

Calibration of Nonlinear Viscoelastic Materials in Abaqus Using the Adaptive Quasi-Linear Viscoelastic Model

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Abstract: This work presents a method for calibrating the built-in Abaqus hyperelastic-viscoelastic material models based on test data from soft material loading and relaxation tests. This approach can be used to represent the uniaxial material behavior with 1-D Abaqus connector elements or by using the built-in 3-D material models. It is based on the Adaptive Quasi-Linear Viscoelastic (AQLV) theory, developed by Nekouzadeh, Pryse, Elson, and Genin, which offers an easy and powerful method for the calibration of highly viscoelastic materials such as soft tissue. In this paper, we first implement the AQLV method in ABAQUS using connector elements. This is done via a process in which the material behavior is calibrated to the test data using the original AQLV formulation implemented within an Abaqus Python script. This script directly specifies the connector properties for the springs and dampers used in the 1-D connector assembly representing the material uniaxial behavior. The AQLV formulation is also used to extract the long-term elastic and the relaxation response from the test data and use it to set up the hyperelastic and viscoelastic definitions of an Abaqus 3-D material model. This allows the flexibility of being able to represent the soft tissue either using 1-D connectors or 3-D elements depending on the modeling needs. The approach is herein illustrated with some example results.

Keywords: Soft Materials, Tissue, Hyperelasticity, Viscoelasticity, Material Modeling, Biomaterials, Quasi-Linear Viscoelasticity.

1. Introduction

Soft tissue usually exhibits nonlinear behavior with large elastic strains and pronounced relaxation response with steep initial fall as illustrated in Figure 1. Modeling such materials within a Finite Element (FE) framework is usually considered a challenging task as it is hard to match the tissue behavior throughout the whole loading-relaxation range. A simple but not very accurate way to set-up such a material model is to use the ramp portion of a ramp-and-hold test as the instantaneous test curve for the elastic definition of a hyperelastic material model, and the hold portion for the viscoelastic relaxation definition. However, the standard ramp-and-hold testing scheme used to get tissue test data usually has a ramp portion of non-negligible duration in which significant relaxation does occur. Thus, it can deviate significantly from the true instantaneous response. The Abaqus material definition provides the option of using a long-term hyperelastic response but it is also not readily available from the ramp-and-hold test. Therefore, an approach to

extract the instantaneous or long-term elastic behavior from the viscous response would be greatly beneficial for tissue material modeling. Such an approach is the Adaptive Quasi-Linear Viscoelastic (AQLV) theory, developed by Nekouzadeh et. al., 2007, which offers an easy and powerful method for the calibration of highly viscoelastic materials. This is a one-dimensional phenomenological model, which is based on Fung's quasi-linear viscoelastic (QLV) model (Fung, 1993). Based on the results from the AQLV formulation, a one-dimensional Abaqus connector assembly can be created that accurately matches the response of the tissue test. This connector assembly consists of springs and dampers connected together and can be used to represent the tissue where 1-D element representation is appropriate. The 1-D model can further be utilized to set-up a 3-D hyperelastic-viscoelastic material model for use with solid elements when needed.

Fung's QLV model makes a good basis for soft tissue modeling because it identifies a class of quasi-linearity that is appropriate for tissues, thus simplifying the material model calibration. Furthermore, the AQLV has comparable or better predictive capabilities than the existing generalizations of Fung's QLV model and is easy to calibrate based on ramp and hold test data.

