

Probabilistic Simulation Applications in Product Design

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Abstract: Each year companies spend millions of dollars for developing new products with high quality and reliability. Highly reliable products require longer test times to verify, and usually takes a few iteration of design-test-fix cycle. Development time can be minimized by (1) doing accelerated testing (ALT) and (2) reducing the design-test-fix cycle by developing methods to predict and test for reliability in simulation environment. Finite element modeling and analysis provides an excellent alternative in evaluating designs to improve on reliability. In this paper, a probabilistic simulation methodology is proposed using a combination of simulation modeling and statistical techniques to predict and improve the drop reliability of a product under repeated random loading. Two examples are illustrated involving two failure modes in two different products a screw pull out and a magnesium housing cracking. Explicit dynamic finite element phone drop simulations were performed in Abaqus to predict the forces in the screw and the principal tensile stress on the housing for various simulation parameters. DOE and Response surface modeling was used to develop regression equations for stress as a function of drop angle. Using the statistical techniques, a probabilistic model was developed by combining the RSM model & statistical distribution of drop angle, to estimate the distribution of the stress. In conjunction, strength degradation models were developed to reflect the housing degradation with each impact. Finally, Monte Carlo Simulations are used in conjunction with the stress-strength interference theorem to predict product reliability. In conclusion, a powerful and practical methodology is proposed that integrates the FEA with statistical methods to predict up front, the reliability of a product

Keywords: FEA, Probabilistic Simulation, Design Of Experiments, Response Surface Modeling, Stochastic Simulations, Monte Carlo Simulations, Stress-Strength Interference Theorem, Reliability, dynamic probability density function, Strength degradation functions.

1. Introduction

Each year companies spend millions of dollars while developing a new product. If the reliability expectation from the product is high, extensive testing is required to demonstrate that the product reliability meets the expectations. Testing requirements inevitably increases the development cycle especially for highly reliable products. One way to reduce this cycle time is to do Accelerated Life Testing (ALT) where test units are subject to high stress environments simulating the stresses that the product encounters in normal operating conditions.

The ALT is intended to detect the potential failure modes, so that the design can be improved upon, prior to the product reaching the market. The product development groups typically go through multiple design iterations and ALT testing. This approach tends to make the product development cycle time long and costly due to prototype tooling and testing for the various design iterations. Also, this process usually takes place at a much later stage of product development at which point millions of dollars are already invested in tooling. Ideally, the product development teams should determine the problems in design, at a much earlier phase than the ALT testing, in order to avoid expensive retooling and redesign phase. Finite element modeling and analysis provides an excellent alternative in evaluating designs to improve on product reliability. However, FEA simulations to date are largely deterministic and provide good point estimates on product performance. The real world product design and development is subjected to a whole host of sources of variation arising from geometry, material, loading etc. that can make or break the product design. The challenge that the product development faces is making the design robust under these host of variations. While deterministic FEA simulations are good at providing point estimates on the product performance, they cannot assess the design robustness. This is where probabilistic/stochastic simulation methods - combination of simulation, experimental and statistical techniques - can play a vital role in designing robust products while minimizing the number of design iterations. In this paper we show the application of the probabilistic simulation methods in evaluating two different failure modes on two different encountered in ALT testing. The first failure mode corresponds to screw pull out during drop testing leading to disengagement of the camera module, while, the second failure mode corresponds to magnesium housing cracking under repeated drop testing.

The paper is laid out in four broad sections. The first section lays out the four essential steps for the probabilistic/stochastic simulation modeling and constitutes the main body of the paper. The four main steps involve – a) Identification of random variables/ sources of variation affecting the product design, b) Development of a transfer function that relates the output response of interest (such as stress, force, displacement etc) with the design/random variable of interest, c) Characterization of the statistical distribution of the random variables & d) Monte Carlo simulation of ALT to predict product reliability using stress-strength interference theorems. The second section details how the above methods are applied in investigating the screw pull out failure mode in ALT drop testing and application of these methods in predicting the probability of failure during drop testing. The third section details how the above methods are applied in investigating the magnesium housing cracking during repeated drop testing in ALT and application of these methods in predicting the probability of failure during drop testing. The conclusion section details the findings as well as the challenges and future trends. All the results are reported in the following system of units : Mass – gm, Length – mm, Time – mSec, Energy – mJ, Displacement – mm, Velocity - m/Sec., Acceleration - Km/S², Force – N, Stress, Modulus – Mpa.

2. Probabilistic/Stochastic Simulation Modeling

The four main steps involve – a) Identification of random variables/ sources of variation affecting the product design, b) Development of a transfer function that relates the output response of interest (such as stress, force, displacement etc) with the design/random variable of interest, c)

Characterization of the statistical distribution of the random variables & d) Monte Carlo simulation of ALT to predict product reliability using stress-strength interference theorems.

