \quad (16)$$

where,  $\bar{\boldsymbol{\epsilon}}^{p*}$  is the plastic strain tensor at which (7) is satisfied.

The evolution law for the long-term kinematic hardening variable is assumed to be in the form

$$d\mathbf{x}_j^h = \frac{2}{3} C_j^h d\epsilon_p - \gamma_j^h \mathbf{x}_j^h dp, \quad (\text{for } j = 1, 2). \quad (17)$$

Cyclic hardening observed under proportional loading paths is incorporated into the model by modifying the first short range kinematic hardening variable as

$$d\bar{\gamma}_1 \begin{cases} = w(\bar{\gamma}_1^\infty - \bar{\gamma}_1) dp & F=0 \text{ and } (\partial F / \partial \boldsymbol{\epsilon}^p) : d\boldsymbol{\epsilon}^p > 0 \\ = 0 & F < 0 \text{ or } (\partial F / \partial \boldsymbol{\epsilon}^p) : d\boldsymbol{\epsilon}^p < 0, \end{cases} \quad (18)$$

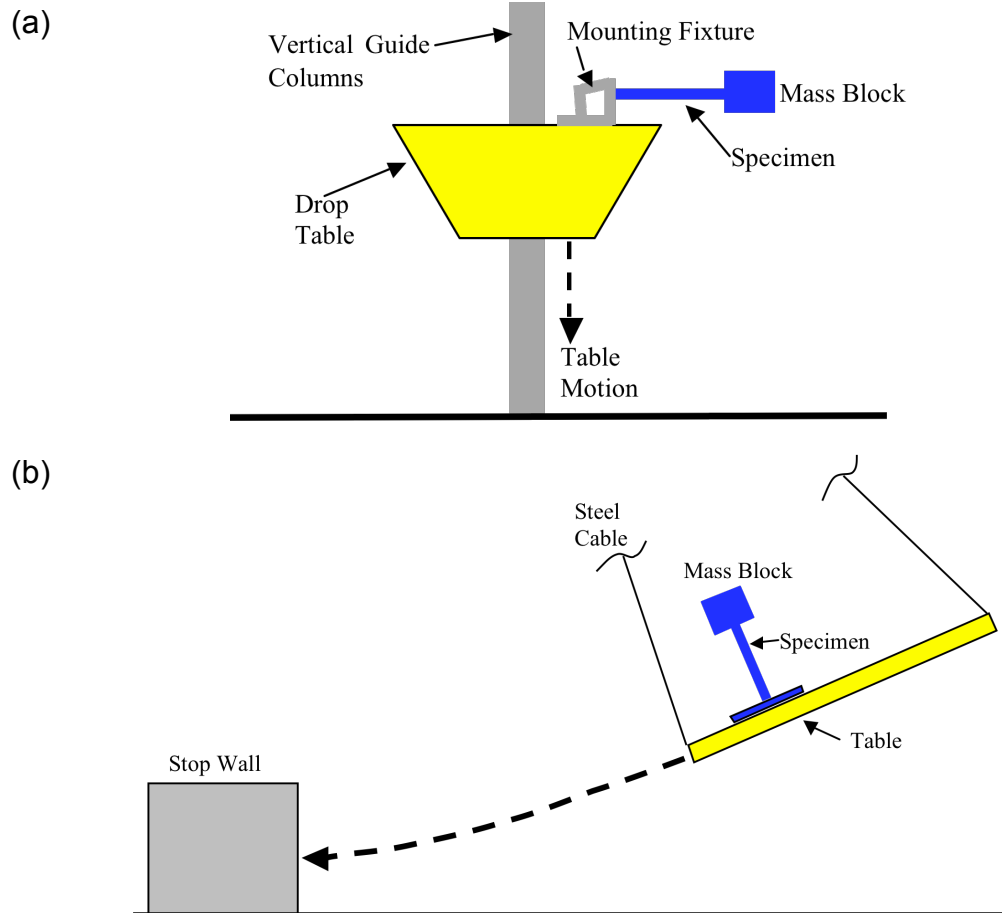
with the initial value  $\bar{\gamma}_1^0 = \gamma_1^{pl}$ . In (18),  $w$  and  $\bar{\gamma}_i^\infty$  are material dependent parameters such that  $0 < \bar{\gamma}_i^\infty \leq \gamma_1^{pl}$ .