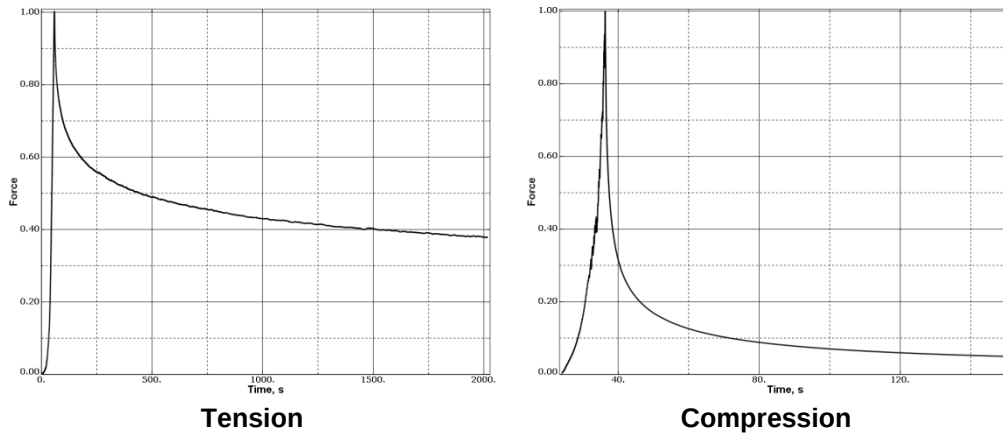


Figure 1. Sample tension and compression curves for tissue material.

2. The adaptive QLV model

The AQLV model as developed by Nekouzadeh, Pryse, Elson, and Genin, is described in detail in Nekouzadeh et. al., 2007. For the purpose of completeness some of the basic relations of the model are herein included. If interested in more details about the model formulation, the reader is further referred to the original publication.

The stresses in the AQLV model are expressed through a “viscoelastic strain”, $V^{(e)}(t)$, in a linear convolution integral:

$$\begin{aligned}\sigma(t) &= k(\varepsilon(t))V^{(e)}(t), \\ V^{(e)}(t) &= \int_{-\infty}^t g(t-\xi) \frac{d\varepsilon(\xi)}{d\xi} d\xi\end{aligned}\quad (1)$$

Here $k(\varepsilon)$ is a pure nonlinear function of strain, and, $g(t)$ is a “reduced” relaxation function that can be expressed as a summation of exponentials with different time constants. $V(t)$ represents the dependence of the model on loading history. The nonlinearity of the model lies in $k(\varepsilon)$, which converts the viscoelastic strain to stress by a simple multiplication. Equation 1 generates proportional stress relaxations for different amplitudes of instantaneous strain. To overcome this proportionality restriction degrees of freedom have been added to the model by allowing a different nonlinear behavior:

$$\begin{aligned}\sigma(t) &= \sigma_0(\varepsilon(t)) + \sum_i k_i(\varepsilon(t))V_i^{(e)}(t), \quad i = 1, 2, \dots, \\ V_i^{(e)}(t) &= \int_{-\infty}^t g_i(t-\xi) \frac{d\varepsilon(\xi)}{d\xi} d\xi, \quad i = 1, 2, \dots,\end{aligned}\quad (2)$$

Here $\sigma_0(\varepsilon)$ is a pure function of strain representing the long-term fully relaxed elastic response. Each $g_i(t)$ could be any relaxation function such that $g(0) = 1$ and $g(\infty) = 0$. In this formulation $g_i(t) = e^{-t/\tau_i}$ is chosen to represent the model in terms of parallel Maxwell elements. Figure 2 shows one such representation.

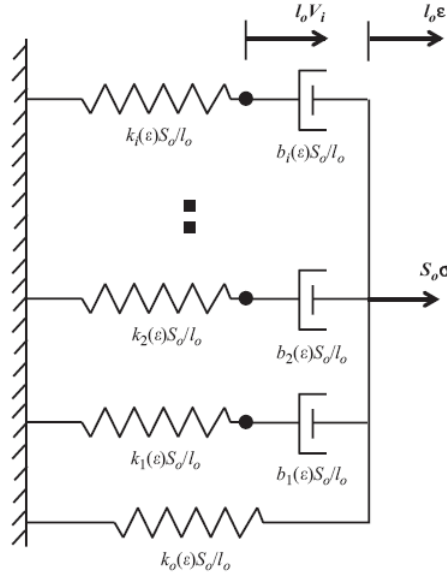


Figure 2. Maxwell element representation of the AQLV model.