2.1 Identification Of Random Variables/ Sources of Variation

The first step in probabilistic simulation modeling involves identifying the critical design factors/sources of variation affecting the robustness of the product design specific to a failure mode.. This is usually determined based on the physics of the problem as well as collective inputs from various subject matter experts that have insights into the design problem. Variability is the essence of all product design. It is usually a good idea to list all potential factors that can affect the design robustness and then use a screening process to Pareto out the most important factors that affect the response, as the typical modeling schemes will not facilitate accounting for numerous factors in an easy manner. The real world product design and development is subjected to a whole host of sources of variation arising from geometry, material, loading etc. that can make or break the product design. It is usually a good idea to map out a fish bone diagram that bucketizes potential sources of variation. Figure 1 shows a typical fishbone diagram for drop testing of cellular products.

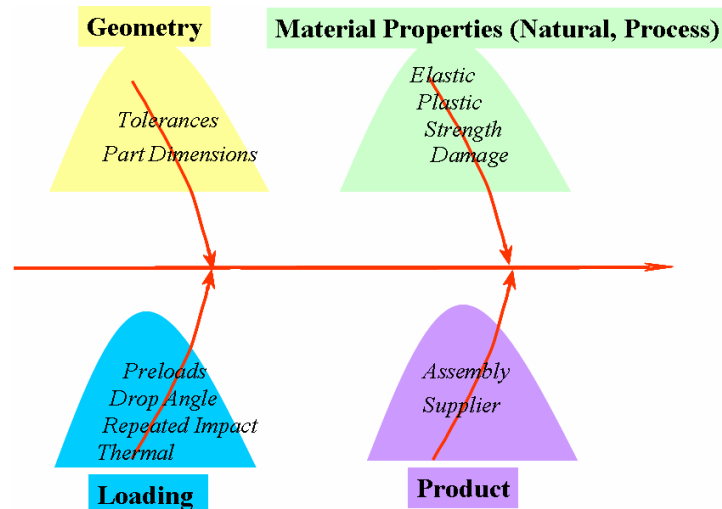


Figure1. System Sources Of Variation – Fishbone diagram.

2.2 Development of Transfer Function

Development of transfer function involves developing a mathematical model that correlates the response variable of interest (variables that would be used in assessing reliability, design robustness) with the input design/random variables. Typical structural response variables can be stress, strain, force, displacement etc. The input design/random variables are the variables that

affect the structural response and also are subject to variations. These typically can be factors related to geometry, material or loading as shown in the fishbone diagram in figure 1. The methods for generating the structural response itself can be through FEA, instrumented testing or analytical equations based on principles of mechanics. The motivation for generating a transfer function comes from the fact, that stochastic simulation modeling calls for computing the structural responses for thousands of permutation and combination of the random variables. While FEA is attractive in making point estimates for specific values of the random variables, in general, it is not practical to compute the responses for the various permutations of the random variables through FEA. In order to establish a transfer function, one of the key steps is to compute the responses for certain combinations of the random variables determined through a design of experiments or a custom factorial design. Determining the combinations requires some degree of understanding of the governing nonlinearities in the problem, as this subsequently determines the fidelity of the transfer function. Finally response surface/regression modeling is required that establishes the mathematical model (such as shown below) that correlates the response variable of interest with the input design/random variables. A lot of details in this section can be worked through statistical software such as JMP, Minitab etc.

$$F = F(X_1, X_2, X_3 \dots X_n)$$

2.3 Characterizing Variations/ Determining Uncertainty of Random Variables

The goal of this section is to establish statistical distributions of the input factors/random variables that govern the design response. These distributions are subsequently used to drive the Monte Carlo simulations to compute design reliability and robustness. Statistical distributions can be established through careful measurements of the parameters of interest. Typically, atleast 30+ sample measurements are needed to provide a decent estimate of the mean and standard deviation of the distribution. In cases where measurements are not feasible or practical, some form of distribution may be assumed based on inputs from subject matter experts. Statistical distributions themselves can be normal, lognormal, Weibull, exponential etc. and statistical software can help to determine the appropriate distribution for a random variable. A typical distribution is shown in figure2 below.

2.4 Monte Carlo Simulation of ALT to predict Design Reliability

The interest in simulation methods started in early 1940's for the purpose of developing inexpensive techniques for testing engineering systems by imitating their real behavior. These methods are commonly called Monte Carlo simulation techniques. The principle behind the method is to develop an analytical model, which is computer based, that predicts the behavior of the system and repeat it many times under all possible conditions.

The last step in the probabilistic simulation methodology is the Monte Carlo Simulation and has essentially two objectives – 1) Establish the statistical stress distribution by combining the transfer function with the statistical distribution of the random variables – ie: convert the deterministic response (transfer function from FEA) model into a stochastic model. Figure 3 below illustrates the concept. 2) The second step involves simulation of virtual ALT to determine product design

reliability as well as robustness through the numerical computation of the stress-strength interference (discussed below).