#### 4. Numerical Simulations

Trimble and Krech (1997) documented a series of experiments, in which cantilever beam specimens made of HY-80 grade steel were subjected to transient dynamic loads. The test program was designed to study the nonlinear dynamic response of simple structures, and consequently to investigate the ability of elastic-plastic analysis techniques to predict dynamic inelastic response of components made of HY-80 steel. Some of the tested specimens in the above-cited study were used as benchmark verification examples to demonstrate the accuracy of the constitutive model proposed.

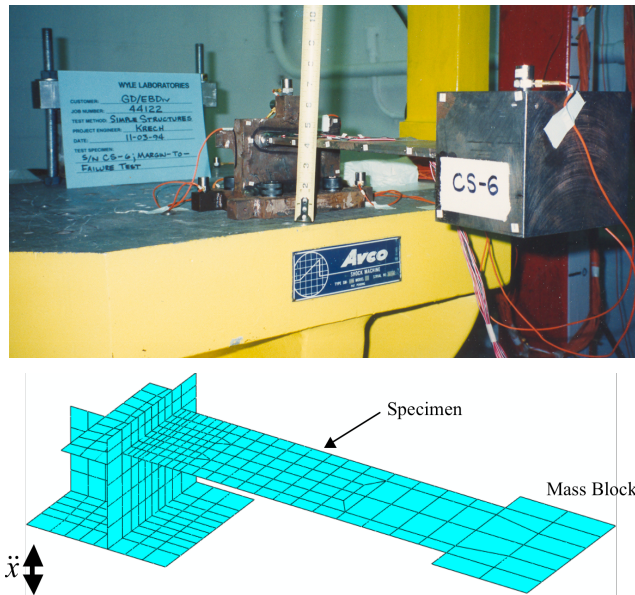
The schematic presentation of the experimental setup documented by Trimble and Krech (1997) is shown in Figure 3. The assemblies used in the experiments consisted of horizontal [Figure 3(a)], and vertical [Figure 3(b)] cantilever beams, made of HY-80 grade steel, attached to drop tables. In the experiments, the horizontal cantilever beams were subjected to short-duration pulses, where as the vertical cantilever beams were subjected to long duration pulses. The specimens were designed to produce surface strains in the range of 2~3%. During the experiments the acceleration and the deformation of the cantilever tip and the

surface strain at different locations were measured. In this study, one horizontal (referred to as CS-2) and two vertical cantilever beams (referred to as CL-1 and CL-3) are considered.

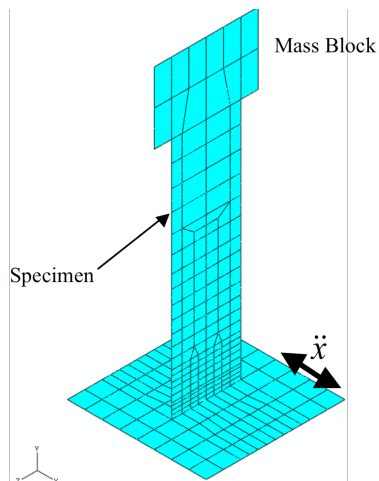


**Figure 3. Schematic presentation of the experimental setup documented by Trimble and Krech (1997); (a) for the horizontal cantilever beams; (b) for the vertical cantilever beams.**

Figure 4 depicts a photograph of the instrumented horizontal cantilever beam assembly used in the experiments, and the corresponding finite element (FE) model used to represent the assembly. Figure 5 depicts the FE model used for the vertical cantilever beam assembly.



