

User subroutine for compressive failure of composites

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Abstract: For high strength carbon fiber reinforced polymers, the design criteria are often specified by the compression strength of the composite materials component. This is due to the fact that the compression strength of unidirectional composites is as low as 50 to 60 percent of the tensile strength. One important compressive failure mode in composite is kink-band formation which for a great deal is governed by the waviness of the fibers and the yielding properties of the matrix material. Therefore, in order to make proper simulation of the failure modes in composites, it is necessary to take these effects into account. One approach is to model the actual fiber/matrix system using a micromechanical based finite element model. For realistic composite structures with a large number of fibers, approaches which will result in extremely large numerical models including a great deal of unwanted details. An alternative is to base the simulation on a smeared out composite model where the nonlinear properties of the constituents are taken into account. A finite element implementation of such a model is presented. The model is implemented as an Umat user subroutine in the commercial finite element program Abaqus and used to predict kink-band formation in composite materials.

Keywords: Compressive failure, Composites, Kink-band, User subroutine, Smeared out composite laws

1. Introduction

Fiber composites loaded in compression parallel to the fibers may fail by localization of strains into kink bands. Structural components based on composite materials, which are subject to changing load histories, are often designed with this failure mode as the most critical. Figure 1 shows an idealization of the kink band geometry.

The formation of kink bands was initially studied in geology. One of the first papers on kink band formation in fiber composites was by Rosen (1965) where a linear elastic analysis was carried out treating the fiber/matrix interaction by a planar model of beams on an elastic foundation. The critical stress for kink band formation was estimated as

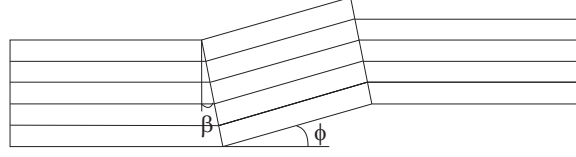


Figure 1. Idealization of kink band geometry with kink band orientation β and fiber rotation ϕ .

$$\sigma_{11} = G \quad (1)$$

where G is the elastic shear modulus of the composite material. Later, Argon (1972) formulated a planar model, which emphasised the importance of two effects not taken into account in Rosen's analysis: Misalignments of fibers relative to the direction of loading, and plastic deformation of matrix material. Budiansky (1983) unified the approaches of Rosen (1965) and Argon (1972) which allowed for later extensions to multi axial loading conditions etc. see for instance (Fleck, 1991 and Slaughter, 1993). The critical stress for kink band formation was obtained as

$$\sigma_{11} - \sigma_{22} + G + 2\sigma_{12} \tan \beta + E_T \tan^2 \beta = 0 \quad (2)$$

where E_T is the transverse modulus of the composite. The works of (Argon, 1972, Budiansky, 1983 and Fleck, 1991) had shown that fiber misalignments and plastic deformation was important for kink band formation and a constitutive model was developed by Christoffersen and Jensen (1996), which allowed for these effects. The conditions for kink band formation were modelled as loss of ellipticity of the governing incremental equilibrium equations (Rice, 1976). In the limit of infinitely stiff fibers, an exact solution for the critical stress for kink band formation could be obtained, which when specialised to $\beta = 0$ results in

$$\sigma_{11} - \sigma_{22} + \frac{1}{c^m} \left(L_{1212}^m - \frac{\sigma_{11}^m - \sigma_{22}^m}{2} \right) = 0 \quad (3)$$

where L_{ijkl} are incremental moduli for the matrix material relating Jaumann rates of Kirchhoff stresses to strain rates as explained in greater detail in the following section.

2. The constitutive model

A planar model is formulated for the composite material in Figure 2 based on the assumptions that the fibers and the matrix material are described by time-independent plasticity theories relating Jaumann rates of Kirchhoff stresses $\hat{\tau}_{ij}$ to strain rates ε_{ij} in the form

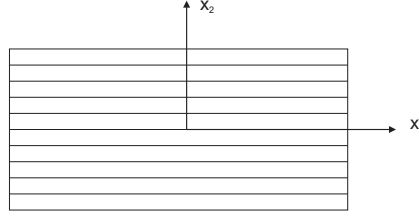


Figure 2. Block of planar composite material with alternating layers of matrix and fiber material

$$\begin{aligned} \hat{\tau}_{ij}^f &= L_{ijkl}^f \varepsilon_{kl}^f & \text{or} & & \varepsilon_{ij}^f &= M_{ijkl}^f \hat{\tau}_{kl}^f & \text{and} \\ \hat{\tau}_{ij}^m &= L_{ijkl}^m \varepsilon_{kl}^m & \text{or} & & \varepsilon_{ij}^m &= M_{ijkl}^m \hat{\tau}_{kl}^m \end{aligned} \quad (4)$$

where $\varepsilon_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i})$, v_i denoting displacement rates and a comma denoting partial differentiation. The superscripts (f, m) refer to fibers or matrix material, and in the following constitutive properties without a superscript denote composite properties. In eq. (4), L_{ijkl} denotes the tensor of instantaneous moduli and M_{ijkl} denotes the tensor of instantaneous compliances. Furthermore, the summation convention is adopted for repeated index. In the applications we set Kirchhoff stresses equal to Cauchy stresses and thus assume that local changes in density due to elastic deformations are negligible.