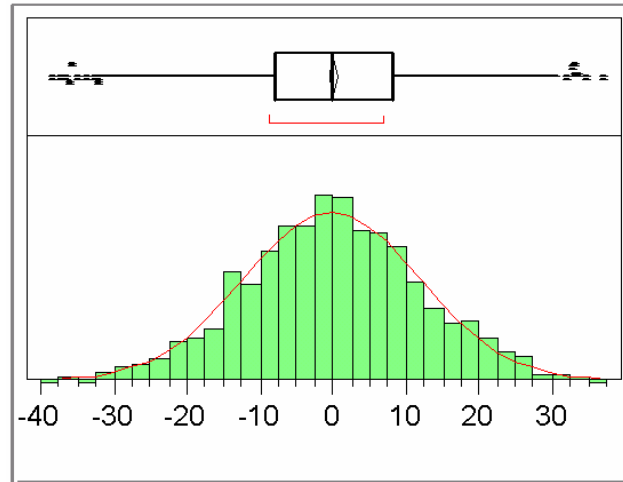


Figure2. Typical Statistical distribution of a random variable.

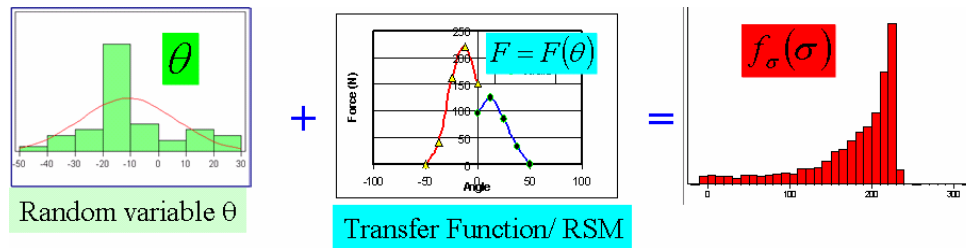


Figure3. Establishing stress distribution by combining transfer function and random variable distributions.

When the distribution of stress and strength are known, the stress-strength interference theorem can be used in calculating the failure probability of the design. This concept is illustrated in Figure 4 for hypothetical stress and strength distributions. The area common to both distributions is shaded. It denotes the set of all conditions under which the stress exceeds the strength and therefore is the probability of failure of the design. Denoting the stress parameter as σ and the strength parameter as S and their respective distributions (p.d.f.'s) by $f_{\sigma}(\sigma)$ and $f_S(S)$

The reliability of the system (strength exceeding stress) can be computed mathematically as

$$R(S > \sigma) = \int_{-\infty}^{+\infty} f_S(S) \left[\int_{-\infty}^S f_{\sigma}(\sigma) d\sigma \right] dS$$

The probability of failure is then given as $P=1-R$, and is expressed mathematically as

$$P = 1 - \int_{-\infty}^{+\infty} f_s(S) F_\sigma(S) dS$$

Where F_σ denotes the cumulative density function of stress, and this can be further simplified as

$$P = \int_{-\infty}^{+\infty} [1 - F_\sigma(S)] f_s(S) dS$$

The above expressions for the reliability and probability of failure of the systems while complex, can in general be evaluated analytically or numerically using software such as MathCad, Mathematica, Matlab etc, when the p.d.f.'s for the stress and strength are known. If both the stress and strength distributions are normal, lognormal, exponential, Weibull or different combinations of normal (exponential), normal (Weibull) analytical solutions exist in literature. If the distributions are unknown or arbitrary, Monte-Carlo Simulations (MCS) presents a very powerful technique to solve the above equations. Figure 5 shows the flowchart for Monte-Carlo simulations. A simple scalar comparison of stress to strength determines the occurrence of failure. Repeating the simulations thousands of times enables computation of probability of failure (area of the shaded region in figure 4) and other statistical measures of the design performance can be inferred from the above simulations.

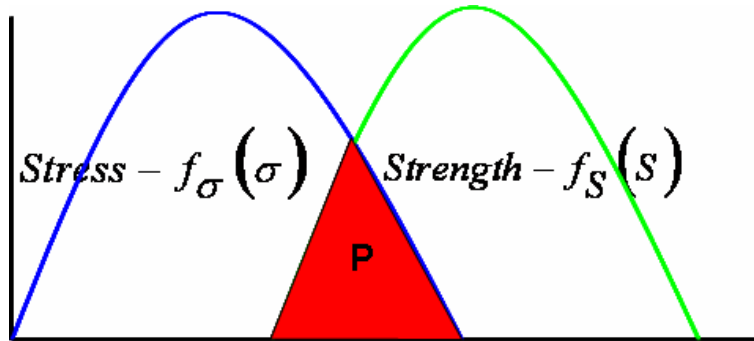


Figure4. Stress-Strength Interference

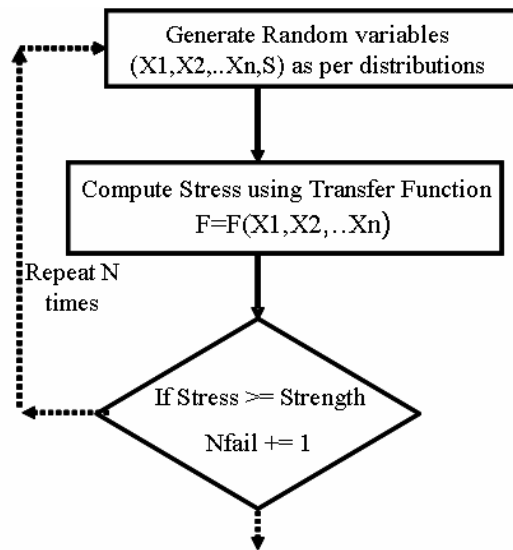


Figure5. Flow chart for Monte Carlo simulations

3. Case Study I : Insert/Screw Pullout during Drop Testing

The first case study presents the application of stochastic simulation modeling in computing the probability of occurrence of an insert/screw pullout during drop testing. Figure 6 shows the failure mode where a heat staked brass insert is pulled out of the housing leading to the disengagement of the camera module during drop testing.

