Fatigue reliability analysis using Latin hypercube sampling method

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Abstract: Fatigue is considered to be one of the main causes of the degradation of mechanical properties in mechanical parts. Probability and reliability methods are appropriate for fatigue life prediction using the uncertainties which exist in fatigue material or process parameters. In this context, evaluating the risk on the fatigue simulation outputs according to these parameters represents the most important challenge. To run reference simulation model, mean values of input parameters are used. So after, the Monte Carlo Latin Hypercube Sampling MC-LHS algorithm will be applied successively for a pre-selected combination of sampling parameters. This allows us to predict by reliabilities methods a safe domain within a fatigue life response surface plotted in the space of tow random variables. The automation simulation is conducted by a main Fortran routine that couple sample input parameters and Abaqus/Standard. The database of pre-selected outputs is built for each Abaqus simulation. A failed simulation will be detected directly thereafter following combination random values will be used instead. This allows quantifying efficiently parameters sensitivities, failure probability according to a limit state function. This work focuses on the reliability simulation of upper arm of a vehicle suspension system. Two main classes of parameters cause the major part of the fatigue damage induced in vehicles. The first is related to the materials properties while the second concerns the road excitation or the applied force of passenger numbers and vehicle weight.

1. Introduction

The estimated fatigue life of mechanical structures uses different fatigue models. These models are based on numerical and experimental methods. The number of parameters involved in the estimation of fatigue life is increasing continuously and it partakes of the degradation of the quality of the results. The error and inaccuracy associated with these results are frequently upgraded for an improved fatigue life in a given mechanical structure. This enhancement is reflected in practice by oversized structures and the choice of material. While it is impossible to predict the fatigue life of structures with any exactitude, the error itself, however, can be minimized in such a way as to contribute to significant financial savings and a safety parts.

Based on these considerations, it becomes necessary to identify the various aspects and parameters describing the fatigue behavior of mechanical structures. This identification consists of a set of theoretical, numerical, and experimental analyses of fatigue problems. In fact, the lack of precision displayed by some parameters such as material behavior laws, hardening coefficients, or physical environments tends to increase the error on the calculated duration of fatigue life. Currently, however, there are several methodologies establishing tight tolerances on the values of the parameters involved in fatigue structure behavior.

2. Reliability analysis

Current used reliabilities techniques allow to calculate the failure probability (safe probability) of pieces or systems in various manners. Especially, this study aims to specify the effect of uncertainties of some parameters on the fatigue life of an upper arm of the vehicle suspension system. Several damage scenarios have been considered by solving finite element model for each combination of some chosen parameters. A Python routine allows getting the maximum strain density energy SENER among all elements of the structure. Once this SENER was ordered in a database according to the whole time step, it's possible to extract Rainflow cycles and determine the fatigue life *Nf*. According to *Nf* and input random variables, the reliability analysis can be made by establishing a surface response, evaluating failure probability and computing sensitivity (Figure 1).

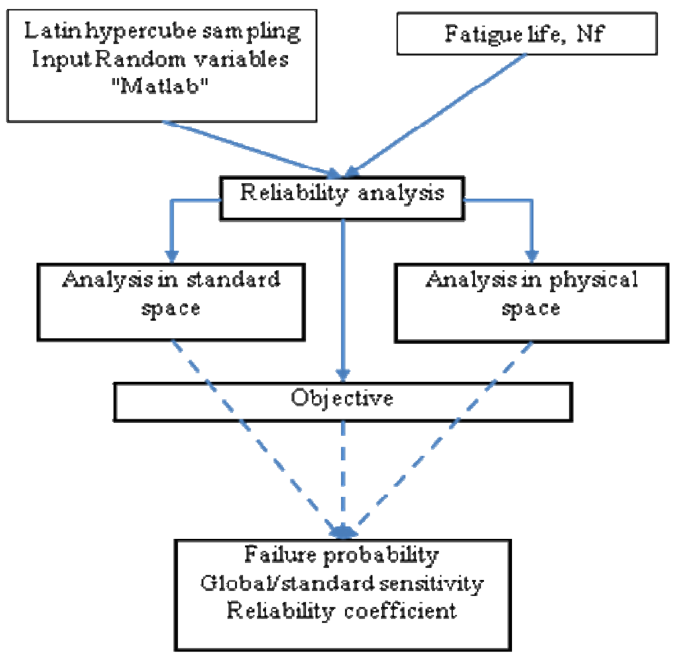

Figure 1. Flowchart describing the main reliability analysis approach.