For each Maxwell element:

$$\begin{cases} \dot{V}_i + \frac{V_i}{\tau_i(\varepsilon)} = \dot{\varepsilon}, & i = 1, 2, \dots, \\ \sigma_i = k_i(\varepsilon(t)) V_i(t), & i = 1, 2, \dots, \end{cases} \quad (3)$$

where $\tau_i(\varepsilon) = \frac{b_i(\varepsilon)}{k_i(\varepsilon)}.$

For the single spring element which represents the long-term fully relaxed elastic response:

$$\sigma_0 = \sigma_0(\varepsilon)$$

The above nonlinear viscoelastic model becomes quasi-linear by assigning each pair of spring stiffness and dashpot coefficients to be proportional to the same nonlinear function of strain:

$$\begin{cases} k_i(\varepsilon) = \eta_i \psi_i(\varepsilon), \\ b_i(\varepsilon) = \beta_i \psi_i(\varepsilon), \end{cases}$$

where $\psi_i(\varepsilon)$ are arbitrary nonzero functions, making each time constant τ_i independent of strain:

$$\tau_i(\varepsilon) = \frac{b_i(\varepsilon)}{k_i(\varepsilon)} = \frac{\beta_i}{\eta_i} = \tau_i$$

The first order differential equation in Equation 3 becomes linear, and its solution can be calculated from a linear convolution integral, which for constant stretch rate has a closed solution of the form:

$$\dot{V}_i + \frac{V_i}{\tau_i} = \dot{\varepsilon} \Rightarrow V_i(t) = \dot{\varepsilon} \tau_i (1 - e^{-t/\tau_i}), \quad i = 1, 2, \dots$$

As the original authors point out, imagining the model as an assembly of Maxwell elements helps in its physical interpretation as different Maxwell elements represent different relaxation time scales, each perhaps from a different physical source and with a different nonlinear response. Each element models a tissue-level strain-dependent relaxation mechanism and therefore the model parameters (spring constant and dashpot coefficients) arise as functions of overall tissue strain.

3. Abaqus implementation of the AQLV model

3.1 1-D implementation using connector elements

The AQLV model is easy to calibrate based on a series of ramp-and-hold tests. The authors of the original formulation have developed a Matlab script to calibrate the model, which uses the data from a test with four equally spaced ramp-and-hold sections as seen in Figure 3. Within the current work, this script was modified to be able to perform the calibration based on a test with a single ramp-and-hold section. Furthermore, the calibration script was reprogrammed into Python so it could be executed from within Abaqus/CAE. Thus the modified script can utilize tests as the ones presented in Figure 1.

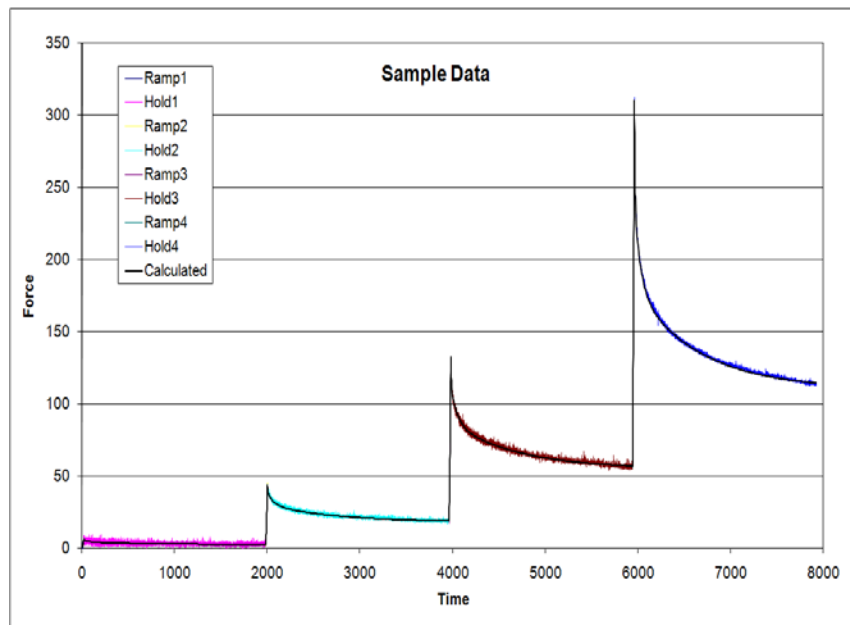


Figure 3. A sample 4-ramp-and-hold test with equally spaced holds for the original AQLV calibration script.

