

FEM Evaluation of Mechanistic Cutting Force Coefficients Using ALE Formulation

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Abstract: *The investigation to obtain the cutting force coefficients has been carried out over the past two decades by many researchers. However, the outstanding disadvantages of voluminous experimental data acquisition and its analyses still exist. This paper demonstrated the use of an efficient and accurate numerical tool (i.e. FEA), in simulating the cutting process and determining the cutting force coefficients. In this approach, an Arbitrary Lagrangian Formulation (ALE) was employed in the FEM simulations. This formulation has been gaining more recognition in structural analysis for its combined advantages of both Lagrangian and Eulerian formulations in a single model. Cutting force predictions using evaluated coefficients are shown to perfectly match experimental results.*

Keywords: *Cutting Force Coefficients; FEM Simulation of Machining; End Milling; High speed Milling; Mechanistic Modelling; Cutting Forces; Simulation of Cutting Process; Flat End Mill; Metal Cutting; Orthogonal Metal Cutting.*

1. Introduction

In machining, the final surface finish is greatly influenced by the undulation and the magnitude of the cutting forces and other factors such as changes in tool geometry (through tool wear), temperature generation, heat flow and so on. The knowledge of the cutting forces allows for the following: prediction of tool deflection and wear, workpiece deflection, onset and possible regeneration of chatter, research into chip formation hence built-up edge (BUE) studies and so on. The prediction of cutting force components is also essential for the jig and fixture strength requirements and determining power requirements. To this end, the cutting force models have been studied extensively after Taylor identified chatter as the most obscure problem in machining (Taylor, 1907).

Limited or no knowledge of the cutting forces expected in machining (for a given cutting condition), compels process planners on most occasions to resort to a trial and error approach or the use of available database (on recommended cutting conditions). The reliance of either of these approaches is heavily dependent on the individual's experience and/or how relevant to the available database the work is, bearing in mind further perturbations induced by vibrations. Even with the most experienced individuals, these selected trial conditions may still result in unexpected excessive cutting forces. The cutting forces are periodic and induce both dynamic and static

deflection on the workpiece – tool system, which in a feedback cycle further modulates the cutting forces. Hence, the prediction of the cutting forces in milling is highly necessary more so because understanding of the cutting forces is required in nearly all studies on machining.

There have been a number of proposed cutting force models in the literature (as explained in section 2), some of which lack a good agreement with experimental results especially when a wide range of cutting conditions is considered. This is owing to the sheer complexity and highly nonlinear nature of the milling process.

2. Background

Research into metal cutting has enjoyed a very rich history. Due to the nonlinear and complex nature of the cutting process in milling, some of the first pioneering work carried out by Merchant (Merchant, 1945) and Lee and Schaffer (Lee, 1951) idealised the principal shear zone to a single plane extending from the cutting edge to the surface of the workpiece. The shear plane was however first introduced in a model by Timme in 1870, in which the chip was said to be due to brittle fracture. Tresca later assumed it to be due to plastic deformation. Other analyses of metal cutting prior to the development of the Finite Element Method (FEM) were mainly based on these two models (Piispanen, 1948, Oxley, 1963, Fenton, 1969, Hastings, 1980). These models were however focused on orthogonal and oblique machining/metal cutting.

Different approaches or methods have since been developed to simulate milling process. The analytical approach is one of the most favoured of these methods. This is because obtaining results is relatively quick. However its usage requires an expert knowledge of the model and usually evaluation of tool dependent constants. Arguably the first in-depth analytical analysis of the milling process was carried out by Martellotti (Martellotti, 1941). This study introduced the kinematics of milling, the tooth path during milling, chip formation and the finish of the surface. Most importantly was the analysis he carried out on the thickness of the undeformed chip, as this is the mathematical representation used to date (as is in this paper) by most of the mechanistic models to calculate the undeformed chip thickness. Koenigsberger and Sabberwal later carried out an analytical study on the cutting force during milling process and developed equations for milling forces using mechanistic modelling (Koenigsberger, 1961). In their study, the cutting force at any instant was assumed to be equal to the area of the chip multiplied by a ‘specific cutting pressure’, K_s .