2.1 Parameter sampling

Computer models based on experience plan are now used to investigate physical phenomena. Many of these models are expensive to run. Thus it's important to make a study on the most efficient models used in dynamics applications. The encountered problems related to a bad representation of random parameters such as in standard Monte Carlo (MC) [5,6] are minimized by using MC-LHS [2,3,4]. The goal is to avoid the non-performance plans and the non-space filling, it will be possible to fill up space and have no particular phenomena alignments in projections. Moreover, the quality in terms of space filling is strongly linked to the choice of initial distribution. This allows efficient evaluation of systems reliability by establishing a homogeneous surface response and correct limit state function according to practical's and theoretical observations.

2.2 Limit state function

Limit state function can be evaluated only by calculations, in particular with finite element reliability methods (*FERM*). Classical reliability techniques remain requires costs computers resources and high simulations number for the study of a failure mode. These techniques become non useful for a complex structural systems. In order to reduce the evaluations number of the limit state function into a reasonable level, an approach consists to establishing "artificially" the limit state function by using practical's and theoretical observations.

Performance function *G* is chosen according to some theoretical observations on acceptable fatigue life. In the present case, possible failure modes must be avoided. The parameters involved in numerical simulation model are based on a regular use of the vehicle (road excitation, passenger's number ...). This lead to a secure choice of the cycle's number *Nf* applied to the upper arm.

2.3 Sensitivity analysis

The sensitivity analysis examines how random variability of the model inputs generates perturbations on the response [1]. Otherwise the sensitivity analysis aims to classify the system parameters according to their importance and effect on the system reliability. Otherwise if the variability of random parameter has great effect on the reliability, the parameter must be rigorously controlled during engineering design and manufacture. While if the variability of random parameter has a negligible effect on the reliability, it can be assumed as a deterministic value in order to reduce the complexity of the general reliability analysis.

Consider a mathematical model in which a set of input random variables forming a deterministic function, and a set of output random variables. This model can be written as follows:

$$\begin{aligned} f &: \Re^{P} \rightarrow \Re \\ X &\rightarrow Y = f(X) \end{aligned} \quad \text{(Eq.1)}$$

Numerical simulations are used to evaluate this function when *f* is complex and difficult to be represented by analytical model. The set of input variables $X = (X_1,...,X_p)$ is formed by all independent model quantities that are considered as random. All output variables is supposed to be reduced to a single variable *Y*. Subsequently *Y* will signify the fatigue life *Nf*.

The local sensitivity analysis is used in this work. It's a method of quantitative analysis establishing hierarchy within the input variables according to their influence on the variability of the response. Moreover this method give an order of magnitude differences in this hierarchy. The method of local analysis is the most typical approach used to estimate the sensitivity indices defined by:

$$S_i = \frac{\partial y}{\partial x_i}\left(x_1^0,..., x_p^0\right) \quad \text{(Eq.2)}$$

3. Numerical formulation

The fatigue model developed was applied to an upper suspension arm of a vehicle, as shown in Figure 2. The road excitation cycles are described by applying power spectral density (PSD) [11]. This type of excitation is due to the irregularity of the floor profile considered as a random function. Figure 3 shows the road profile which depend on the displacement versus time for three types of road. The excitation spectrums of the road through the tire, the spring reaction, and the shock absorber are calculated based on the enslavement relationship between the suspended and unsuspended mass.

a- b-

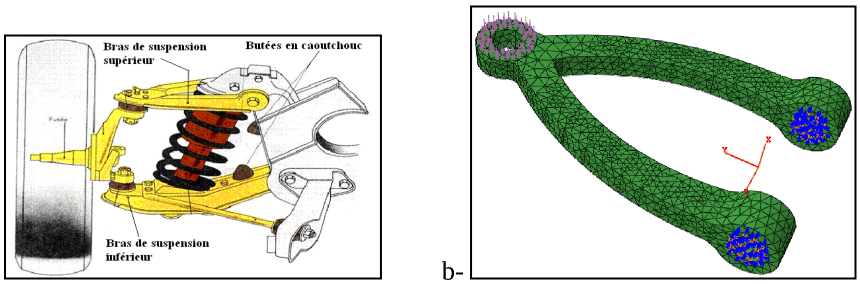

Figure 2. Suspension system of a vehicle showing the upper arm used in the present investigation; (a) real use of upper arm, and (b) finite element model.

