Fatigue equivalent loads for visualization of multi-modal dynamic simulations

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Abstract: *Transient dynamic simulations gain importance in the automotive industry and modern fatigue postprocessors are apt to evaluate the fatigue damage. However, additional insight into a structure's behaviour may be obtained from observing the displacements. Displacement patterns are important for design engineers in order to improve the structure. With proportional static loads it is trivial to display and understand the displacements, but the displacements in dynamic simulations are often very complicated. This paper describes a novel method for visualising the transient displacements of mode-based analyses. Based on the modal displacements a new simple, yet fatigue equivalent, modal displacement is computed and visualized instead. Applications from commercial vehicles are used as examples.*

Keywords: *modal dynamics, equivalent load, fatigue, post-processing, Basquin's law*

1. Introduction

The displacement history of a finite element model of an arbitrary structure may be expressed as a linear combination of fixed displacement vectors multiplied with scalar time varying functions. In many engineering applications it is possible to discern a few displacement vectors that adequately span the entire displacement space. Typically the first few eigenmodes of the structure are sufficient. This paper is concerned with fatigue loads on vehicle components and for this type of application a linear representation with only a few eigenmodes is often sufficient to represent the entire displacement history.

Modern fatigue post-processors, whether strain- or stress based, are capable of evaluating the fatigue load from a time varying linear combination of fixed vectors. The fatigue damage is computed as a local damage on a critical plane in every point/node of the structure. As an example; for a number of vectors in the order of 10^1 and a number of nodes in the model in the order of 10^6, the time for fatigue evaluation is counted in hours on an ordinary computer. This type of fatigue evaluation of linear dynamic simulations is currently a regular routine at many companies in the vehicle industry.

However, the fatigue life of a component is not alone sufficient for developing new engineering solutions. If only a weak point is indicated the remedy is usually to add more material (read cost and weight) to that region. Most often the clever solution to a problem requires a good understanding of the critical displacement patterns. This understanding is difficult to obtain in transient (non-stationary) dynamics. It is here proposed to form a fatigue equivalent harmonic stationary displacement and use this displacement to comprehend the critical components of the transient dynamics.

2. Fatigue damage of transient dynamics

What is sought is a displacement history that is easy to visualize. Assuming that the actual displacement may be approximated with

$$u_i(x,t) = \sum_r q_r(t)\Phi_{ri}(x), \tag{1}$$

where q is the modal amplitudes and Φ the modal vectors. For linear elastic structures it then follows that the strain is given by

$$\varepsilon_{ij}(x,t) = \sum_r q_r(t)\Phi_{ri,j}(x) \tag{2}$$

and the stress by $\sigma_{ij} = D_{ijkl}\varepsilon_{kl}$.

Suppose that the fatigue stress is of uniaxial type, such that for each material point, (or at least for the critical material point) the fatigue damage F of a stress cycle is a monotonic function of the stress amplitude measured in a <u>single fixed</u> spatial direction ***b***, referred to as the critical direction. Suppose furthermore that rainflow counting and Palmgren-Miner's summation rule are valid techniques for computing the damage of complex load histories.

The stress projected on ***b*** is

$$B_{ij}\sigma_{ij} = \sum_r q_r(t)B_{ij}D_{ijkl}\Phi_{rk,l} \ , \tag{3}$$

where B is a constant tensor that depends only on the material point. It is useful to simplify Equation (3) with

$$B_{ij}\sigma_{ij} = \sum_r q_r(t)C_r \ , \tag{4}$$

where $C_r = B_{ij}D_{ijkl}\Phi_{rk,l}$ is the critical stress component of mode r. Introducing now the normalized weight factors c_r such that

$$c_r = \frac{C_r}{\|C\|} \text{ giving } B_{ij}\sigma_{ij} = \|C\|\sum_r q_r(t)c_r \ . \tag{5}$$

A rainflow counting of the stress history gives a set of load cycle amplitudes

$$RFC\left(B_{ij}\sigma_{ij}\right) \rightarrow \left\{\sigma^{A}\right\}. \tag{6}$$

However, because the rainflow counting is a proportional operator the normalized weight factors introduced in Equation (5) may be used:

$$RFC\left(B_{ij}\sigma_{ij}\right) = \left\|C\right\| RFC\left(\sum_{r} c_r q_r(t)\right). \tag{7}$$