Two classical models for calculating the constitutive response of the composite material for small strain and small rotations are obtained by assuming identical strains in the fibers and matrix (the Voigt model) or by assuming identical stress in the fiber and the matrix (the Reuss model) leading to

$$\begin{aligned} L_{ijkl} &= c^f L_{ijkl}^f + c^m L_{ijkl}^m \quad (\text{Voigt}) \\ M_{ijkl} &= c^f M_{ijkl}^f + c^m M_{ijkl}^m \quad (\text{Reuss}) \end{aligned} \quad (5)$$

where c^f and c^m denote relative volume fractions of fibers and matrix material. For unidirectional fiber composites, the Voigt model is reasonable for properties in the fiber direction, while the Reuss model is reasonable in the perpendicular direction.

When introducing the constitutive relations, which the present work is based on, and which are valid for finite strains and rotations, it is convenient to formulate the constitutive relations as the relationship between nominal stress rates and displacement gradient $v_{i,j}$ in the form

$$\dot{s}_{ij} = C_{ijkl} v_{l,k}, \quad i, j, k, l \in \{1, 2\} \quad (6)$$

The following relations hold between L_{ijkl} and C_{ijkl}

$$L_{ijkl} = C_{ijkl} + \frac{1}{2}\delta_{il}\sigma_{kj} + \frac{1}{2}\delta_{ik}\sigma_{lj} + \frac{1}{2}\sigma_{il}\delta_{kj} - \frac{1}{2}\sigma_{ik}\delta_{lj}, \quad i, j, k, l \in \{1, 2\} \quad (7)$$

Here, σ_{ij} denotes the Cauchy stress tensor. Let us introduce the following alternative notation for the constitutive equations (6).

$$\dot{\mathbf{s}}_\alpha = \mathbf{C}_{\alpha\beta} \mathbf{v}_{,\beta}, \quad \alpha, \beta \in \{1, 2\} \quad (8)$$

so that the vectors $\dot{\mathbf{s}}_\alpha$ and $\mathbf{v}$ contain the components of the nominal stress rates and the displacements according to

$$\dot{\mathbf{s}}_1 = \begin{pmatrix} \dot{s}_{11} \\ \dot{s}_{12} \end{pmatrix}, \quad \dot{\mathbf{s}}_2 = \begin{pmatrix} \dot{s}_{21} \\ \dot{s}_{22} \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (9)$$

and the matrices $\mathbf{C}_{\alpha\beta}$ are given by

$$\begin{aligned} \mathbf{C}_{11} &= \begin{pmatrix} C_{1111} & C_{1112} \\ C_{1211} & C_{1212} \end{pmatrix} & \mathbf{C}_{12} &= \begin{pmatrix} C_{1121} & C_{1122} \\ C_{1221} & C_{1222} \end{pmatrix} \\ \mathbf{C}_{21} &= \begin{pmatrix} C_{2111} & C_{2112} \\ C_{2211} & C_{2212} \end{pmatrix} & \mathbf{C}_{22} &= \begin{pmatrix} C_{2121} & C_{2122} \\ C_{2221} & C_{2222} \end{pmatrix} \end{aligned} \quad (10)$$

The fibers are aligned with the x_1 -axis as indicated in Figure 2. A compromise between the assumptions in the Voigt and Reuss models is introduced by assuming that material lines parallel with the fibers are subject to a common stretching and rotation, and planes parallel with the fibers transmit identical tractions. According to this,

$$\begin{aligned} \mathbf{v}_{,1}^f &= \mathbf{v}_{,1}^m = \mathbf{v}_{,1} \\ \dot{\mathbf{s}}_2^f &= \dot{\mathbf{s}}_2^m = \dot{\mathbf{s}}_2 \end{aligned} \quad (11)$$

Furthermore,

$$\begin{aligned} c^f \mathbf{v}_{,2}^f + c^m \mathbf{v}_{,2}^m &= \mathbf{v}_{,2} \\ c^f \dot{\mathbf{s}}_1^f + c^m \dot{\mathbf{s}}_1^m &= \dot{\mathbf{s}}_1 \end{aligned} \quad (12)$$

It was shown in (Christoffersen, 1996) that equations (11) and (12) along with (8) for the constituents allowed the composite moduli to be written in the form

$$\mathbf{C}_{\alpha\beta} = c^f \mathbf{C}_{\alpha\beta}^f + c^m \mathbf{C}_{\alpha\beta}^m - c^f c^m \left(\mathbf{C}_{\alpha 2}^f - \mathbf{C}_{\alpha 2}^m \right) \mathbf{C}_{22}^{*-1} \left(\mathbf{C}_{2\beta}^f - \mathbf{C}_{2\beta}^m \right) \quad (13)$$

where

$$\mathbf{C}_{22}^* = (c^m \mathbf{C}_{22}^f + c^f \mathbf{C}_{22}^m) \quad (14)$$