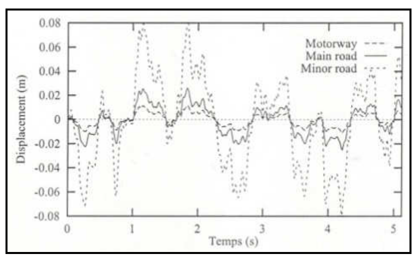

Figure 3. Road profile in time domain for three road type.

For the purposes of this study, MC-LHS in conjunction with the dynamical model of the upper suspension arm of a vehicle were used. The empirical fatigue model used was based on the multiaxial strain energy density criterion which is equivalent to the uniaxial model [9].

3.1 Model development, background

The complexity of the problem of fatigue has led many researchers to approach the subject from different viewpoints using several methods. Bazergui et al. [7] based their work on the S-N fatigue curves S and two empirical models of fatigue to prevent rupture. They report rotational bending as a cheaper standardized test. To improve the ability of predicting the fatigue life of a part, they practised on standard test-tubes before installing the operational part on the apparatus. In practice, stress variation is often periodic, but it is not always sinusoidal and the average value of the stress is not null because of the static contribution from the weight of the part and premature tightening. Cervello [8] analyzed and studied the design of railway wheels which produced weak noise.

A numerical procedure was used for the calculation of the loss factor. This procedure checked on plates by means of experimental modal analyses. Elmarakbi [9] studied the validity of the multiaxial criterion of fatigue failure based on the density of the energy equivalent to the uniaxial case; this researcher succeeded in equalized the energy density of the multiaxial case obtained from finite element commercial software. The uniaxial case was calculated by exact integral, since it is two of the same order of tensors: that is to say, zero order. Mars [10] based his study on the same criterion as Elmarakbi, namely, the density of the energy of cracking. March performed fatigue and tension-torsion tests and compared the results with the energy density criterion. The model aimed to predict the fatigue life of rubber particles using two approaches. One focused on crack initiation, providing the history of some parameters such as the stress state and deformation.

3.2 Fatigue model formulation

Development of Road Profile Model: The excitation caused by the road profile is calculated analytically from the PSD acting on the upper arm suspension of a vehicle through a tire of stiffness with a constant, *Kt*, where the return chain is made up of a linear shock absorber and a linear spring. The motion equation of mass mu is :

$$m_u \ddot{x}_1 + k_t(x_2 - x_1) = 0 \quad \text{(Eq.3)}$$

where x_2 is calculated from the PSD formulation. This type of second order ordinary differential equation (ODE) can be transformed into a system of two first order ODEs which can be solved numerically using the fourth order Runge-Kutta method.

Fatigue model: The uniaxial criterion equivalent to the multiaxial case is used as suggested by Elmarakbi *et al*. Since the aluminium alloys present a plastic rather than an elastic behavior, the nonlinear profile is provided by the Ramberg-Osgood equation as

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{1/n'} \quad \text{(Eq.4)}$$

where $K' = \frac{\sigma'_f}{\varepsilon_f'^{n'}}$ is the coefficient of the endurance limit, $n' = \frac{b}{c}$ is the cyclic hardness exponent, σ'_f is the coefficient of the fatigue strain, c is the exponent of the fatigue ductility, *b* is the fatigue strain exponent, and ε'_f is the coefficient of fatigue ductility. In the present study, a three-dimensional elastic finite element analysis of the model was used. The value of the strain energy density obtained from the analysis by means of the three-dimensional finite element method (FEM) is equalized to the strain density energy of the uniaxial case which constitutes the plastic and elastic deformation energy. The total uniaxial strain energy per unit of volume obtained by an exact integration is represented as

$$U_a = U_{ae} + U_{ap} = \int_0^{\varepsilon_a} \sigma_a \, d\varepsilon_a = \frac{\sigma_a^2}{2E} + \frac{\sigma_a}{n' + 1}\left(\frac{\sigma_a}{K'}\right)^{\frac{1}{n'}} \quad \text{(Eq.5)}$$

The resolution of Equation 5 provides the necessary constraint for producing the same energy density as in the uniaxial case. Substituting the stress value in Equation 4 provides the corresponding deformation. Based on knowing the deformation, fatigue life can thus be predicted by using the Manson-Coffin relation as in Equation 6, as follows,

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f^{'}}{E}(2N)^b + \varepsilon_f^{'}(2N)^c \quad \text{(Eq.6)}$$

3.3 Latin hypercube sampling

Two independent random variables were chosen for the reliability study, these include the applied load (due to the structural weight, the effects of baggages and passengers), and Young's modulus of the upper arm. In fact, these two variables directly affect the life of several vehicle pieces. It is, thus, necessary to characterize these mechanical pieces using quality limitations in order to produce a confidence interval and to secure the vehicle. The initial analysis is done by using mean values of these tow random parameters. Latin hypercube sampling (LHS) method is adopted for produce random variables samples as shown in Figure 4. The LHS is very efficient to obtain a homogeneous filling space witch minimize the error to calculate the coordinates of the design point (Hasofer and Lind, β_{HL}) defined as the shortest distance from the origin of the reduced variable space (standard space) to the limit state function $G=0$.