Thrusty and MacNeil later showed that force prediction could be made by assuming the tangential cutting force is proportional to the chip load, while the radial cutting force is proportional to the tangential cutting force (Thrusty, 1975). This was then the basis of Kline’s work (Kline, 1982), in which he combined chip thickness equations with empirical equations relating cutting forces to chip load. He also proposed the methodology of discretising the tool into segments (disk elements) along the axial as shown below in. This is still the preferred approach when analysing the forces acting on the tool. The forces on the elements are combined to give the total cutting forces on the tool in the global coordinate. Other later studies have made use of this model or its variant to predict the cutting forces in milling (Kline, 1982, Sutherland, 1986). However these studies were only for flat end tools.

Following a research to develop a CAD/CAM software for predicting the force components for different machining operations including peripheral and face milling, the cutting force acting on each flute was split into edge (ploughing) and cutting forces (Armarego, 1983, 1985). The mechanics of metal cutting was also included in the force models, in an approach called the Unified Mechanics of Cutting (Armarego et al., 1983, 1985). The approach involved modelling the tooth elements as oblique cutters and an orthogonal database had to be established for the given tool-workpiece material combination. The force models proposed in these papers so far have been for cylindrical end mills.

Perhaps one of the first studies to present a force model for ball end mill was by Koch et al. (Koch, 1990). They developed a CAE-Module to calculate the cutting forces in 3-axis milling using both a flat end mill and a ball-end mill. This report however came short of an experimental validation. Tai and Fuh (Tai, 1995) used the intersection of a spherical surface and a skew plane to define the cutting edge and identified how oblique cutting theory could be applied to model the force system in ball end milling. Feng et al. developed a force model similar to that of DeVor et al. for ball-end mills (Feng, 1993), where the coefficients (radial and tangential) were approximated by polynomials for the ball end section part of the tool. Altintas and Lee (Altintas, 1996) proposed a model, which allows for evaluation of a general helical cutting tool geometry and prediction of the chip thickness. This geometric model was improved and was well defined for different types of end mill shapes by Altintas and Engin (Altintas, 2001).

Contrary to analytical approach is the FEM approach to simulate the milling process. Orthogonal and oblique cutting processes have been extensively analysed using FEM based techniques. FEM simulations are based on three main formulations:

The first of these is the **Lagrangian (Total or Updated) Formulation**, which was the basis of Klamecki's study (Klamecki, 1973). He was the first to introduce FEM technique into machining, using a three-dimensional elastic-plastic finite element method. Shirakashi and Usui (Shirakashi, 1974) later applied the elastic-plastic finite element method to orthogonal metal cutting process. K. Iwata (Iwata, 1984) also used a rigid-plastic finite element method to consider the effect of friction between the tool rake and face. The first analysis to simulate the movement of the tool into the workpiece and continuous chip formation along a predefined "parting line" was by Strenkowski and Carroll (Strenkowski, 1985). They used a finite element program (NIKE2D) adopting the Updated-Lagrangian formulation (ULF) and they also proposed a separation criterion to simulate chip formation. Shih J et al. (Shih, 1990) carried out a study on the effects elasticity, viscoplasticity, temperature, strain-rate and large strain have on the stress-strain relationship and the effects large friction has on the tool-chip interface. Other studies based on a Lagrangian formulation were reported in (Shih, 1996, Shet, 2000, Mamalis 2001, Baker 2002, Carrino, 2003, Rosa, 2007).

Opposite to the Lagrangian formulation is the **Eulerian Formulation**, which is more suitable for fluid flow problems involving a control volume. However its first application to metal cutting was reported by Usui et al. (Usui, 1978) and Lajczok (Lajczok, 1980). In the study by Lajczok, the tool forces and geometry obtained experimentally were applied to the workpiece surface, thereby omitting the chip in the model. Strenkowski and Moon (Strenkowski, 1990) analyzed a steady-state orthogonal cutting with the capability to predict chip geometry and chip-tool contact length. Moriwaki et al. (Moriwaki, 1993) developed a rigid-plastic finite element model to examine the

effects of the tool edge radius to the depth of cut in micro cutting process. Other studies carried out with the use of pure Eulerian formulation are reported in (Strenkowski, 2002, Athavale, 1997,).

The third formulation is the **Arbitrary Lagrangian-Eulerian ALE formulations**. The ALE formulation was introduced to finite element method by Donea et al. (Donea, 1977) and has been used extensively mainly to model processes involving large deformations (e.g. metal forming, metal cutting and metal forging). Some studies using ALE to model metal cutting are reported in (Ozel, 2005, Ozel, 2007). The ALE formulation combines advantages of both the Lagrangian and Eulerian formulations, rendering it a more versatile formulation. As it is known, the main advantages of Lagrangian are the main disadvantages of the Eulerian and vice versa.