**Figure 4. Photograph of the instrumented horizontal cantilever beam assembly and the corresponding finite element model.**



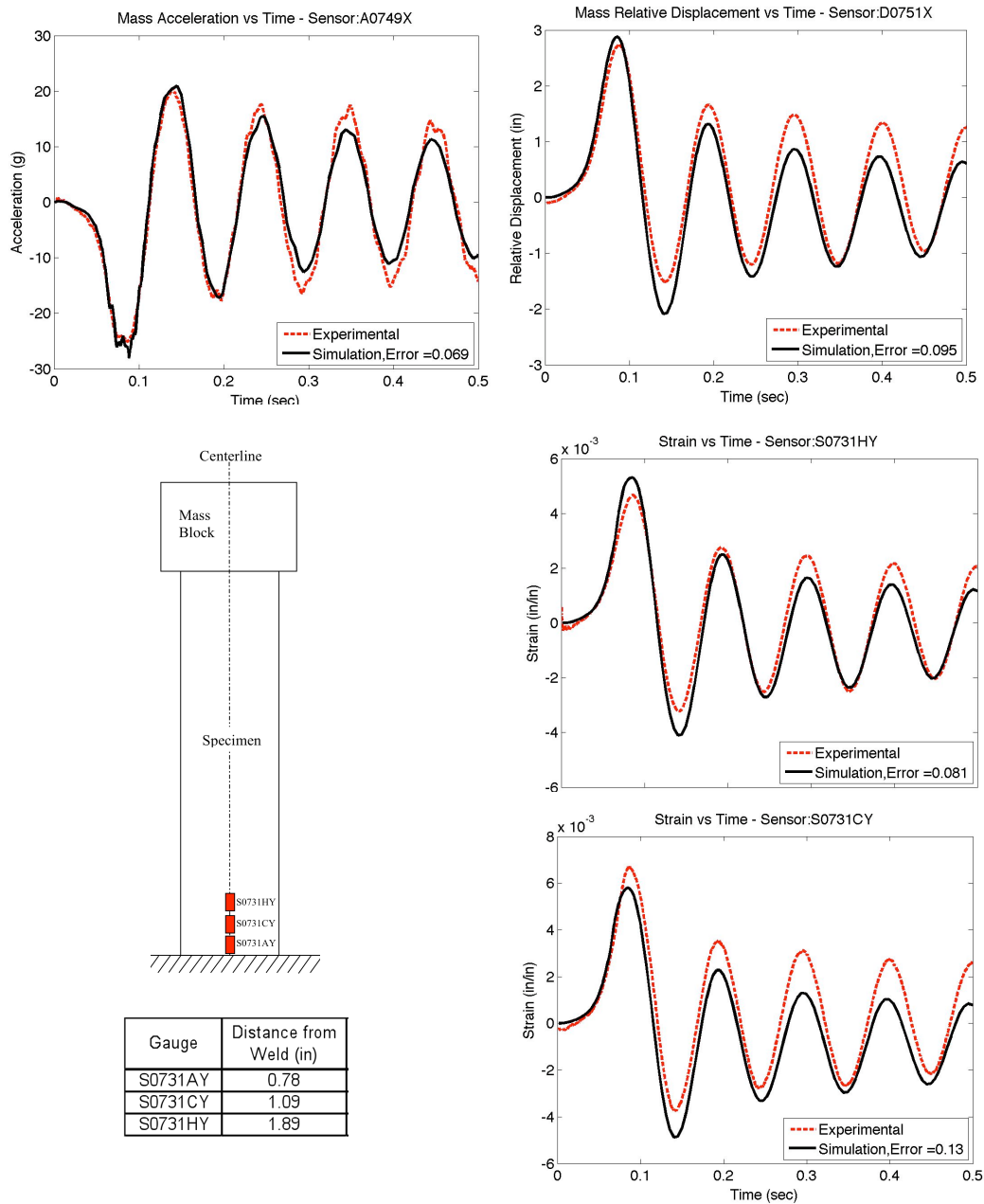
**Figure 5. Finite element model for the vertical cantilever beam assembly.**

In the FE models the plates, mass blocks, and support plates are mathematically represented by 2<sup>nd</sup> order quadratic reduced integration shell elements (Abaqus element type S8R5). The models are constructed using the nominal construction dimensions of the specimens, and excited at the cantilever base with the actual transient accelerations.