It was also shown in (Christoffersen, 1996) that the moduli L_{ijkl} in eq. (7) with C_{ijkl} given by (13) satisfy the symmetries

$$L_{ijkl} = L_{jikl} = L_{ijlk} \quad \text{and} \quad C_{ijkl} = C_{klij} \quad (15)$$

The constitutive relations given by eq. (13) form the basis for the present study. Note that the first two terms in (13) can be considered as a generalisation of the Voigt model (5) to finite strains, and that the last term is a correction to this due to the static conditions in (11) and (12).

Each constituent may now be described by arbitrary time-independent plasticity theories. Here, J_2 -flow theory is used to model the behavior of both constituents. Experimental results in (Kyriakides, 1995) showed indications of relative weak fiber nonlinearities. The effects of this were investigated in (Jensen, 1997) showing some influence on the critical stress for kink band initiation. The J_2 -flow theory for the matrix can be formulated as the following incrementally linear relation between Jaumann rates of Kirchhoff stresses and strains (McMeeking, 1975)

$$\begin{aligned} \hat{\tau}_{ij} &= L_{ijkl} \varepsilon_{kl} \\ L_{ijkl} &= G \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right) + \left(K - \frac{2}{3} G \right) \delta_{ij} \delta_{kl} - \frac{4}{3} (G - G_t) m_{ij} m_{kl} \end{aligned} \quad (16)$$

where superscript $()^m$ for the matrix has been omitted and δ_{ij} denotes the Kronecker delta. In eq. (16), G and K are the elastic shear modulus and bulk modulus, respectively, and G_t is the shear tangent modulus, which along with m_{ij} are given by

$$G = \frac{E}{2(1+\nu)} \quad , \quad K = \frac{E}{3(1-2\nu)} \quad , \quad \frac{1}{G_t} = \frac{3}{E_t} - \frac{1-2\nu}{E} \quad (17)$$

and

$$m_{ij} = \frac{1}{2\sigma_{eq}} \left(\sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} \right) \quad (18)$$

Here, E_t is the uniaxial tangent modulus, which requires a uniaxial true stress versus logarithmic strain relation to be specified and σ_{eq} the effective von Mises' stress given by

$$\sigma_{eq} = \sqrt{\frac{3}{2} \sigma_{ij} \sigma_{ij} - \frac{1}{2} \sigma_{ii} \sigma_{jj}} \quad (19)$$

In the present study, a standard power-law hardening material is used

$$\varepsilon = \begin{cases} \frac{\sigma}{E} & , \sigma \leq \sigma_y \\ \frac{\sigma_y}{E} \left[\frac{1}{n} \left(\frac{\sigma}{\sigma_y} \right)^n - \frac{1}{n} + 1 \right] & , \sigma > \sigma_y \end{cases} \quad (20)$$

3. Implementation in UMAT

In the following section the implementation of the model in Abaqus is described. This implementation is an alternative to the application of the model in (Pane, 2004) and to the individual discretization of fiber and matrix material described in (Hsu, 1998). For a more detailed description on how to implement a constitutive model in Abaqus, see (Dunne, 2005).

The subroutine UMAT (User MATerial) is written in FORTRAN and is used to define the constitutive behavior of a material. Abaqus provides the deformation gradient, total strains and strain increments and the subroutine must then return the material Jacobian matrix $\partial\Delta\sigma/\partial\Delta\varepsilon$ for the constitutive model along with updated stresses. In this case the material behavior of the composite is simulated by mixing the properties of two materials each described by a power law hardening.

3.1 Calculation steps

The UMAT subroutine used in the present work contains the following steps:

1. Calculate the gradient of velocity from deformation gradient. The deformation gradient F is provided by Abaqus and the velocity gradient, which describes the spatial rate of the velocity, is found from $v_{i,j} = \dot{F}_{ik} F_{kj}^{-1}$
2. Calculate the effective von Mises' stress for matrix and fiber according to (19)
3. Calculate tangent modulus from uniaxial true stress versus logarithmic strain curve (20)
4. Calculate L_{ijkl}^f and L_{ijkl}^m according to (16)
5. Calculate C_{ijkl} from (7) and (13)
6. Calculate stress increments, $\dot{\sigma}_{ij} = C_{ijkl} v_{l,k} - \sigma_{ij} v_{k,k} + v_{i,k} \sigma_{kj}$, see (Jensen, 1997)
7. Update stresses
8. Update yield stress
9. Update plastic strains
10. Return state variables, see the following subsection
11. Return material Jacobian matrix $\partial\Delta\sigma/\partial\Delta\varepsilon$

All variables are updated using a forward Euler procedure. In order to ensure sufficiently small increments, a convergence study has been performed.