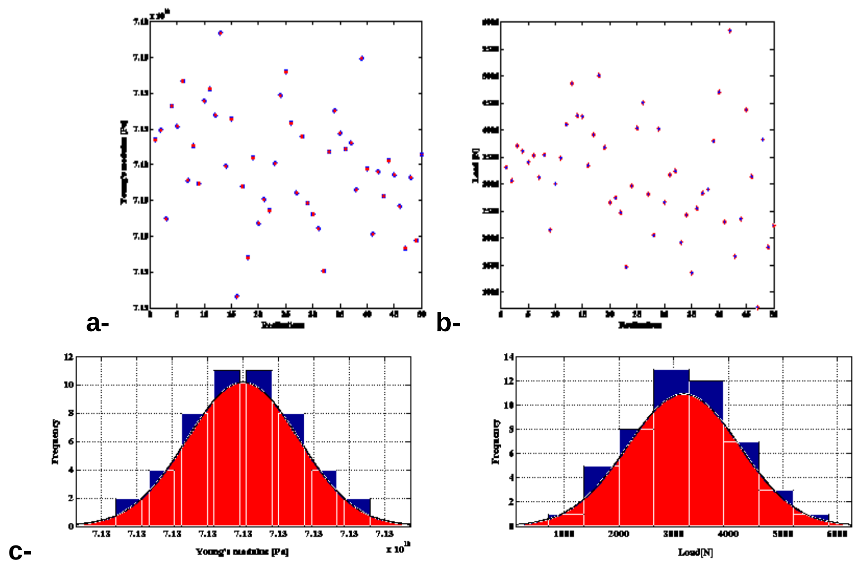

Figure 4. Latin hypercube sampling, a - Young's modulus, b - Load, c- Frequency of input variation variables.

4. Implementation

This section addresses the computer implementation of fatigue life estimation of the upper arm in a vehicle suspension system made of an aluminum alloy. The main idea is to make a scan of the combination values random parameters so as to calculate the corresponding fatigue life. An intuitive approach indicates using the Latin Hypercube Sampling coupled with Monte Carlo method for making deterministic simulations by Abaqus/standard.

The algorithm for the implementation of this approach in a computer program can be summarized as follows:

- Initial properties and configuration :

1. Specify deterministic properties and finite element model of the upper arm;

2. Specify initial statistical distribution and generate random variables (Young's modulus *include material file and Load *include loading file) by Matlab and Fortran routine.

- Fatigue life prediction and reliability analysis:

3- calculate the strain energy density "SENER" in the mechanical part of the suspension system of a vehicle subjected to random excitation described by the power spectral density (PSD) by Abaqus/standard;

4- filter the results and identifying the critical elements by Python routine (RPT File);

5- evaluate the fatigue life of the upper arm (by Matlab);

6- describe the fatigue life of the part in a confidence interval (response surface), by Excel;

7- calculate sensitivities parameters and failure probability by Excel.

Figure 5 shows the general structure of MC-LHS algorithm and how it is implemented by computer.