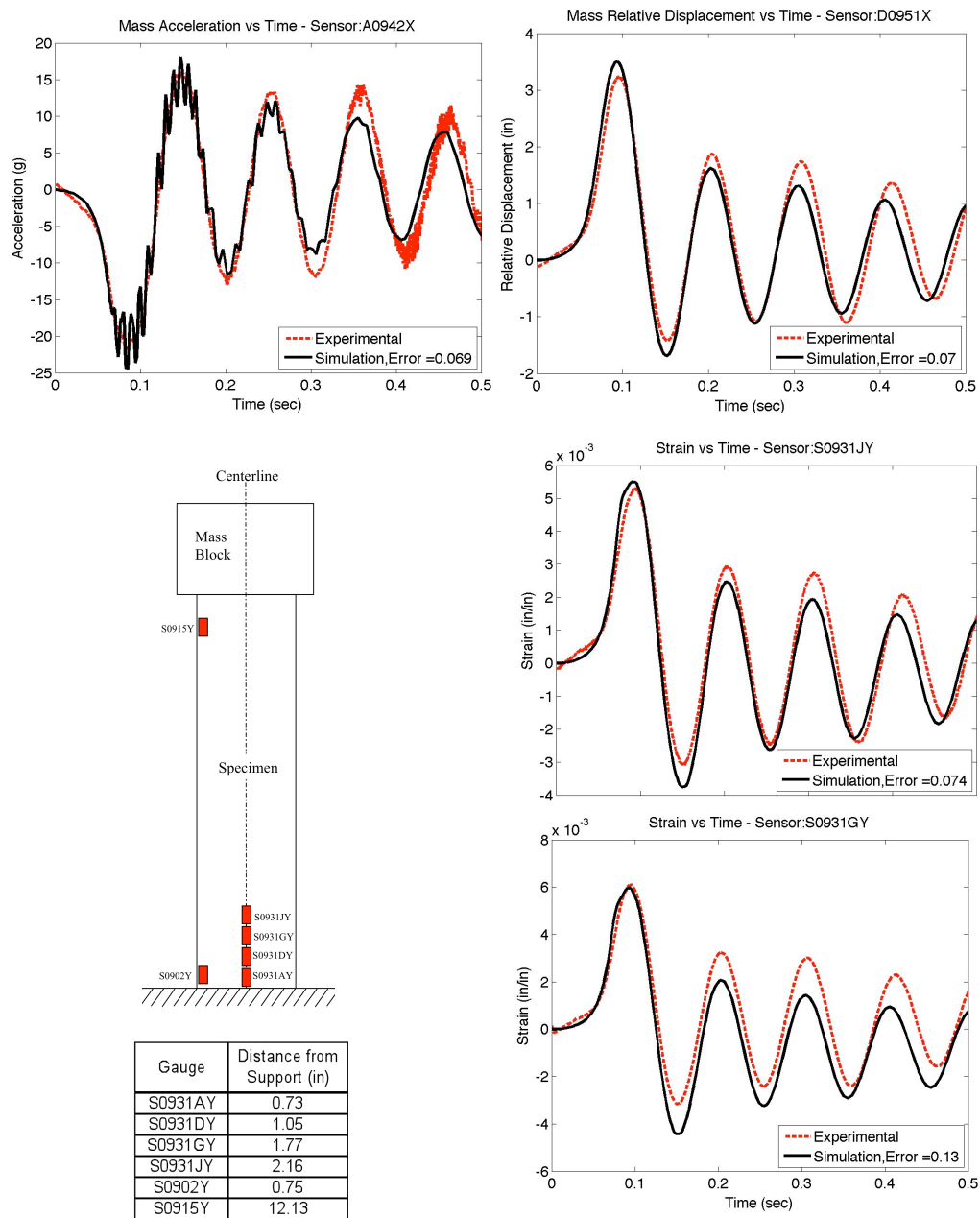
The material dependent properties used for the proposed model are calibrated using the experimental data reported by Arpin and Trimble (1997), and Hodge et al. (2003). The simulated material response under uni-axial tension and constant amplitude proportional cyclic loading are presented in Figure 1 and Figure 2 respectively. A comparison of the experimentally obtained results with the simulations shows that the model can capture the essential characteristics of the material. The plateau response, curvature of the monotonic loading curve, the shape of the hysteresis curves, and the cyclic characteristics of the material are correctly simulated.