There are only a few reports of the use of FEM to model end milling and even fewer adopting the ALE formulation. The majority of these studies have focused on orthogonal metal cutting. Analysis and predictions of end milling cutting forces have been solely based on the use of the mechanistic approach. The mechanistic approach has been proven to produce very accurate results, however these results are as accurate as the force coefficients used. Obtaining accurate coefficients is encumbered with a large amount of experiments. Therefore, this paper proposes an approach that is used to extract these coefficients from cutting forces results obtained through FEM simulations. The accuracy is shown by comparing predicted cutting forces with experimental results.

3. Mechanistic Model

The mechanistic model is the model favoured by companies and researchers alike when it comes to predicting the cutting forces in end-milling. Even though its implementation requires programming or the use of a commercial package, its results can be very accurate as have been reported by numerous of studies (Gradisek, 2004, Altintas, 2001).

Based on Kline's work (Kline, 1982) the tool is discretized into segments and the differential tangential (dF_t), radial (dF_r) and axial (dF_a) cutting forces acting on the infinitesimal cutting edge segment are given as:

$$dF_{t,j}(\phi_j) = K_{te}dS + K_{tc}h(\phi_j)db \quad (1a)$$

$$dF_{r,j}(\phi_j) = K_{re}dS + K_{rc}h(\phi_j)db \quad (1b)$$

$$dF_{a,j}(\phi_j) = K_{ae}dS + K_{ac}h(\phi_j)db \quad (1c)$$

The coefficients $K_{\bullet e}$ are cutting force coefficients due to the sear and $K_{\bullet c}$ are edge force coefficients due to the ploughing force of the tool edge and flank on the workpiece. Care should be taken when using this equation if the tool is not a flat end mill tool. The uncut chip thickness was defined by Martellotti (1938) in his study approximately as,

$$h = s_t \sin(\phi_j) \quad (2)$$

If there is helix angle, run-out or the tool is not a flat end tool, then the chip load at the different segments are not the same. In order to simplify the equations, a non helical flat end milling tool with no runout was used in this paper. To analyse all general tools, the equations will require special treatment. While this is not discussed in detail in this paper, the readers are referred to the work by Gradisek et al. (Gradisek, 2004). The cutting forces in the cylindrical coordinates are transformed into global Cartesian coordinates. Therefore the average milling forces for one tooth period, is defined as,

$$\bar{F}_q = \frac{1}{\phi_p} \int_{\phi_{st}}^{\phi_{ex}} \int_{z_1}^{z_2} dF_q(\phi, z) d\phi \quad (3)$$

where, $\phi_p = 2\pi/N$, $q = x, y, z$

From the paper by (Altintas, 2000) the average forces per tooth period are:

$$\bar{F}_x = \frac{Nas_t}{8\pi} [K_{tc} \cos 2\phi - K_{rc} (2\phi - \sin 2\phi)] - \frac{Na}{8\pi} [K_{te} \sin \phi - K_{re} \cos \phi] \Big|_{\phi_{st}}^{\phi_{ex}} \quad (4a)$$

$$\bar{F}_y = \frac{Nas_t}{8\pi} [K_{tc} (2\phi - \sin 2\phi) + K_{rc} \cos 2\phi] - \frac{Na}{2\pi} [K_{te} \cos \phi + K_{re} \sin \phi] \Big|_{\phi_{st}}^{\phi_{ex}} \quad (4b)$$

$$\bar{F}_z = \frac{Na}{2\pi} [-K_{ac}s_t \cos \phi + K_{ae}\phi] \Big|_{\phi_{st}}^{\phi_{ex}} \quad (4c)$$

The classical approach to obtaining the six coefficients is by conducting various experiments at different feedrates s_t , and a constant radial and axial depth of cut. Therefore by assuming a linear relationship between the cutting forces and the feed per tooth, the cutting forces are,

$$\bar{F}_q = \bar{F}_{qc} \cdot s_t + \bar{F}_{qe} \quad (5)$$

where $(q = x, y, z)$

Equations 4 (a, b and c) and 5 are equated to obtain the cutting coefficients (reference should be made to the paper by Altintas (2000) for an in-depth review of this approach. The entry angles (Figures1) of the tool for downmilling are determined using,

$$\phi_{entry} = \pi - \cos^{-1}\left(1 - \frac{w}{R}\right) \quad (6)$$

where $a.d$ is the radial depth and $r.d$ is the radius of the tool.