For a single load cycle it is assumed that the fatigue damage is a function of the stress amplitude only

$$F^{cycle} = F^{cycle}\left(\sigma^{A}\right). \tag{8}$$

An example may be Basquin's law,

$$F^{cycle}\left(\sigma^{A}\right) = \left(\frac{\sigma^{A}}{\sigma_o}\right)^{m}, \tag{9}$$

where σ_o and m are material parameters. For a complex load history the damage of the individual cycles are summed according to the Palmgren-Miner hypothesis

$$F = \sum_{A} F^{cycle}\left(\sigma^{A}\right). \tag{10}$$

With Basquin's law we may rewrite the fatigue damage using Equation (7, 9-10)

$$F = \left(\frac{\left\|C\right\|}{\sigma_o}\right)^{m} \left[RFC\left(\sum_{r} c_r q_r(t)\right)\right]^{m}, \tag{11}$$

and there should be no ambiguity as to the interpretation of the notation of *RFC* raised to the power of m, c.f. Equation (6).

In the general case no specific knowledge of the structure is available and there is no chance of knowing which linear combination of the eigenmodes that is critical. However, using the normalized weight factors allows for rapid evaluation of all linear combinations. The maximum damage of any structure is sure to be located on the surface S

$$S : \left\|c_r\right\| = 1 \tag{12}$$

Consider as an example a structure with two relevant eigenmodes Φ_1 and Φ_2 with corresponding modal displacements q_1(t) and q_2(t). Different material points in the structure will suffer different damages. This is because the modal stresses are different at different points and also the critical direction ***b*** is different. For some points only the first mode is important. The damage on these points is evaluated with $[c_1, c_2] = [\pm 1, 0]$. For some other points the two modes are equally important, these points are captured with $[c_1, c_2] = [1, \pm 1]/\sqrt{2}$. It is quite possible that the structure in question contains no points that are influenced by only the second eigenmode. We have no knowledge of this, but we compute the damage also for the linear combination $[c_1, c_2] = [0, 1]$. When all linear combinations on the surface are evaluated all the *RFC* of all possible points in the structure are evaluated.

Discretizing the *S* in *n* points allows expressing the fatigue damage for these points with Equation (11). The fatigue damage of point *i* on the surface *S* being

$$F^i = \left(\frac{\left\| C^i \right\|}{\sigma_o} \right)^m \left[RFC\left(\sum_r c_r^i q_r(t) \right) \right]^m . \tag{13}$$

The numerical evaluation of the *RFC* is rapid, thanks to modern rainflow counting algorithms.

In the particular case of two eigenmodes, the angle $\theta = \text{atan}(c_2/c_1)$ is a useful representation *S*. When all angles θ in $[0, \pi]$ have been evaluated the *RFC* is determined for all points. An example of the rainflow count on *S* for the case of two eigenmodes using modal displacement data from a dynamic simulation of a spare wheel is presented in Figure 1(b and c).

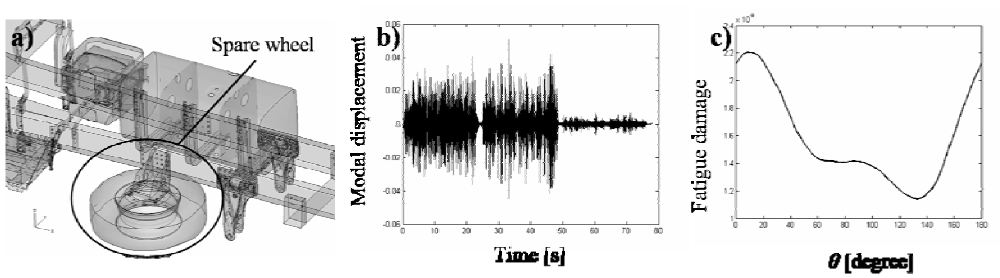

Figure 1. a) Schematic of a commercial vehicle seen from the side. The spare wheel is encircled. b) Simulated transient modal displacements for the two eigen-modes that are most important to the spare wheel bracket. Mode 1 indicated with a black line and mode 2 with a grey line. c) Non-normalized rainflow count of the displacement for points on the surface *S* using Basquin's law with exponent *m* = 7.

3. Fatigue equivalent harmonics

It is here proposed to search a fatigue equivalent displacement in the form

$$\tilde{u}_i(x,t) = \sum_r A_r \sin(\omega t + \varphi_r) \Phi_{ri}(x), \tag{14}$$

where A_r is the fatigue equivalent modal amplitudes and φ_r is the phase shift of mode *r*. This displacement is clearly easy to visualize and understand because all the modal displacements have the same frequency and constant amplitude. It remains to show if and how the parameters *A* and φ may be chosen to assure fatigue equivalence.