Failure Mode : Insert pull out during Drop Testing

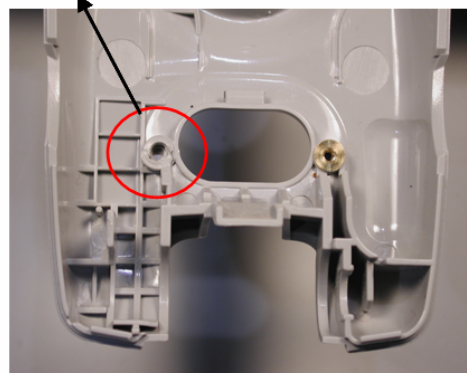


Figure6. Failure Mode showing Insert Pullout during drop testing.

3.1 Identification Of Random Variables/ Sources of Variation

Based on the physics of the problem, the key factors affecting the product design were determined to be the drop impact angle in the two planes (X_1 & X_2) as well as the strength of the insert. The response variable of interest is the tensile force in the screws as this was identified to be the root cause resulting in the heat staked insert being pulled out of the housing. It was known from ALT testing as well as the physics of the problem that the insert pull out during drop was primarily occurring in the top orientation. However, since the units are hand dropped in ALT, it was clear that they were most likely to land at some arbitrary angle. We denote the orientation in the front view as X_2 (Φ_{x2}) and the orientation in the side view as X_1 (Φ_{x1}). The goal of the screening design is to identify if one or both planes are statistically significant in determining the tensile forces in the screws. An initial screening DOE was used to assess the relative importance of the design variables. The Pareto plot below shows the relative significance of two drop planes.

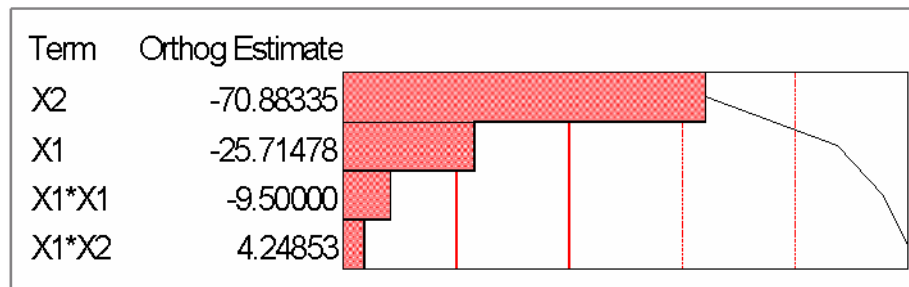


Figure7. Pareto plot of transformed estimates.

3.2 Development of Transfer Function

Development of transfer function involves developing a mathematical model that correlates the response variable of interest (variables that would be used in assessing reliability, design robustness) with the input design/random variables. FEA model of the product was developed to simulate the stress environment in the ALT. An IGES representation of the entire phone assembly from PRO/E was imported into HyperMesh, where the finite element was developed. The finite element model consisted of all the components in order to accurately capture the distribution of the inertial mass. A beam element was used to model the screw and the section forces in the element are used to monitor the tensile force in the screw.

Dynamic finite element drop simulations were performed (dropping is part of the ALT testing that is being simulated by FEA) in Abaqus/Explicit to predict the tensile force in the screw. Figure 8 shows a typical tensile force history in the screw during drop. All response variables - calculated or experimentally determined - were coded in order to protect proprietary information, which only affects the scale of the distributions

From the experimental study of the drop angle distribution as detailed in the next section, it was realized that the drop angle can vary from from -50 to +50 degrees. Drop simulations were

performed in this window in increments of 12.5 degrees. Figure 9 shows the response surface of force as a function of drop angle for both the screws.