The exit angle for down milling is π according to the convention used in Figure1. Using the boundary conditions (entry and exit angles) used in this paper, the cutting coefficients are defined (ignoring coefficients for z-axis) as,

$$K_{tc} = \frac{1}{Na} (7.4674 \cdot F_{yc} + 7.9133 \cdot F_{xc}) \quad (7a)$$

$$K_{rc} = \frac{1}{Na} (7.9133 \cdot F_{yc} - 7.4674 \cdot F_{xc}) \quad (7b)$$

$$K_{te} = -\frac{1}{Na} (4.7989 \cdot F_{xe} + 3.1416 \cdot F_{ye}) \quad (7c)$$

$$K_{re} = \frac{1}{Na} (3.1416 \cdot F_{xe} - 4.7989 \cdot F_{ye}) \quad (7d)$$

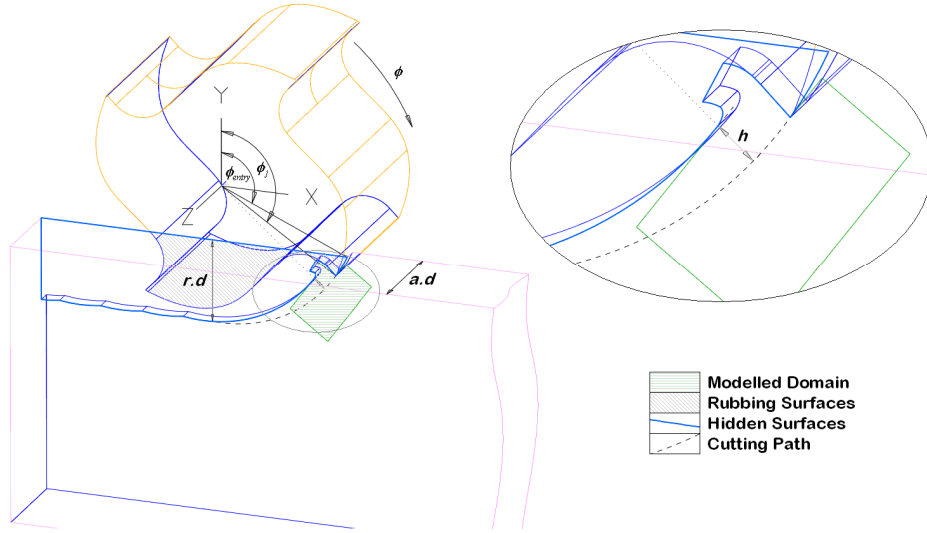


Figure 1 - Undeformed chip thickness, radial and axial depth, angle of rotation

4. Model Formulation

4.1 Proposed Simulation Approach

In this paper, a new approach (other than the use of calibrations test) is proposed in order to obtain the cutting coefficients required for the mechanistic model. The actual milling process was simulated using Finite Element Analysis (FEA). The simulation stage involved modelling the cutting process at different angles of rotation to obtain the instantaneous cutting forces at these respective angles. The simulation is carried out for the maximum feed and from the results, the cutting forces for lower feeds can be predicted easily.

Therefore using Martellotti's (Martellotti, 1938) approximation the undeformed chip thickness for a specific angle of rotation is calculated. The FEA simulation is conducted (simplify the domain to a 2-dimesnional domain, Figure1) and the forces obtained corresponds to the instantaneous radial and tangential cutting. Following transformations from cylindrical coordinates to Cartesian coordinates, the global cutting forces are obtained.

To calculate the cutting coefficients the cutting forces at different feeds are required as this is used to obtain the relationship between the feed and cutting forces (Equation 5). Therefore assuming the FEM simulation was conducted for a feed of $37.5\text{ }\mu\text{m}$, if the radial depth of cut is the tools radius, then the undeformed chip thickness at the entry angle would be $37.5\text{ }\mu\text{m}$. The forces at this entry angle are obtained using FEM simulations as explained. The tool is then rotated through various angles and cutting forces obtained until it is past its exit angle. This will define the full cutting force for the feed of $37.5\text{ }\mu\text{m}$. To use the same results to obtain the cutting forces for a smaller feed of $20.0\text{ }\mu\text{m}$, the undeformed chip thickness corresponding to $20.0\text{ }\mu\text{m}$ (which is $20.0\text{ }\mu\text{m}$ for a radial depth of the tool radius) is calculated. Then the cutting forces for this thickness/angle and all subsequent angles can be obtained from results for equivalent undeformed chip thickness for feed of $37.5\text{ }\mu\text{m}$ (explained in Figure2).