3.2 List of solution-dependant variables

The solution-dependant variables are variables that are updated as the analysis progresses. For instance, in order to be able to return the material Jacobian and to update the overall stresses in the composite material, it is necessary to keep track of the individual stresses in the fiber and matrix material. The UMAT utilizes a total of 16 state variables, passed from Abaqus through the array STATEV(NSTATV), each containing information about every integration point. The state variables are:

- The updated yield stresses of the fiber and matrix materials - both modelled as power hardening materials
- The effective plastic strain in fiber and matrix
- Two variables, β^f and β^m which will have the value 1 or 0 depending on if the material yields or not
- The stresses in the matrix material, $\sigma_{11}^m, \sigma_{22}^m, \sigma_{33}^m, \sigma_{12}^m$
- The stresses in the fiber material, $\sigma_{11}^f, \sigma_{22}^f, \sigma_{33}^f, \sigma_{12}^f$
- The initial direction of the fibers and the current rotation

3.3 List of Material Constants

To the matrix and fiber material, a total of nine different material properties are associated: two Young's moduli, E_m and E_f , two Poisson's ratios, ν^m and ν^f , two initial yield stresses, σ_y^m and σ_y^f , two hardening parameters, n^m and n^f and finally the fiber volume fraction c^f . These properties are passed to UMAT by Abaqus in the array PROPS(NPROPS). In the input-file, the keyword *USER MATERIAL is used to specify material constants. In the present study, the fiber volume fraction is assumed to remain constant $c^f = 0.6$ throughout the deformation.

3.4 User Subroutine ORIENT

The Abaqus user subroutine ORIENT is used for defining local material directions. In the kink band analysis described in the following section, parameters describing the geometry of the kink band are placed in a datafile which is being read from the ORIENT subroutine. In ORIENT these parameters, B, ϕ_m, α and β are subsequently used to calculate the initial local fiber direction. The subroutine is called by Abaqus at the start of the analysis at each material point.

4. Results

The constitutive model described in the previous section has been used to simulate the formation of kink bands in fiber reinforced composites. If not other is indicated, the material parameters of the matrix material is chosen to be given by $\sigma_y^m / E = 0.013$, $\nu^m = 0.356$ and $n^m = 4.5$, see Kyriakides (1995) while the fiber material for simplicity is taken to be elastic in all simulations.

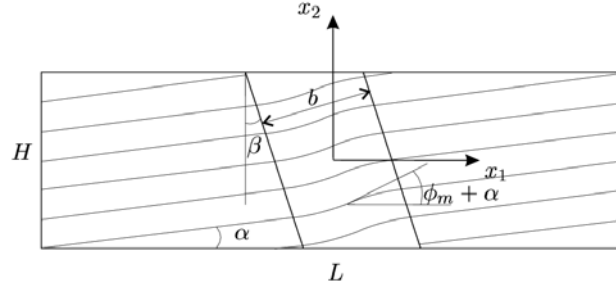


Figure 3. Kink band geometry

The kink band geometry is sketched in Figure 3. A block of material, see Figure 2, is subject to compressive stresses under plane strain conditions. The block has the dimensions height $H = 3$ and length $L = 10$ and in a band of width b and at an angle β the direction of the fibers is given a small imperfection. The direction of the fibers outside the band is given by the angle α and inside the kink band the fibers are assumed to be at an angle ϕ , and this angle is given by the expression:

$$\phi(x_1, x_2) = \frac{1}{2}\phi_m \left[\cos\left(\frac{2\pi \cos \beta}{b}(x_1 + x_2 \tan \beta)\right) + 1 \right] + \alpha \quad (21)$$

so that a small imperfection is added to the fiber angle α inside the band, and ϕ_m is the value of the maximum imperfection in the middle of the kink band. Furthermore, the displacements u_1 and u_2 satisfy the boundary conditions

$$\begin{aligned} u_1 &= 0 & \text{on} & & x_1 &= -\frac{L}{2} \\ u_2 &= 0 & \text{on} & & (x_1, x_2) &= \left(-\frac{L}{2}, -\frac{H}{2}\right) \end{aligned} \quad (22)$$

In the following, the width of the kink band has the value $b = 2$, and in all simulations the fiber volume fraction is 0.6. Furthermore, analysis has been restricted to two different fiber/matrix stiffness ratios, $E^f/E^m = 35$ and $E^f/E^m = 100$, which qualitatively correspond to a glass-fiber reinforced polymer and a carbon-fiber reinforced polymer, respectively.