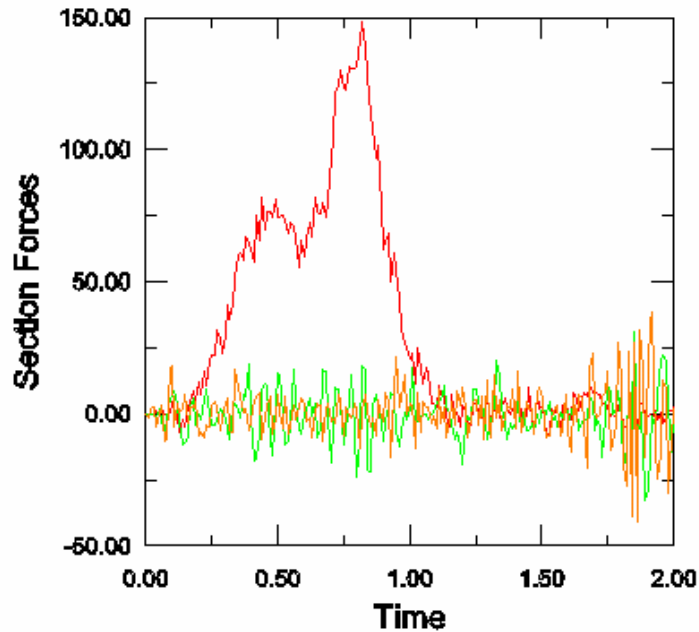


Figure8. Tensile force history in the screw.

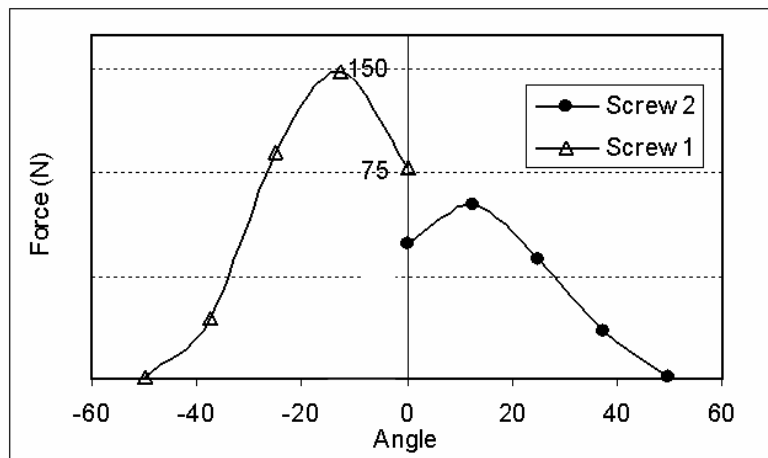


Figure9. Peak Tensile Force variation as a function of drop angle.

The force on the left screw (Screw1) is shown to span from -50 to 0 degree angle, designated as (-50, 0) in the remainder of the text. This force rapidly drops to an insignificant value for positive values of angle (X2 variable). Similarly, the force in the right screw (Screw2) is shown to span in

the (0, 50) drop angle, as this force rapidly drops to an insignificant values for negative drop angles. The regression equations are shown below for both screws as a function of drop angle along with their goodness of fit. In both the cases, a cubic polynomial is found to be an excellent fit to the data given in Figure 4. The goodness of fit for both are larger than 0.99.

$$\text{Force}_{\text{SC1}} = 78 - 14.4 * X_2 - 0.79 * X_2^2 - 0.009 * X_2^3, X_2 \leq 0$$

$$\text{Force}_{\text{SC2}} = 23 + 5.8 * X_2 - 0.34 * X_2^2 + 0.004 * X_2^3, X_2 \geq 0$$

The above equations represent a mean tensile force as a function of drop angle. The distribution of the drop angle, X_2 in the above equations, is empirically developed from experimental work as detailed in next section. These two regression equations are used in Monte-Carlo simulation in estimating the probability of failure. To estimate the product reliability in ALT testing, the stress distribution (through regression equalitons) and the strength distribution need to be developed. The next section shows the experimental work in empirically developing the drop angle distribution

3.3 Characterizing Variations/ Determining Uncertainty of Random Variables

Experimentally units were dropped multiple times on the plane of the interest. Each time the angle of impact at the instant of impact was measured in the front plane (X_2) using a high speed video and motion tracking software. The figure below shows the distribution for the drop angle. A normal distribution was fit to the data. The mean and the standard deviation of the data were estimated to be -10.7 and 17.2 degrees, respectively.

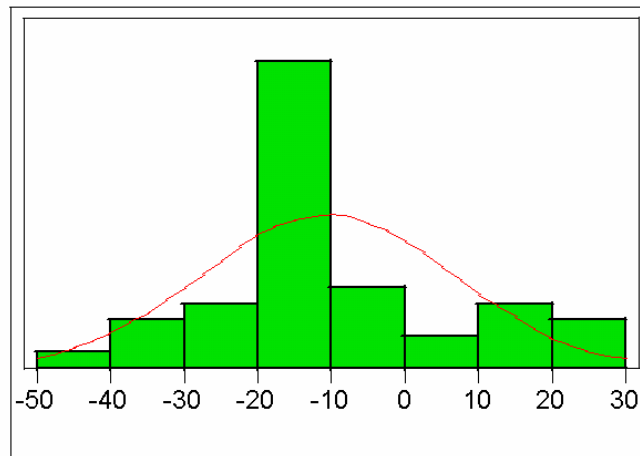


Figure10. Drop Angle Distribution.

Similarly, insert pull out tests were carried out with 20 samples and the force required to pull the screw out was measured. The results shown in figure11 have a mean of 116.5 N and standard deviation of 29.7 and a normal distribution is found to fit the data.