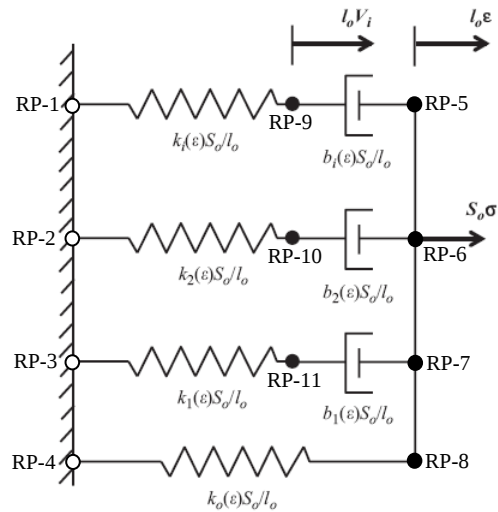


Figure 4. Schematic of the Abaqus connector assemblage.

In Abaqus the tissue is being represented by a connector assembly, consisting of one single spring and three spring-damper elements connected in parallel. The AQLV model script performs a functional minimization of a scalar function of the three time constants, τ_i , and based on the minimal values, determines the properties of each of the connector springs and dampers. The schematic of the Abaqus connector assembly is shown in Figure 4. Within this connector assembly the reference points on the left side on Figure 4, RP-1 thru RP-4, share the same spatial location. Similarly the right-hand side reference points, RP-5 thru RP-8, plus the intermediate reference points, RP-9 thru RP-11, also share the same location at the opposite end of the connector. This makes all the springs have equal length, which is the length of the whole connector, and the three dampers have zero initial length. As shown in Figures 4 & 5 all spring and damper parameters are functions of the strain of the connector assembly (connector deformation). The single spring is defined in Abaqus as a nonlinear spring, and its force vs. displacement curve is directly input from the AQLV model. Since during the connector deformation the three intermediate points which connect the corresponding springs and dampers would move, the same cannot be applied to the three springs in the Maxwell elements and the three dampers. They are defined as linear springs and linear dampers with field-dependent stiffness and field-dependent damper constants, respectively. To implement the connector deformation dependence through a field variable in Abaqus, a sensor is defined giving the deformation of the connector assembly. Within a user subroutine VUSDFLD the field variable controlling the spring and damper constants is evaluated through a user-defined amplitude definition. It is thus being set equal to the current sensor value, which is in fact the connector deformation. The VUSDFLD is very simple to program and the same subroutine can be reused for different models & different materials with very minor changes. These changes require no significant programming experience from the user. The whole connector model can be built in Abaqus/CAE and, if necessary, the corresponding keywords can be added to existing .inp files.