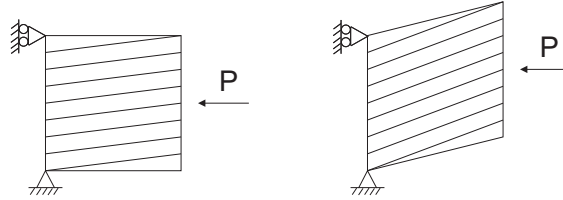


Figure 4. Deformation of a single element

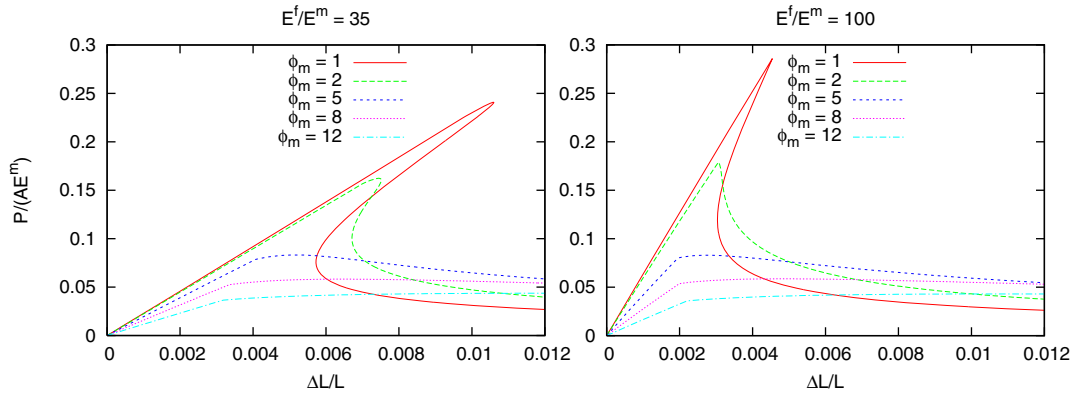


Figure 5. Plastic deformation of one element

First, the deformation of a single element, Figure 4, is investigated. A 4-node plane strain element (CPE4) is used and the initial fiber angle is chosen to be constant throughout the element. The resulting load versus displacement curves are shown in Figure 5. Curves are shown for various angles α of the fiber misalignment for the two different stiffness ratios of the fiber/matrix system. For sufficiently small fiber misalignments the response is essentially linear until a critical load is reached after which material softening and snap-back behavior is observed.

The maximum loading capacity is summarized in Figure 6a. Even though both constituents are modelled as hardening materials, the composite shows extensive softening behavior after the maximum loading capacity is reached with snap-back behavior for small initial fiber inclinations as shown in Figure 5. The snap-back behavior is in the finite element code Abaqus modelled using the Riks method. In experimental measurements a snap-back material behavior would manifest as a dynamic material response. A detailed description of the Riks method can be found in (Crisfield, 1991).

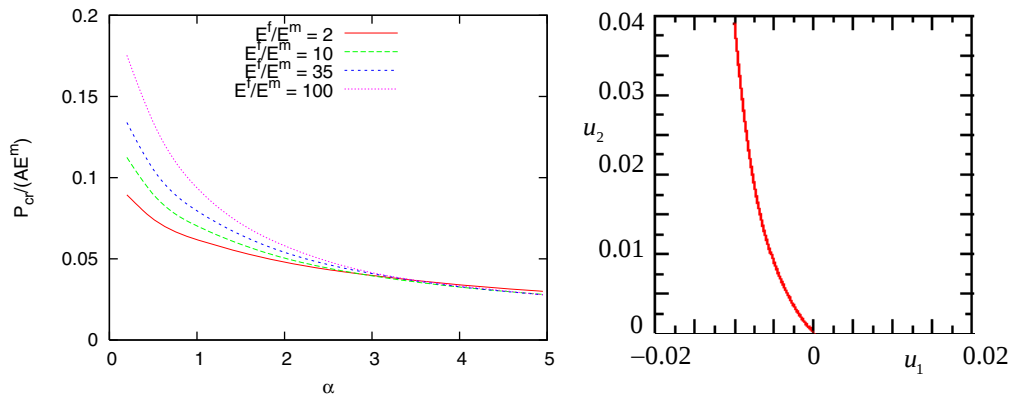


Figure 6. (a) Peak load for one element and (b) transverse versus horizontal displacement of lower left corner

The observed overall composite material softening which is required for strain localization in structural components is caused by an interaction of the non-linear response of the matrix material with an increasing rotation of the fibers during compression as illustrated in Figure 4 , during compression, the block of material will, in addition to increasing axial compression, show an increasing shearing tendency in the direction of increasing fiber rotation. For the cases studied here, if the matrix material plasticity is suppressed so both composite constituents follow a linear elastic material response, the fiber rotation itself does not cause material softening.

The deformation of one element is further investigated in Figure 6b. This figure shows the vertical displacement u_2 versus the horizontal displacement u_1 for node number 2 (the lower right corner in Figure 4) for the case where the fiber angle is $\alpha = 1^\circ$ and $E^f/E^m = 100$. The results show a shift in the direction of the displacement vector from initially equal amounts of the two displacement components towards displacement mainly in the vertical direction.