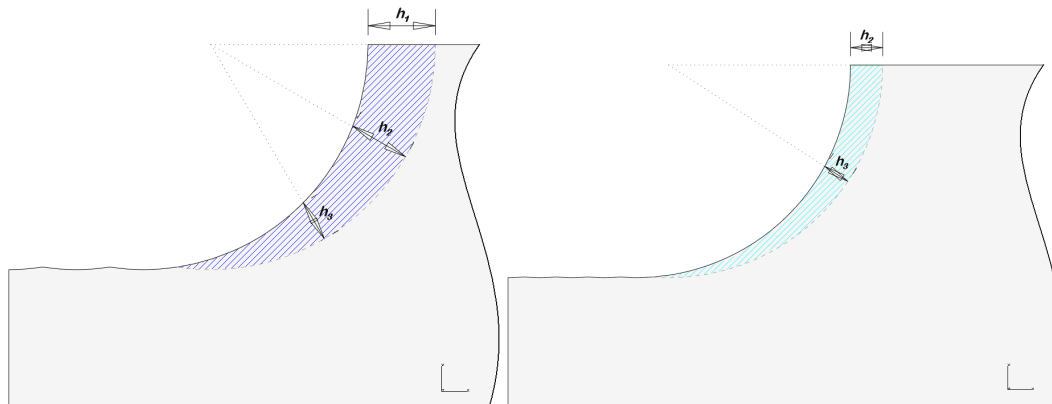


Figure 2. Interpolating lower feed data from higher feed

In order for this approach to be easily and accurately implemented, enough simulations are carried out in order to obtain a linear or polynomial relationship between the tangential and radial cutting forces and the undeformed chip thickness. In this paper, the least squares method is used to fit the data to a 6 order polynomial model. This is used to predict the corresponding forces for any undeformed chip thickness hence any angle or rotation. Care should be taken however, to ensure the initial simulation covers the maximum and the smallest thickness as much as possible in order to make the polynomial approximation as correct as possible.

4.2 Modelling Assumptions

The actual tool modelled has a helix angle of 30 degrees, however due to such a small axial depth of cut of 0.5mm it was postulated that this angle is negligible and taken as zero. In order to model a tool with helix a three-dimensional domain would be required. This allowed for the forces in z-direction be taken as zero. If the helix was included in the model then the simulation would have to be carried out as a full 3-dimensional simulation.

When simulating the end milling cutting process as a two-dimensional domain, the friction experienced on the face rubbing against the base of the machined surface (this surface is highlighted in Figure1) is not included in the simulations. This can also contribute to some force in the axial direction. To include this in the simulation would also require using a three-dimensional domain and calculating the friction using a contact condition on the surfaces. To capture a considerable amount of frictional effect (due to its relative magnitude), a good mesh density would be required in this region.

During machining as explained earlier the tool and workpiece experience different vibrations, which can grow significantly in what is called regenerative chatter. This in turn affects the undeformed chip thickness. In the simulations, it was assumed that the whole cutting process was stable and the vibrations were negligible. This criterion is also required when conducting experiments to calibrate the tool using the mechanistic approach.

5. Results and Discussions

5.1 FEM Cutting Force Results

The simulations were conducted successfully and an example of the steady stress state of the workpiece is shown in Figure3. The primary and secondary shear zones are seen with the highest stress area and the residual stress is seen trailing further along the machined surface. From these simulations the cutting forces in directions 1 and 2 (refer to Figure3) in the local coordinates which correspond to the tangential and radial cylindrical coordinates respectively are collated and shown in table 1. The cutting forces in the cylindrical coordinates are transformed to the global coordinates and a plot of these are shown below in (Figure4 (a)). The geometry and cutting conditions of the study by Ko et al. (2002) were used for the simulations and the result by Ko et al. (Ko, 2002) is shown in Figure4 (b).