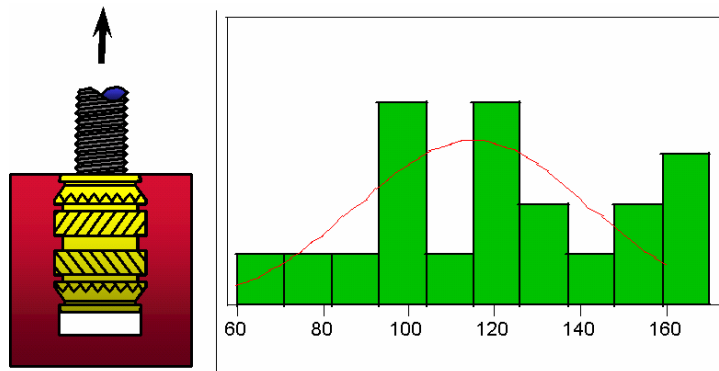


Figure11. Strength Distribution.

3.4 Monte Carlo Simulation of ALT to predict Design Reliability

The last step in the probabilistic simulation methodology is the Monte Carlo Simulation and has essentially two objectives – 1) Establish the statistical stress distribution. 2) The second step involves simulation of virtual ALT to determine product design reliability as well as robustness through the numerical computation of the stress-strength interference.

In previous sections, the transfer function as well as the distributions of drop angle and strength were established. In order to calculate the probability of failure, two normal variables are generated from the known strength and drop angle distributions. Given the random variable for the angle, the regression equation can be used to estimate stress. After generating a pair of random variables for stress and strength, they are compared to each other. These steps are repeated many times and a running total is kept for the failure condition (stress larger than strength is the failure condition). The ratio of number of failures to the number of simulation run is the Monte Carlo estimate of the probability of failure.

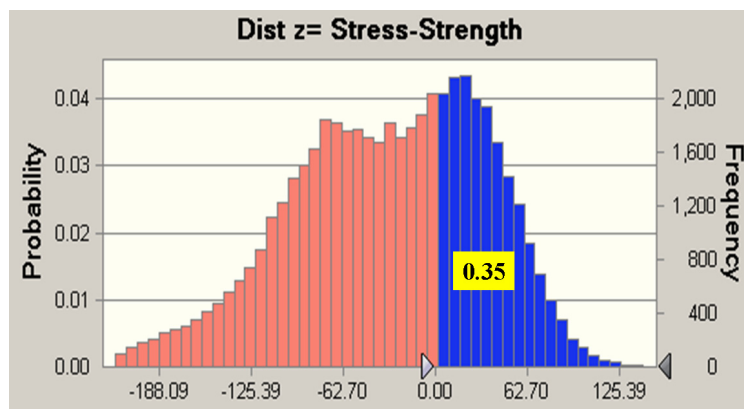


Figure12. Monte Carlo Simulation showing probability of failure

From the simulation, estimated probability of failure for left screw is 0.34792 while the probability of failure for the right screw is 0.00188. The probability of failure from screw1 or screw2 is estimated to be 0.34977. The observed failure probability in the ALT was varying from 0.30 to 0.40 which is in good agreement with MCS failure probabilities. Further, the methodology correctly identified the problem to be predominantly with the left screw.

4. Case Study II : Magnesium Housing Cracking during Drop Testing

Drop testing of portable electronic products is a popular test method that is widely used in the industry as a means to improve product reliability, by detecting all possible failure modes at an early stage in the development process. Survivability of a product under repeated drop testing which is random is one of the measures that are used to gauge the robustness of the product. The failure mode encountered is the cracking of magnesium housing under repeated drop testing. The probabilistic simulation methodology presented in this paper is used to predict the survivability of the product under repeated drop testing. The figure below shows the orientation as well as the failure mode which is the cracking of the housing under repeated drop testing.

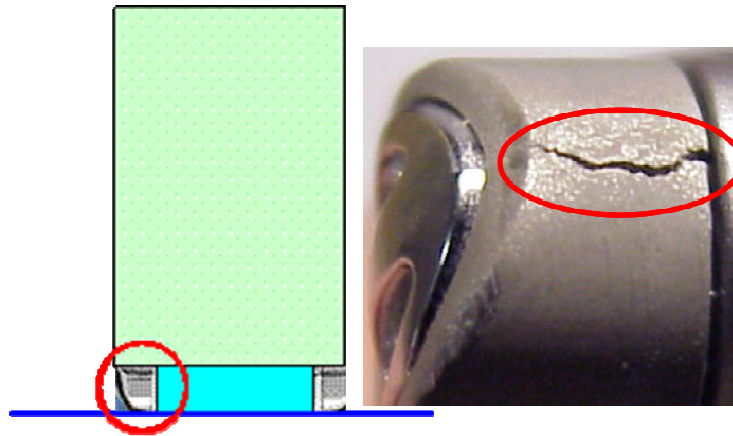


Figure13. Magnesium Housing Cracking during repeated drop testing.