Insertion of Equation (14) into Equation (2-11) gives

$$\tilde{F}^i = \frac{2\pi T}{\omega}\left(\frac{\left\|C^i\right\|}{\sigma_o}\right)^m \left[\frac{1}{2}\max_t\left(\sum_r c_r^i A_r \sin(\omega t + \varphi_r)\right) - \frac{1}{2}\min_t\left(\sum_r c_r^i A_r \sin(\omega t + \varphi_r)\right)\right]^m \tag{15}$$

$$\tilde{F}^i = \frac{2\pi T}{\omega}\left(\frac{\left\|C^i\right\|}{\sigma_o}\right)^m ampl\left(\sum_r c_r^i A_r \sin(\omega t + \varphi_r)\right)^m . \tag{16}$$

Equation (16) is a useful representation because it allows comparing the fatigue damage of the original displacement with that of the equivalent displacement. The relative damage at the point *i* on *S* is

$$\rho_i = \frac{\tilde{F}^i}{F^i} = \frac{2\pi T}{\omega}\frac{ampl\left(\sum_r c_r^i A_r \sin(\omega t + \varphi_r)\right)^m}{\left[RFC\left(\sum_r c_r^i q_r(t)\right)\right]^m} . \tag{17}$$

The fatigue equivalent displacement has a ratio $\rho_i = 1$ for every point *i*. This problem is over-determined with a number *n* evaluated points on the surface *S* and a number 2*R* unknown parameters, where *R* is the number of eigenmodes used. The unknown parameters are the amplitudes of the *R* eigenmodes A_r, the *R* - 1 phase shifts φ_r and the number of cycles $2\pi T/\omega$.

Using numerical minimization the best fitting fatigue equivalent amplitudes and phase shifts are sought. Once they are found the fatigue equivalent displacement (and stress) may be written to an output database using the modal vectors and modal stresses. The equivalent displacement is harmonic so only one cycle needs to be written. In this work the equivalent displacements was written to an Abaqus output database using the Abaqus python scripting interface.

4. Example of application

Consider the spare-wheel of a commercial vehicle in Figure 1a. A direct fatigue evaluation using a commercial fatigue post-processor with the transient simulation in Figure 1b together with the modal stresses, Figure 2, gives a damage distribution presented in Figure 3a. It should be remarked that this damage distribution correlates well with experimental results. Failures have been observed on and around the top left and the bottom right frame-bolts and also around the rear pressing. A closer examination reveals that the two most important eigenmodes are dominated by bending in the vertical and horizontal direction, respectively. A visualization of the modal stresses, Figure 2, shows that individually none of these eigenmodes can be responsible for the damage distribution; it is necessary to consider them simultaneously.

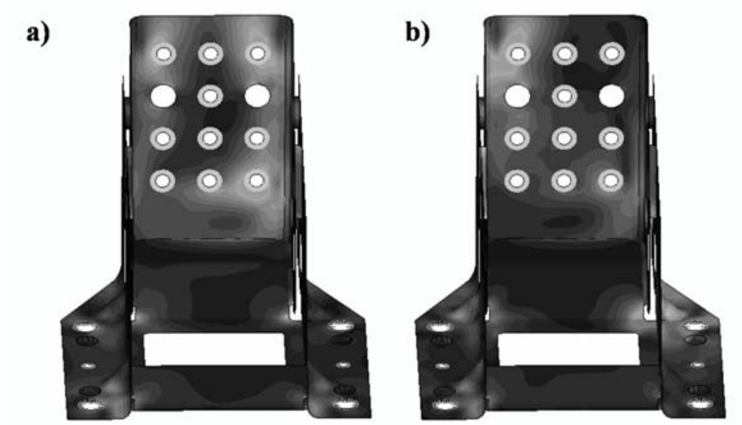

Figure 2. a) Modal stress distribution of the vertical mode. b) Modal stress distribution of the horizontal mode.