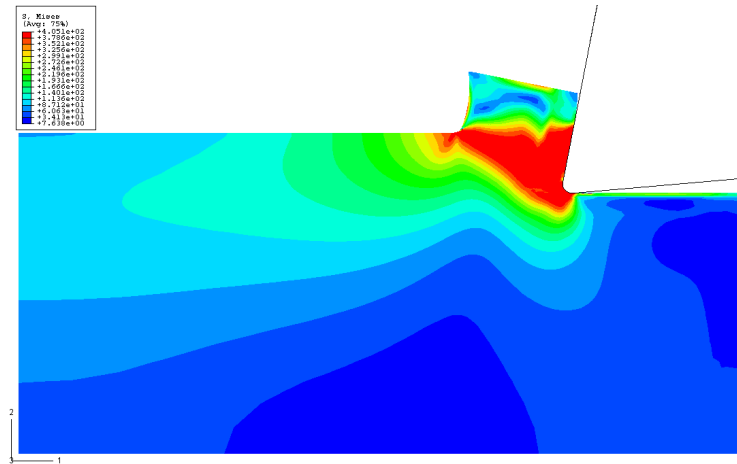
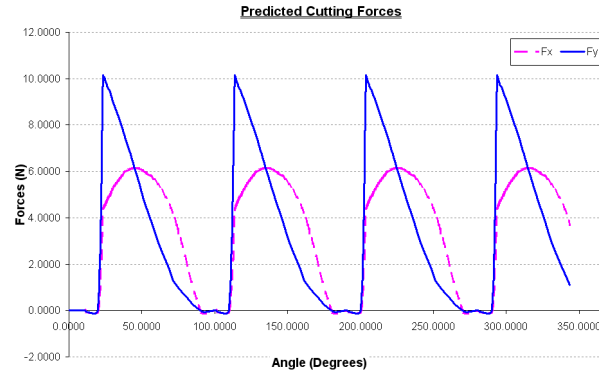


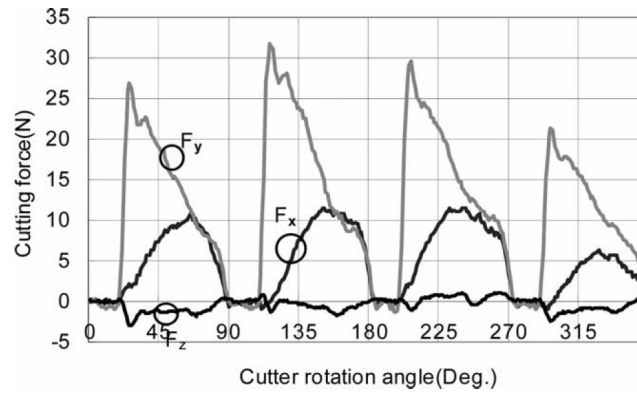
Figure 3. von Mises stress distribution for the milling simulation.

Table 1. FEA and calculated results for cutting forces.

$\Delta\phi$	ϕ (rads)	h (μm)	Tangential Force, F_t (N)	Radial Force, F_r (N)	F_x (N)	F_y (N)
0	5.12	34.369	11.047	0.719	4.419	10.125
1	5.14	34.102	10.960	0.721	4.559	9.967
2	5.16	33.825	10.873	0.724	4.694	9.807
3	5.18	33.537	10.784	0.729	4.825	9.644
4	5.19	33.239	10.694	0.735	4.951	9.479
5	5.21	32.931	10.604	0.742	5.072	9.312
7.5	5.25	32.117	10.369	0.764	5.353	8.881
10	5.30	31.242	10.121	0.790	5.598	8.432
12.5	5.34	30.308	9.855	0.818	5.804	7.965
15	5.39	29.316	9.568	0.846	5.966	7.480
17.5	5.43	28.268	9.256	0.874	6.082	6.977
20	5.47	27.166	8.920	0.901	6.149	6.462
25	5.56	24.810	8.187	0.957	6.139	5.417
30	5.65	22.265	7.403	1.027	5.957	4.395
35	5.73	19.550	6.604	1.114	5.635	3.443
40	5.82	16.687	5.787	1.206	5.182	2.575
45	5.91	13.696	4.892	1.308	4.554	1.787
50	6.00	10.602	3.843	1.536	3.686	1.086



(a)



(b)

Figure 4. (a) Predicted cutting forces (b) Experimental cutting Forces (Ko, 2002)

5.2 Obtaining Coefficients

The force predicted in the simulations is seen to be very low compared to the experimental results however when it comes to extracting coefficients, the magnitude of the cutting force predicted is not not important. The most important factor is if the relationship between the forces (in cylindrical coordinates) and the feed is captured. Figure5 shows the radial and tangential forces plotted against the undeformed chip thickness. This is therefore used to calculate the cutting force for different smaller feeds as explained in section 4. Following the classical approach used for the mechanistic model, the relationship between the forces and the feed is obtained by taking an average of the cutting forces over the tooth period according to Equation 3. Figure6 shows the cutting forces in Cartesian coordinates against the feed. This relationship is different from the one in Figure5 as they are in different coordinates system. The force coefficients evaluated are:

$$K_{te} = 1.35, \quad K_{tc} = 1729.101, \quad K_{re} = 1.288, \quad K_{rc} = 746.332$$