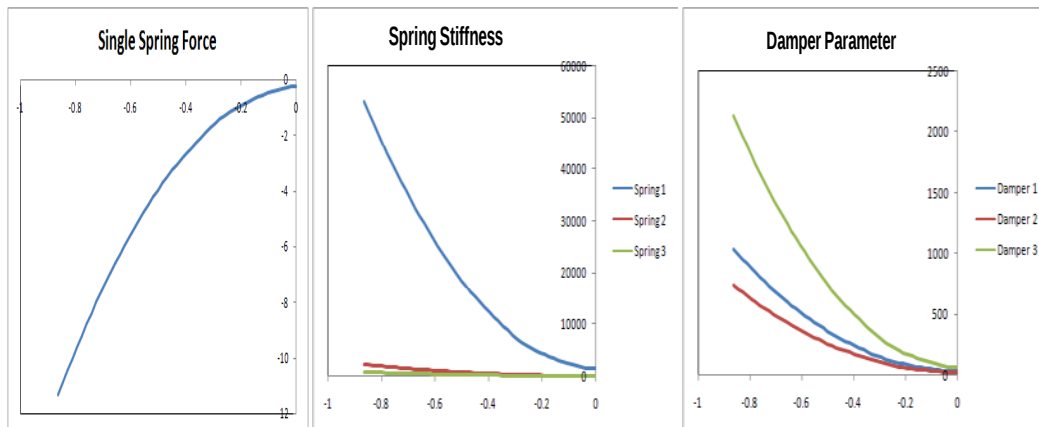


Figure 5. Spring and damper parameters as functions of connector deformation.

3.2 3-D implementation using material models

Once the 1-D Abaqus connector model has been calibrated, the computed long-term response and isolated relaxation response can be used to also set-up a 3-D Abaqus material model. Note that the single spring from the AQLV assembly (lowest spring on Figures 2 & 4) defines the long-term completely relaxed behavior of the tissue. Therefore, transferring that spring force-deformation relation into stress-strain we get the uniaxial nominal stress vs. nominal strain data needed to define the elastic portion of an Abaqus hyperelastic or hyperfoam material model. Within the elastic material definition we also have to specify that the moduli time scale is long-term.

The hold portion of the uniaxial test can be used to define the viscoelastic portion of the material model. We need to specify the data as relaxation test data. However, we need to uniformly scale the curve so it starts at one unit since it defines the compliance or moduli. Also, if the test ramp portion is not very short compared to the relaxation portion we need to perform a virtual test with a shorter ramp portion using the already calibrated Abaqus 1-D model. Using this approach we can relatively easily achieve a good 3-D material definition which matches the uniaxial test response. However, note that depending on the test data we might arrive at a hyperelastic material which is unstable for some strain values. This could be checked by performing a material evaluation within Abaqus/CAE.

4. Abaqus results

One of the main reasons for selecting to implement the AQLV method in this work was its good correlation with test data as shown in the original publication by Nekouzadeh et. al., 2007. We have also tested the model against several tissue compression tests and have in all instances observed an excellent correlation. Figure 6 shows how the AQLV model in its current Python implementation compares to the corresponding data for one of the ramp-and-hold tests. As seen we have a very close fit for both the ramp and the hold portions. The test data is represented in the figure with gray dots and the AQLV curves are solid black. The red dots in the ramp curve represent intermediate points used in the Python script, which can be modified by the user, and are used to improve the ramp curve fit. All of our AQLV models had similar correlation to the corresponding test data.

All of the AQLV models that we converted into Abaqus 1-D connector assemblies also showed excellent correlations between the AQLV model and the Abaqus results. Figure 7 shows a comparison of one of the Abaqus 1-D models versus its corresponding AQLV model. As seen we have a perfect match between the two, which was also the case in all our observations.

Figures 8 & 9 show representative results for the 3-D material models. Several sample and test data from tissue tests were processed using the Abaqus 1-D models to get the necessary material model data. The hyperelastic portions of the materials were evaluated to assess the limits of their stability, and the model with a good correlation to the long-term data and stable response within the desired strain range was selected. The Abaqus hyperfoam material model was also tested together with the hyperelastic models. In all cases the long-term elastic response was directly input from the 1-D models, but the relaxation response required some additional processing. This is due to the fact that some relaxation has already occurred prior to the hold portion during the ramp up loading stage. The 1-D model already calibrated to the corresponding test data was used as a virtual test bench to perform a uniaxial test with shorter ramp portion duration to better represent

the relaxation behavior after an instantaneous loading. The relaxation portion of that test was used for the viscous definition within the 3-D material model. Several iterations were performed with increasing loading speed until the peak of the 3-D ramp step matched the peak of the test curve with enough accuracy. Figure 8 shows one data set where an exact match to the initial data was achieved, while Figure 9 has some acceptable deviation in some areas of the curves for both the hyperelastic and the hyperfoam models.