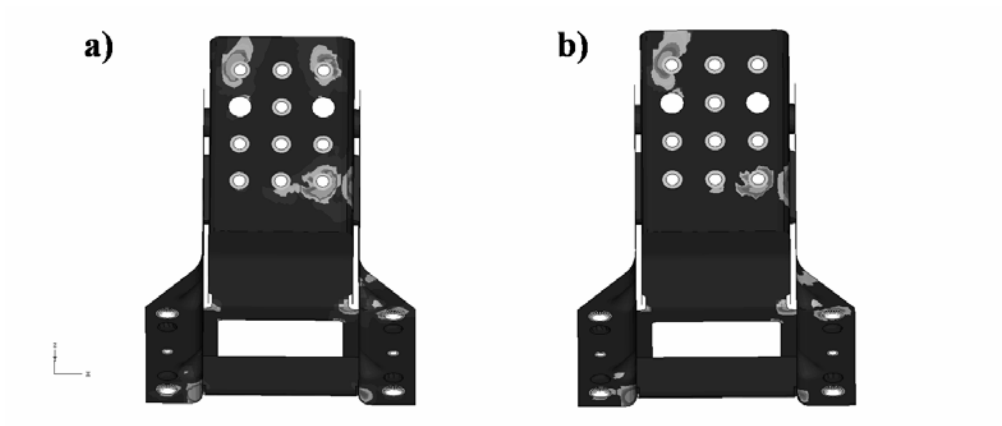

Figure 3. a) Damage distribution using Femfat of the two eigenmodes with transient data. b) Damage distribution using Femfat of the two eigenmodes with equivalent displacement.

Using the proposed method gives an equivalent harmonic displacement in terms of the amplitudes of the two modes, the phase shift and the number of cycles, refer to Figure 4a. The transient- and the equivalent displacements are also compared in a phase-diagram, and the damage of the equivalent displacement is computed for all points on the surface S, Figures 4b and 4c. It is noted that for no point on S is the discrepancy in rainflow count larger than 18 per cent. However, the damage distribution of the equivalent displacement may also be computed in Femfat, Figure 3b. A closer examination and comparison of Figures 3a and 3b reveals that the damage distribution is slightly different between the original and the equivalent displacements. This difference is due to several factors. Firstly, the equivalent displacement is computed for the mostly damaged point residing on S with a specific fatigue exponent m (here $m = 7$ is used). Because the equivalent displacement is strongly influenced by the fatigue exponent it may be a poor representation of the load for material points with different exponents. Secondly, the commercial fatigue post processor uses different assumptions about the damage. In particular it probably uses more sophisticated rules than Basquin's law computed on the stress in a fixed critical direction.

Nevertheless the two damage distributions do have similarities that are important. The asymmetric damage distribution which indicates a greater risk of failure around the top left and the bottom right frame-bolts, and around the inner right wheel nut, would be difficult to obtain if the modal stresses were regarded separately.

A visualization in form of a movie of the equivalent displacement shows that the spare wheel follows an inclined ellipse in the x,z plane. The visualization also provides much needed understanding of the critical displacement of the structure and allows for a faster development of strength increasing modifications.

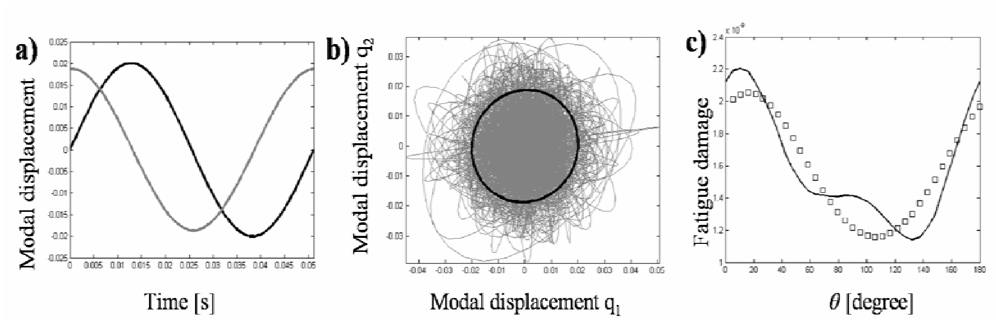

Figure 4. a) Equivalent modal displacement for the two eigenmodes in the time domain, mode one – black line, mode two – grey line. b) Phase plot of the two eigenmodes using transient data - thin grey line, and computed equivalent data - thick black line. c) Non-normalized rainflow count of the transient data – solid line, and the equivalent data – squares, for all possible points on *S.*

4. Summary

The presented method of fatigue equivalent modal displacement is resting on rather strict hypotheses, notably a uni-axial damaging stress state, Basquin's law and linear damage accumulation are assumed valid. Despite these gross simplifications the method provides a useful tool to understand and visualize complex transient time histories from a fatigue perspective.