Figure 6 through Figure 8 depict the correlation between the recorded and predicted acceleration, relative displacement, and strain time histories for vertical and horizontal cantilever specimens. The acceleration and relative displacement time histories were recorded at the tip of the specimens, where as the strain time histories were recorded close to the cantilever base support. The strain gage locations are given in Figure 6 through Figure 8. Here it is important to note that due to the test configuration, the relative displacement of the mass could not be physically measured for specimen CS-2, since there was no place to attach a displacement transducer. In all figures the calculated Russell error factor (Russell, 1997) is also shown for reference. The Russell error factor is an unbiased error measure used for comparing transient signals. Separate error measures are calculated for both magnitude and phase error. These are then combined in a single comprehensive error measure, which accounts for both sources of error. In general, a comprehensive error of less than 0.15 is considered excellent correlation.

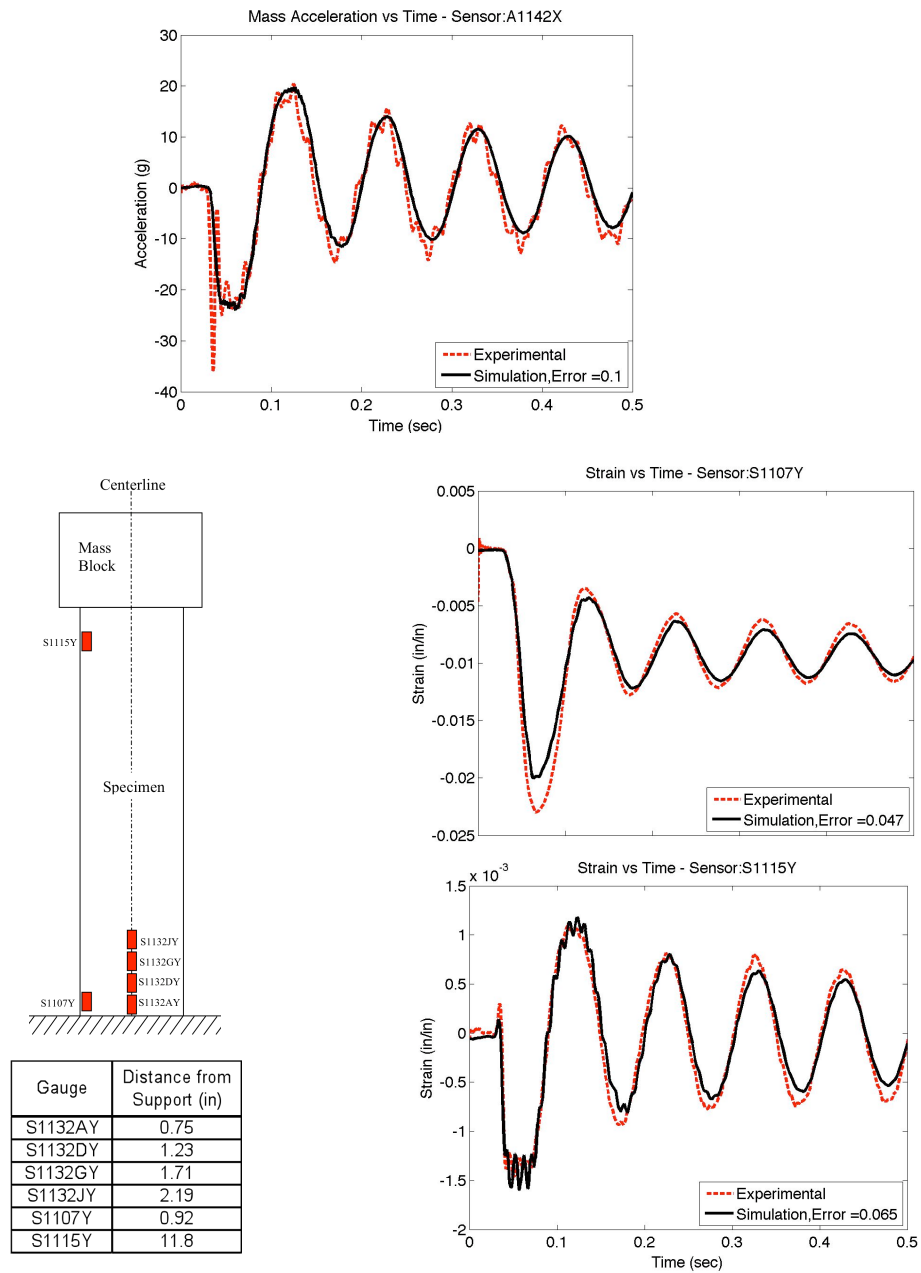
A comparison of the time histories presented in Figure 6 through Figure 8 shows that the predicted results are in good agreement with the experimental ones. The peak acceleration, relative displacement, and strain values are very well captured. Excellent correlation between the recorded and simulated time histories is observed, and the phase of the response is correctly captured. Here it is important to note that the peak strain values the specimens experienced is well above the yield strain, and the response is highly inelastic.