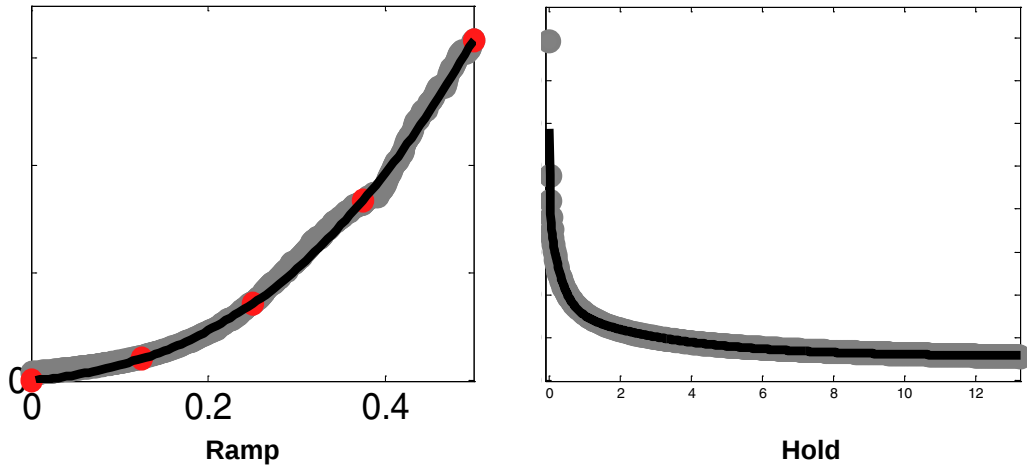


Figure 6. AQLV model results vs. test data
(gray dots – test points, black lines – AQLV model curves).

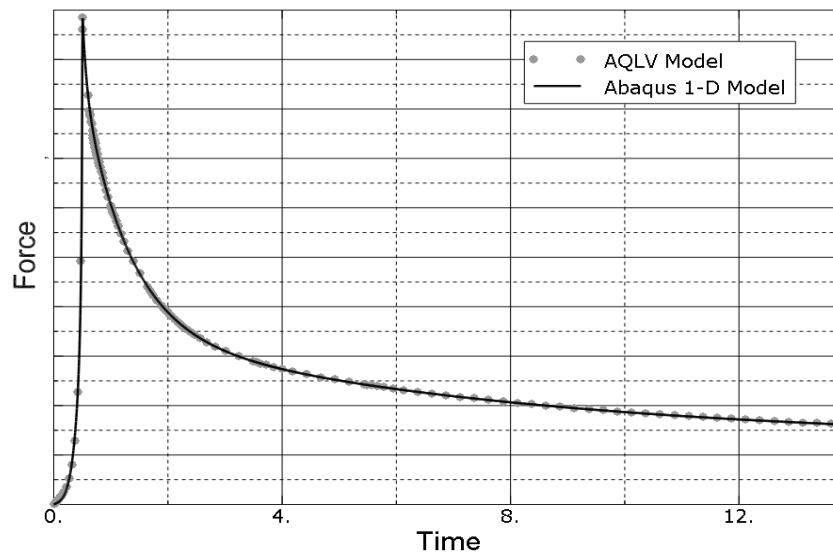


Figure 7. Abaqus 1-D model vs. AQLV results.