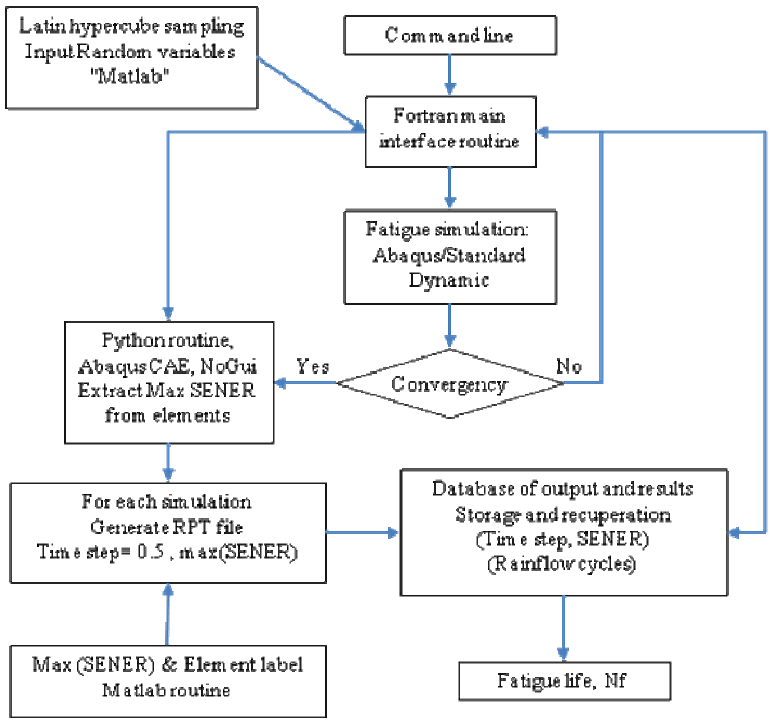

Figure 5. Flowchart describing main structure of computer implementation of MC-LHS method.

5. Results and discussion

The strain energy density (SENER) is extracted from each Abaqus deterministic finite element simulation for storage in a database using Python and FORTRAN routines. Figure 6 show strain energy according to a deterministic simulation using random parameter mean value. Subsequently, six Rainflow cycles may be identified. Once this identification has been made, it will be easy to extract all $N_{MC\text{-}LHS}$ x 6 Rainflow cycles and store them in a database. Moreover, Rainflow time for each of MC realizations does not change. Strain energy related to each Rainflow cycle is extracted by Markov algorithm.

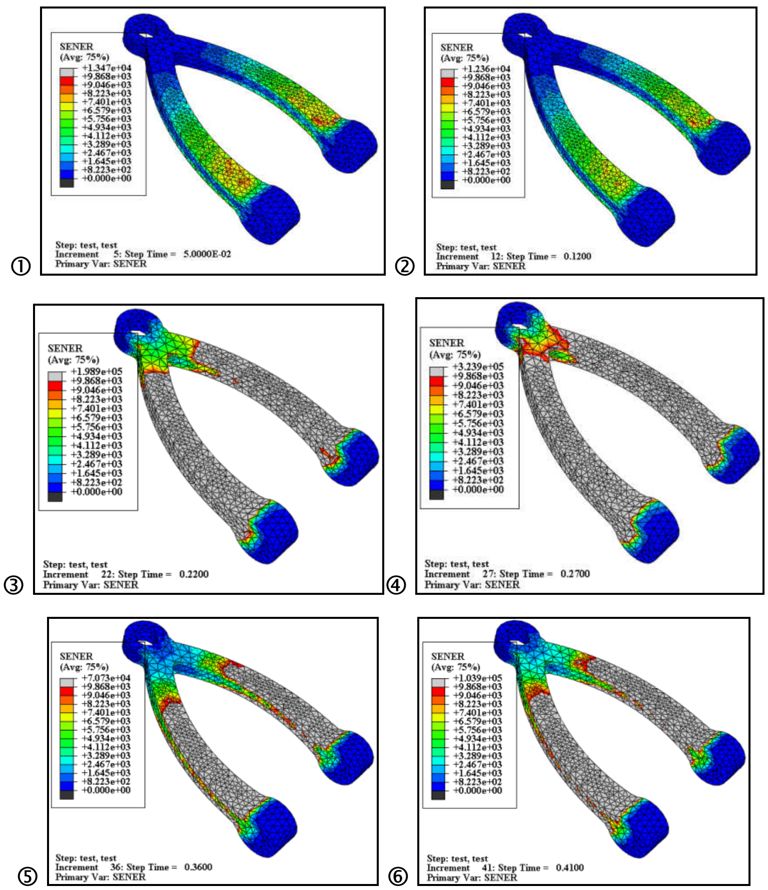

Figure 6. Strain energy profile associated to each of the six Rainflow cycle.

Limit state function G is chosen according to some theoretical observations on acceptable fatigue life. In the present case, possible failure modes must be avoided. The parameters involved in numerical simulation model are based on a regular use of the vehicle (road excitation, passenger's number ...). This lead to a secure choice of the cycle's number N_f applied to the upper arm. The performance function can then be written as:

$$G(young, load) = \begin{cases} 1, \;\; for \;\; N_f \geq \left(\max(N_f) - \sigma_{N_f}\right), & success \\ 0, \;\; for \;\; N_f = \left(\max(N_f) - \sigma_{N_f}\right), & failure \\ -1, \;\; for \;\; N_f \langle \left(\max(N_f) - \sigma_{N_f}\right), & failure \end{cases}$$

Figure 7 show fatigue life frequency and response surface constructed according to the limit state function in physical and standard spaces. The probability of failure P_f is calculated by: $P_f = P[G(young, load) \leq 0] = 0.084$, thus the reliability probability P_r is equal to P_r=1-P_f =0.896

(Probability of undesired simulations = 0.02). Note that an increase in the safety or the success domain implies a decrease of the load and increase of Young's modulus.