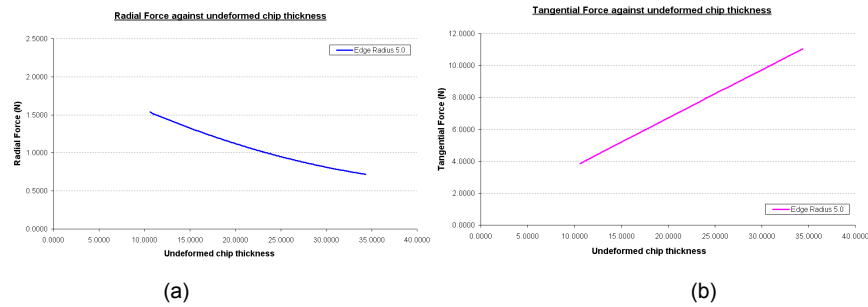


Figure 5. (a) Radial and (b) Tangential forces against undeformed chip thickness

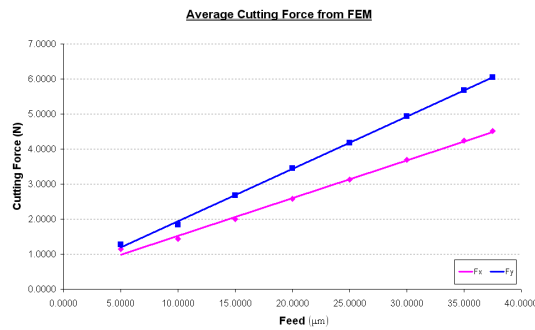


Figure 6. Average Cutting Force from FEM

5.3 Validation

To validate the coefficients, a commercial package (CutPro) is used. CutPro is an analytical and time-domain machining process simulation package. It has an in built module that accepts known coefficients and obtains the cutting forces using the mechanistic model. The alternative to using a commercial package would be to program the equations and the discretization of the tool using for example FORTRAN.

Quick analytical simulations were carried out using the cutting conditions similar to ones in the reported by Ko et al. (Ko, 2000) and the results are shown in Figure 7. It can be seen that the obtained cutting coefficients produce very good agreement with experimental results. To further explore the validity of the coefficients, several cutting conditions reported in the paper by Ko et al. (Ko, 2000) were simulated and the results are compared with experimental results in Figures (8, 9, 10). This was assuming the coefficients were not dependent on the axial depth of cut, which should be the case for a flat end milling tool with a small constant helix angle. These predictions show a very good agreement, which further validates the cutting coefficients obtained using FEA simulations of the end milling process. There are slight over estimations on the F_y 's however it is believed that this is due to the helix angle not being equal to zero on the actual tool as assumed on FEM models.

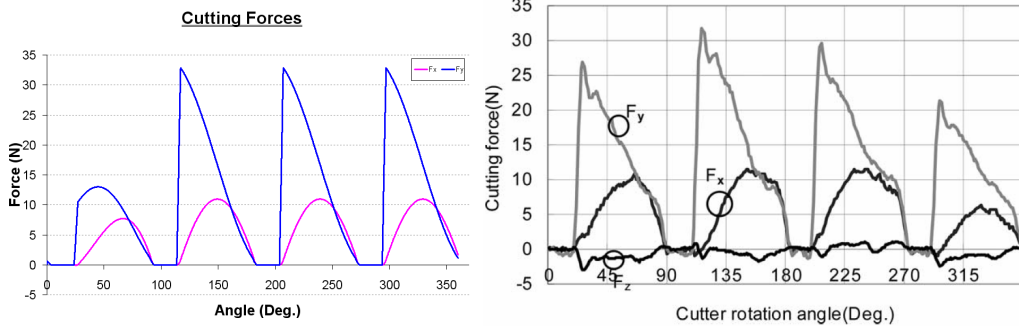


Figure 7 – (a) Predicted and (b) experimental cutting forces for feed 0.0375, r.d = 3.0mm, a.d = 0.5mm, 1000rpm (Ko, 2002)