4.1 Identification Of Random Variables/ Sources of Variation

Since the units are hand dropped in ALT, it was clear that they were most likely to land at some arbitrary angle. Hence orientation was certainly a factor that was expected to have significant impact on the response. In addition, since the magnesium housing was die cast, the process was also expected to be a significant factor. Initial measurements on the part indicated significant variation in the part thickness in the region of failure. Based on the physics of the problem, the key factors affecting the product design were determined to be the drop impact angle, wall thickness of the magnesium housing as well as the strength of the housing. The response variable of interest is the tensile stress on the housing as this was identified to be the root cause resulting in cracking of

the housing. For simplicity, the methodology is presented here with drop angle and strength as the random variables.

4.2 Development of Transfer Function

Dynamic finite element drop simulations were performed in Abaqus/Explicit to predict the tensile stress in the screw. A 3X7 custom factorial was designed involving 3 levels of thickness and 7 levels of drop angle (ranging from 0 to 45 degrees). A total of 21 drop simulation runs were performed to gather the response (principal tensile stress on the housing) at various combinations of the settings. For simplicity, a response surface model is shown below for both the right and left knuckle, in which the stress is a fourth order polynomial of the drop angle ϕ . The figure below shows the stress distribution as a function of the drop angle for both the left and right knuckles. All response variables - calculated or experimentally determined - were coded in order to protect proprietary information, which only affects the scale of the distributions.

$$\text{Stress}_{RK} = 1420.7 + 206.1\phi - 12.63\phi^2 + 0.251\phi^3 - 0.0016\phi^4$$

$$\text{Stress}_{LK} = 1426.3 - 225.8\phi - 17.54\phi^2 - 0.525\phi^3 - 0.0055\phi^4$$

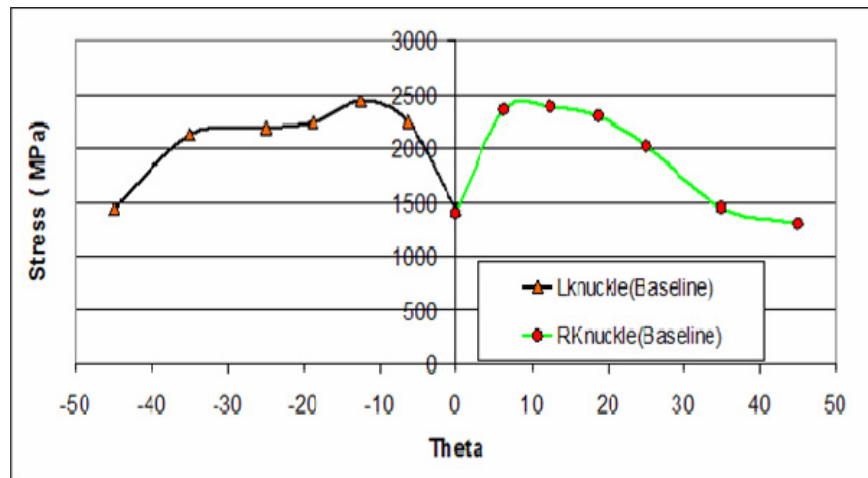


Figure14. Stress distribution on the Knuckles.

4.3 Characterizing Variations/ Determining Uncertainty of Random Variables

The drop angle distributions were determined in a manner similar to the previous section. This section focuses primarily on the development of strength distribution. Survivability of a product under repeated drop testing is one of the measures used to gage the robustness of the product. The number of drops at which the product fails for the failure mode of interest is called drops-to-failure (Dtf) which is one of the metrics of interest in ALT testing. Repeated drop testing however implies that the product sustains some degree of “damage” with each impact, until it leads to

catastrophic failure. However, simulation of repeated drop testing using FEA methods requires enormous computational resources and is not practical since each simulation takes several hours to complete. In this study, strength degradation models are proposed in which the strength of the housing degrades with each impact. These models in conjunction with response surface models and Monte Carlo simulation can then be used to simulate repeated drop testing without actually having to simulate (using FEA) each drop loading.

A “dynamic” probability density function is proposed for strength in which the strength variable from a known initial distribution is degraded after each drop through the strength degradation models. Figure 15 shows the stress-strength interference idea with the concept of dynamic pdf for strength. With each impact, mean of the strength is degraded (through the strength degradation modes discussed below) and the strength distribution shifts to the left, increasing the stress and strength overlap area (failure probability) as shown in the figure. In Figure 15, S_t denotes the virgin strength distribution while S_E denotes the degraded strength distribution. Further, it is assumed that the standard deviation of the strength distributions is not affected by repeated drop testing. Two strength degradation models – logarithmic and exponential are proposed below that account for different mathematical models for strength degradation.