This concludes the study based on the performance of a single element. The following calculations are performed using at first 6×20 8-node plane strain (CPE8) elements. In Figure 7 the normalized load versus end shortening are shown for the low fiber to matrix ratio, $E^f/E^m = 35$, corresponding to glass fibers reinforced polymers and the high ratio, $E^f/E^m = 100$, corresponding to carbon fiber reinforced polymers. Calculations are carried out for different values of the maximum fiber imperfection angle ϕ_m . Outside the imperfection band, Figure 3, the fibers are aligned with the x_1 -axis, i.e. $\alpha = 0$. As also shown in previous studies, see (Sørensen, 2007), the peak load is highly sensitive to the misalignment of the fibers. At sufficiently large imperfection angle, the well defined peak load disappears and the composite structure fails by another mechanism. After the peak load material softening occurs and the curves eventually converge and the previous load history has insignificant effects.

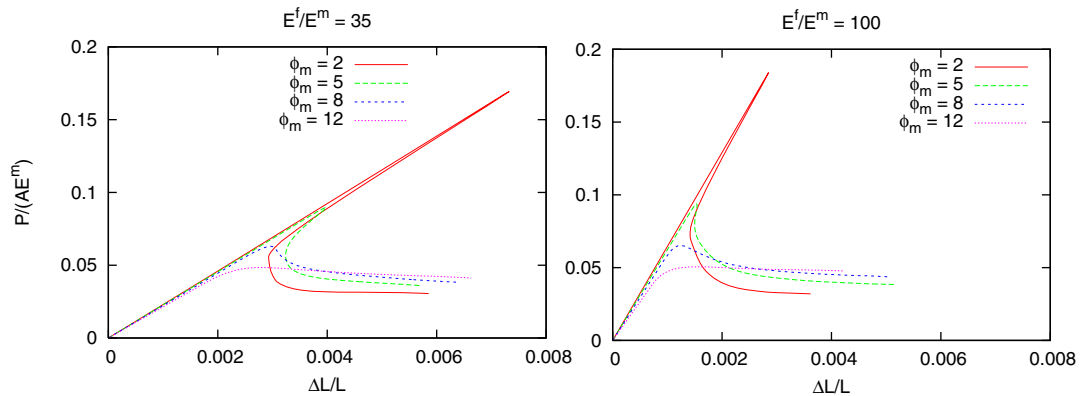


Figure 7. Kink band formation including plastic deformation in the matrix material

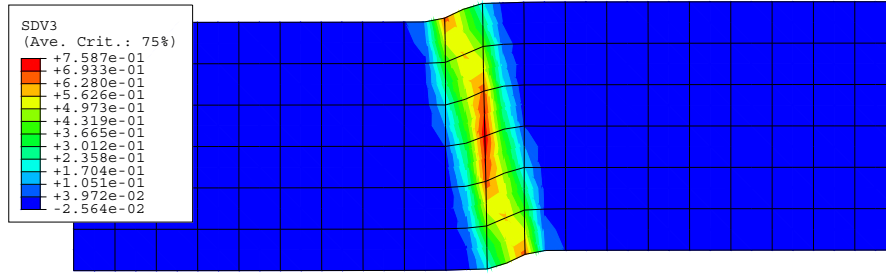


Figure 8. Contour-plot of the effective plastic strains in the matrix material

A contourplot of the plastic strain in the kink band for the case $E^f/E^m = 35$ is shown in Figure 8. The figure shows the tendency for strains to localize into a band inclined relative to the load and fiber direction. The deformations remain almost homogeneous outside the localized band.

The peak load is sensitive to a number of parameters. The sensitivity to the misalignment has been demonstrated and the non-linear response of the matrix material can also play a role. In Figure 9a the load versus end shortening curve is shown for a linear elastic response of the matrix, corresponding to $n^m = 1$. It is seen that a peak load results also in this case, and load vs. end shortening curves are shown in Figure 9b for different values of the hardening exponent n^m for the matrix. For all the curves in Figure 9 meshes consisting of 12×40 CPE8 elements are used. In addition the fiber imperfection angle was chosen to $\phi_m = 3^\circ$ and stiffness ratio to $E^f/E^m = 35$.

The peak stress and the post critical response is seen to be sensitive to the hardening exponent in certain regimes. The peak stress as a function of the hardening exponent n^m and $\phi_m = 3^\circ$ is shown in Figure 10a, where the peak load initially drops by a large amount until convergence towards an elastic-perfectly plastic response of the matrix is achieved. The peak load is less sensitive to the