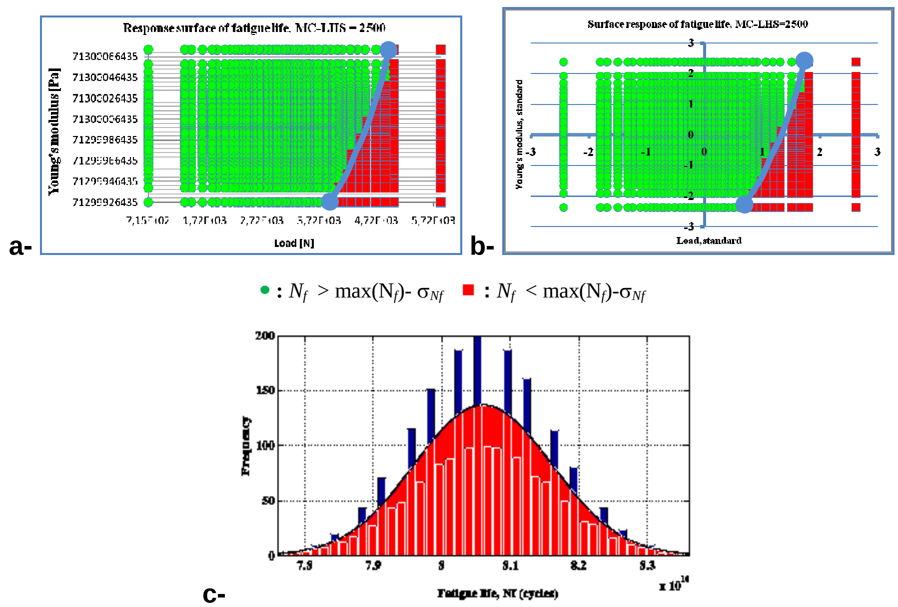

Figure 7. Response surfaces of the fatigue life as a function of the Young's modulus and vehicle load; a- physical space, b- standard space, c- Fatigue life frequency.

In Table 1 the local sensitivities of the input variables with respect to the fatigue life (*Nf*) are shown. The main statistical values of input (Young's modulus and load) output (fatigue life) parameters are also given.

Table 1. Statistical characteristics of the input/output parameters.

	Standard deviation	Mean value	Local sensitivity	Design point β_{HL}
		Inputs		
Load	1012	3.182 E3 [N]	1,018	1.24
Young's modulus	30981	7.13 E10 [Pa]	0,845	
		Output		
Fatigue life	1E13	8.06 E14 [cycles]		

6. Conclusion

This paper gives a study of a reliability approach of fatigue based on Monte Carlo Latin Hypercube Sampling MC-LHS. Input data of random parameters generated by Latin hypercube sampling method are used in finite element reliability simulations. An approach for estimate the output reliability and input sensitivity is presented basing on Abaqus/standard automation. Sensitivity analysis based on local index is used to analyze the impact of the parameters on fatigue life. In particular, we define how to assess the most sensitive parameter in fatigue life according to response surface analysis. Besides, we also present how determine which of the components affects the failure/success probabilities by using a numerical approach based on automation of Abaqus/standard software. Thus, basing on the MC-LHS fatigue simulations of an upper arm of vehicle suspension system under tow random variables, the following conclusions can be drawn:
1 - From the comparison of the obtained fatigue life distribution it appears that the load has a direct and important effect. While the used range of Young's modulus values is provided a negligible variation of fatigue life. Especially, this variation becomes important respectively at separations zones defined by limit state function.
2 - Proposed model based on the strain energy density parameter depends strongly on the obtained Rainflow cycles number.
3 - Homogeneous scatters of the fatigue life in response surface are due to the Latin hypercube sampling method.
4 - The simulation shows that for fatigue life predictions using fixation angle of the upper arm is not very sensitive.
5 - Other fatigue analyses have to be carried out using second order reliability method and polynomial chaos under non-proportional loadings and cyclic mechanical properties.

7. References

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