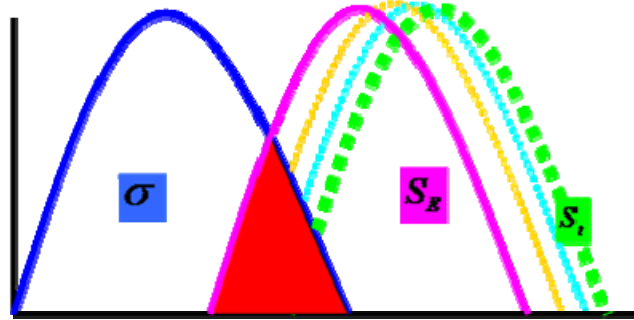


Figure15. Stress(σ) and “Dynamic”-Strength(S) Interference.

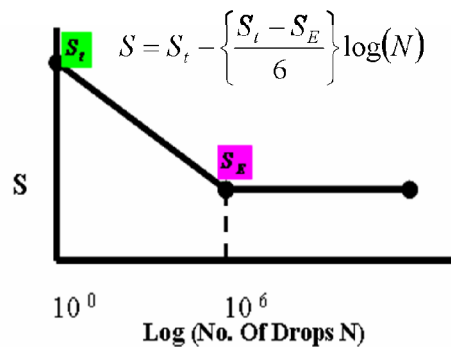


Figure16. Logarithmic Strength Degradation Model.

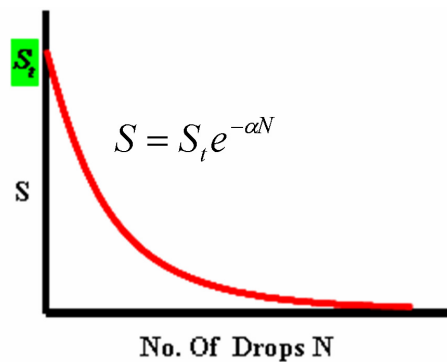


Figure17. Exponential Strength Degradation Model.

The logarithmic strength degradation model is derived analogous to the theory of metal fatigue, in which a material has a limit called the endurance limit (S_E) below which the material can be subjected to millions of loading cycles with out failure. Materials such as steel and plastics tend to obey this kind of material behavior. The exponential strength degradation model is based on the notion that the material does not have an endurance limit. This implies that material strength degrades with each drop irrespective of the magnitude of the stress level during drop. Materials such as magnesium, aluminum and some copper alloys tend to obey this kind of material behavior. Both are two parameter strength degradation models – the determination of which is requires controlled material testing. However, an engineering approach was adopted in this study, in which these parameters were inferred based on observed probabilities of failure in baseline ALT testing.

4.4 Monte Carlo Simulation of ALT to predict Design Reliability

The drop angle distribution in conjunction with the RSM model is used to define the probabilistic response model for stress. Monte Carlo simulation can then be used to perform “Virtual ALT”, wherein thousands of simulation runs can be used to simulate virtual drop testing, by using the statistical distribution created from the probabilistic response model. For each simulation run, two random variables drop angle and strength are generated. The computed stress from RSM model is compared against the strength (random variable) generated. If stress is less than strength, then the strength is degraded using either the logarithmic or exponential degradation models. This process is repeated until the computed stress exceeds the degraded strength or the maximum pre-determined drop number is reached. For each unit, drop number at which the unit is failed is recorded as the drop-to-failure (Dtf) number. Thousands of such units are “Virtually dropped” to generate distributions of Dtf for the baseline and improved designs.

The table below summarizes the Dtf predictions (both mean and percentiles) for the baseline and improved design as compared to the ALT testing, with both the logarithmic and exponential strength degradation models. As can be seen from the table, the mean Dtf score predictions from MCS are in reasonably good agreement with mechanical testing. It is found that there is no significant difference between the two strength degradation models given the low levels of reliability. Further, it is found that MCS predictions without strength degradation over predict the product reliability.

Test Method	Baseline Design		Improved Design	
	Dtf Mcan	Dtf -Percentiles (10, 90)	Dtf Mcan	Dtf -Percentiles (10, 90)
Mechanical Testing	0.04	(0.02, 0.06)	0.15	(0.1, 0.2)
MCS – No Degradation	0.06	(0.02, 1)	0.28	(0.02, 1)
MCS – Log Model	0.05	(0.02, 0.12)	0.1	(0.02, 0.22)
MCS - Exponential	0.054	(0.02, 0.08)	0.09	(0.04, 0.14)

Figure18. Normalized Reliability Predictions using MCS for the Baseline and Improved Designs

5. Conclusions

In conclusion, a powerful and practical methodology has been presented that integrates the FEA with statistical methods to predict up front, the reliability of the product under repeated random loading. Application of this methodology at an early phase in the design process has the potential for reducing developmental cycle times and enabling design for reliability. While deterministic simulations provide good point estimates on product performance, it is incapable of assessing the design robustness. The real world product design and development is very challenging as it is subjected to a whole host of sources of variation. Probabilistic simulations address a lot of shortcomings of deterministic simulations. However, they are computationally expensive as it involves building response surface models or direct Monte Carlo simulations. In addition, decent automation of pre and post processing is highly desirable as it greatly facilitates the application of probabilistic methods in product design.

6. References

Numerous good references on probabilistic simulation methods exist in literature and are not listed here.

7. Acknowledgements

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