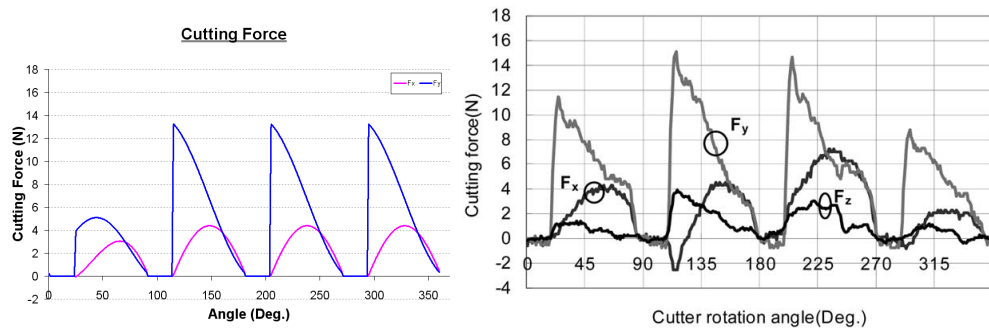


Figure 8 – (a) Predicted and (b) experimental cutting forces for feed 0.0375, r.d = 3.0mm, a.d = 0.2mm, 1000rpm (Ko, 2002)

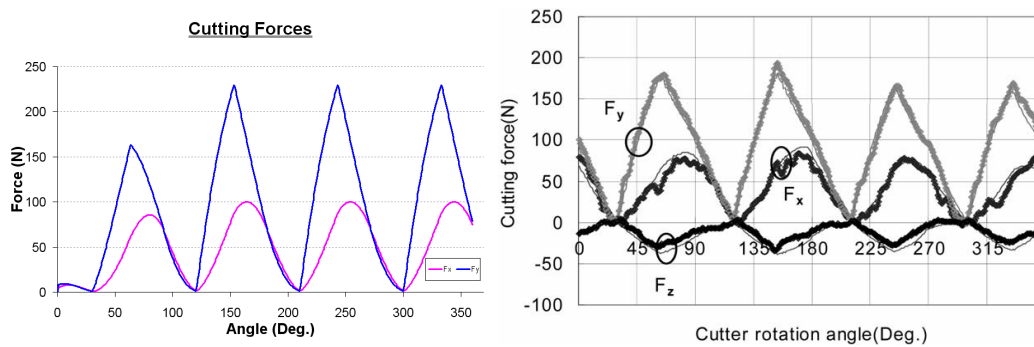


Figure 9 – (a) Predicted and (b) experimental cutting forces for feed 0.0375, r.d = 2.5mm, a.d = 5.0mm, 2000rpm (Ko, 2002)

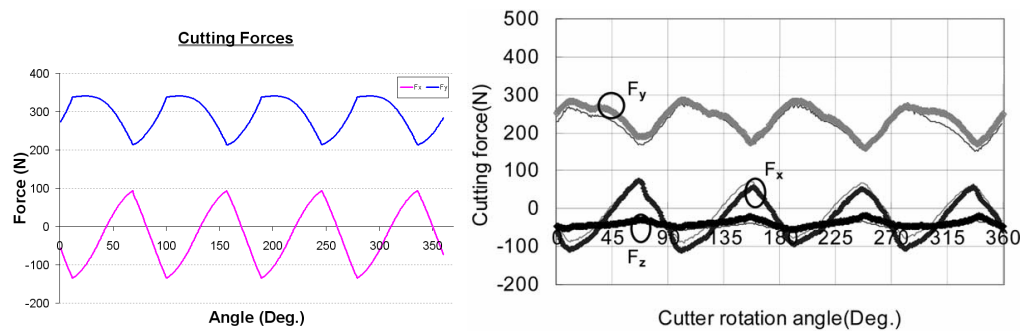


Figure 10 – (a) Predicted and (b) experimental cutting forces for feed 0.0375, $r.d = 7.0\text{mm}$, $a.d = 5.0\text{mm}$, 1000rpm (Ko, 2002)

6. Conclusions

In this paper, the results obtained from FEM simulations of the end milling process was used to evaluate the force coefficients. A new quick and efficient approach to evaluating the force coefficients using FEM results was proposed. This approach is easily implemented and takes the burden and cost off experimental calibrations. The coefficients obtained were validated by comparing predicted cutting forces with experimental results taken from study by Ko et al. (Ko, 2002). Several comparisons were made to check on reliance over a wide range of cutting conditions. A very good correlation can be seen in the comparisons.

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