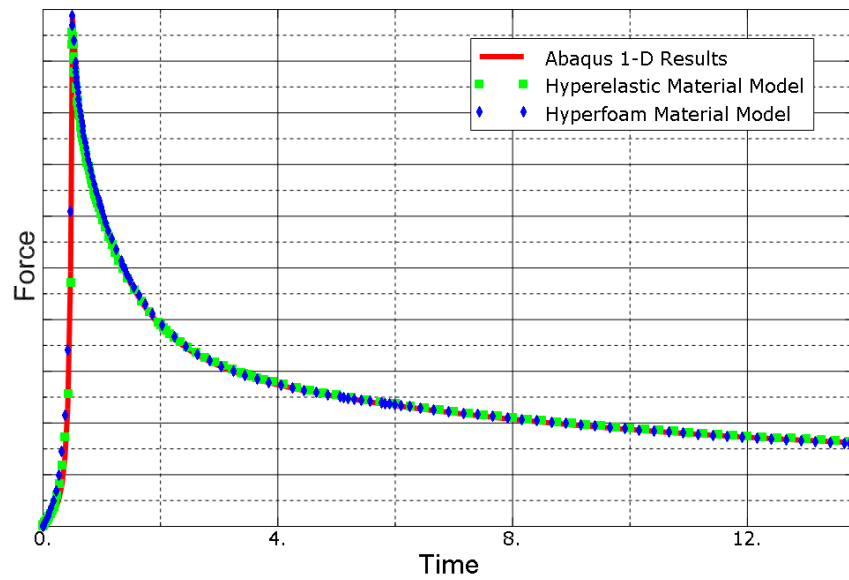


Figure 8. Abaqus hyperelastic and hyperfoam 3-D material model vs. 1-D results.

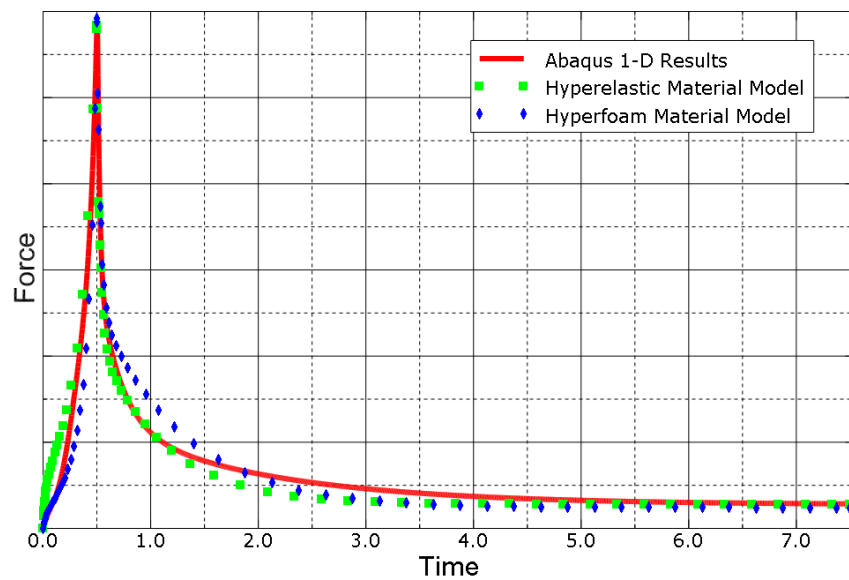


Figure 9. Abaqus hyperelastic and hyperfoam 3-D material models vs. 1-D results.

5. Conclusion

The adaptive QLV model on which this work is based proved to fit the large strain behavior of soft tissue very well. The model possesses extra degrees of freedom compared to Fung's QLV model, enabling it to capture non-proportionality in the relaxation curves at different strains. The model is easier to calibrate and use than the Fung QLV model and its results data is easy to transfer and implement into an Abaqus 1-D connector model. Furthermore, this 1-D model proves very useful when calibrating an Abaqus 3-D constitutive model to match the material response.

The Python script that implements the AQLV formulation greatly improves transferring the data into an Abaqus model. The script could easily be modified to directly build the whole 1-D connector assembly within Abaqus/CAE with all the corresponding spring and damper definitions. It could, furthermore, be expanded to also build and evaluate the corresponding 3-D material definitions. This could be cast in an Abaqus/CAE plug-in to simplify its utilization. The authors are currently working on this enhancement. The current Python script together with a sample tutorial is available upon request from the authors.

6. References

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2. Nekouzadeh, A., Pryse, K. M., Elson, E. L., and Genin, G. M., "A Simplified Approach to Quasi-Linear Viscoelastic Modeling", Journal of Biomechanics, vol. 40, pp. 3070–3078, 2007.

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