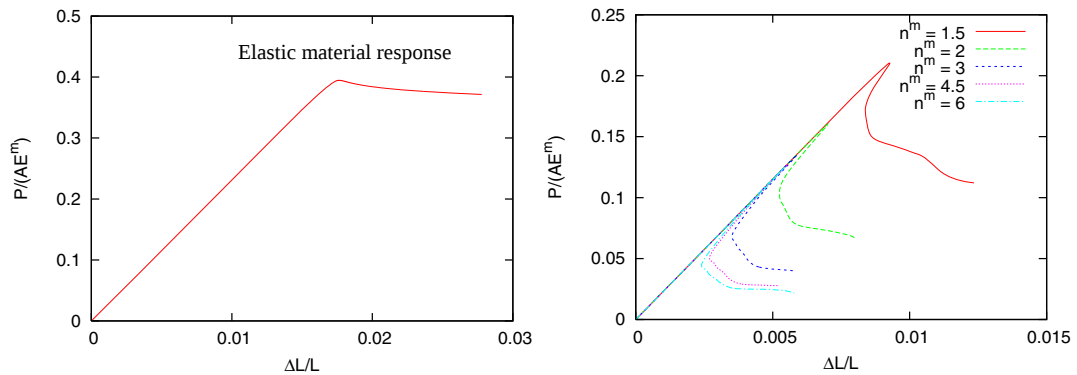


Figure 9. Load versus shortening curves depending on the hardening exponent n^m .

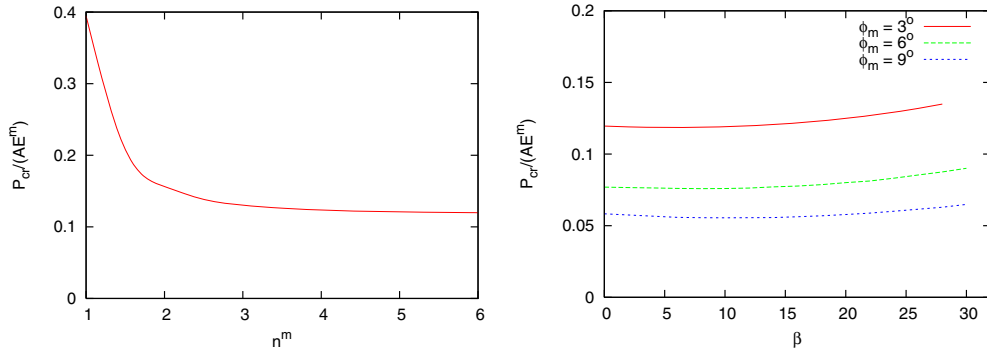


Figure 10. Peak load versus the (a) hardening exponent n^m and (b) the kink band orientation β .

kink band inclination angle, β . In Figure 10b, the critical load as a function of β is shown for three different misalignment angles. The peak stress is seen to vary only moderately with the angle, β , while the angle, ϕ_m , plays a more significant role. Again, the mesh is 12×40 CPE8 elements.

The effect of the applied load not being in the same direction as the fibers is studied by changing the value of α . Load-deformation curves for various α are given in Figure 11a. The fiber angle is $\phi = 2^\circ$, $E^f/E^m = 35$ and the mesh consists of 6×20 CPE8 elements. It is seen that the snap-back effect only exists for small angles and that the load-deformation curves cease to have a peak value when the value of α increases. A material length-scale is not included in the implementation of the constitutive model and as a consequence, the width of the kink band is undetermined and mesh dependent. As a result of this, the rate of convergence of load versus end-shortening curves, see Figure 11b with decreasing element size in the mesh is somewhat slow, especially in the immediate post kinking regime. The peak stress and the response far into the post critical regime seem to converge faster. Figure 12 shows contourplots of the effective plastic strain in the 4 different meshes used in the mesh-dependency studies. It is seen how the width of the localized band decreases as the size of the elements decreases. In the finite element model, the

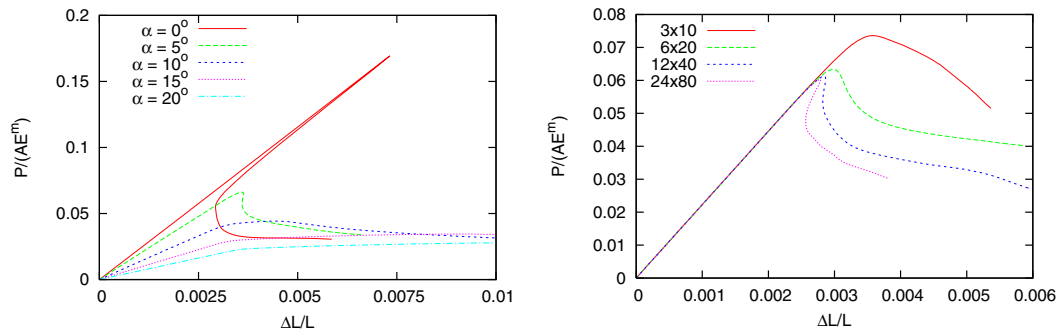


Figure 11. Load versus shortening curves for (a) various α and (b) different element sizes