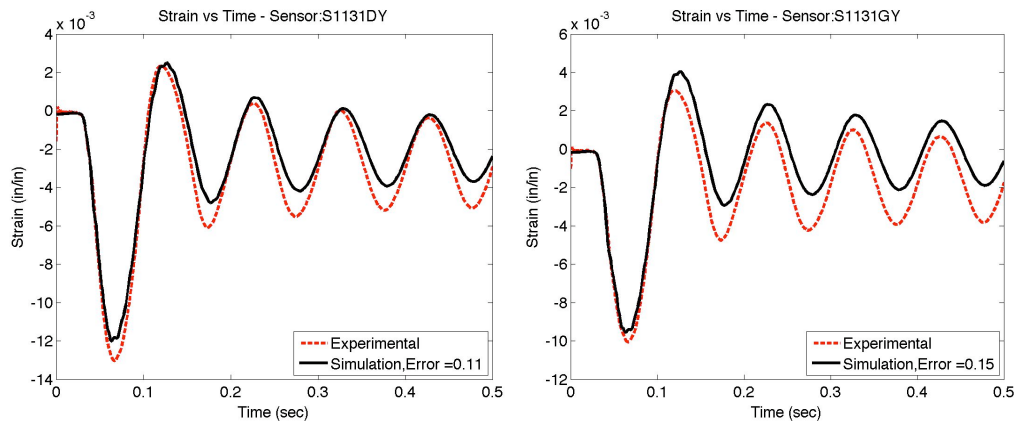
**Figure 6. Specimen CL-1: recorded and predicted acceleration, relative deformation and strain time histories**



**Figure 7. Specimen CL-3: recorded and predicted acceleration, relative displacement and strain time histories**



**Figure 8: Specimen CS-2: recorded and predicted acceleration and strain time histories**



**Figure 8: Specimen CS-2: continued.**

For all the specimens, the Russell comprehensive error factors calculated is below 0.15. For specimen CL-1 and CL-3 the error factors range from 0.069 to 0.13, and for specimen CS-2 the observed error factors are between 0.10 and 0.15. Generally, for the strain time histories, the error factor increases, as the gage gets closer to the fixed support. This can be attributed to the welds at the support, and initial stresses caused by the welds, which are not explicitly modeled. Never the less, for all specimens the observed Russell comprehensive error factor is below 0.15, which indicates the correlation is excellent.

## Conclusions

A constitutive model is outlined for HY-80 steel, used in naval applications. The model can correctly capture both the yield plateau and the progressive cyclic hardening observed in the experiments. The proposed model is validated against published experimental data. It is shown that the constitutive model can accurately replicate the inelastic dynamic response of simple structures under short and long duration impulse loading.

## 5. References

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