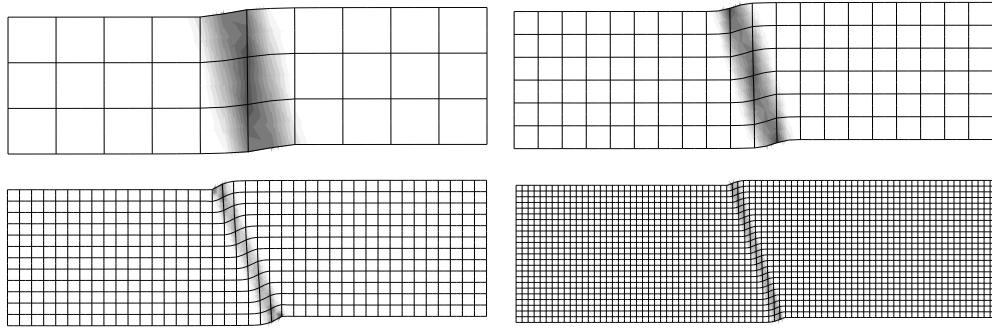


Figure 12. Four different deformed meshes with contour plot of effective plastic strains included.

element size represent a length scale and can be used to represent the microstructure in the composite material. A more proper way would be to include the missing material length scale using a non-local strain theory. Using an integral formulation is expected to be rather straight forward, while an implementation based on a gradient dependent plasticity model would be more complex.

Up until this point all results have been produced using a mesh consisting of rectangular elements. To investigate how a 'distorted' mesh affects the results, a mesh as shown in Figure 13 is introduced. The elements (12×40 CPE8R elements) are aligned with the initial imperfection at an angle of $\beta = 8^\circ$. The results are shown in the two figures in Figure 13 where the load-deformation curves for a 'regular' and a 'distorted' mesh are compared. In Figure 13a the fiber angle is $\phi = 4^\circ$ and $E^f/E^m = 35$, in Figure 13b the fiber angle is $\phi = 5^\circ$ and $E^f/E^m = 100$. The critical load is apparently unaffected by the change of the mesh but the immediate post-kinking behavior is seen to be different.

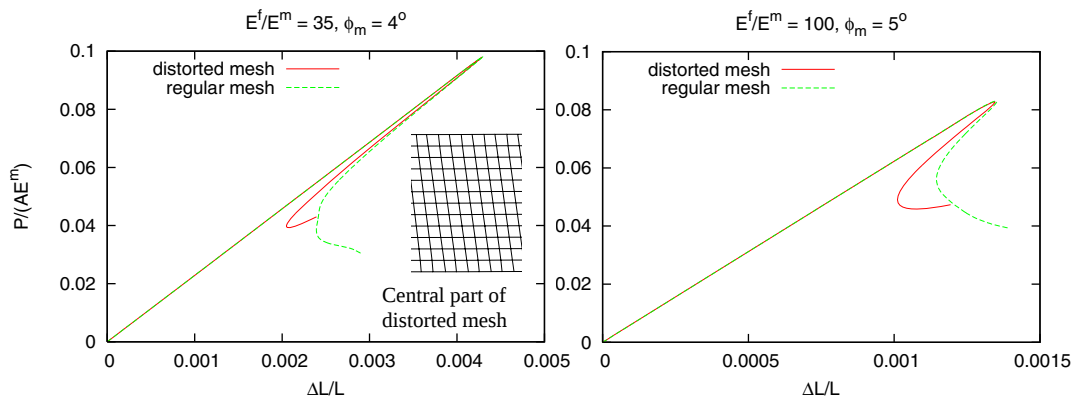


Figure 13. Load versus deformation for regular and distorted mesh.

5. Discussion and Conclusion

A plane constitutive model for fiber reinforced composites has been implemented in Abaqus as a user subroutine. The model has been tested under cases where failure by kink band formation is a relevant failure mechanism. At the material level, the model is found to predict a reasonable behavior for a single finite element. The behavior of a rectangular block of material divided into a number of finite elements is then investigated. The focus in this study is on the performance of the model when implemented in Abaqus with respect to reliability and rate of convergence. In agreement with previous results (Sørensen, 2007) it is seen that the critical stress for kink band formation is rather insensitive to the initial kink band orientation and that the kink band undergoes a distinct rotation in the post kinkband regime. In the model, a continuously varying imperfection in the initial fiber orientation is specified. This field can vary with the spatial components to mimic real defects in the production of laminated structures. The imperfection of the composite material applied in the present study is an assumed band of material of a specified width in which the fiber orientation gradually changes from the orientation outside the band to a maximum deviation from this and back to the original orientation again. The band can be arbitrarily oriented relative to the fiber direction. No variation of fiber rotations has been assumed in the direction of the band although this is not a general restriction in the model.

The constitutive model has no intrinsic length scale and consequently the width of the kink band is mesh dependent. As a result the rate of convergence of load versus end-shortening curves with decreasing finite element mesh size is somewhat slow especially in the immediate post kinking regime. The peak stress and the response far into the post critical regime seem to converge faster. It is demonstrated that an initial alignment of the finite element mesh in the direction of the orientation of the kink band may have the effect of increasing the rate of convergence.

The model is applied in a study of the effect of initial misalignment of the fiber and load direction. This effect has previously been studied in an approximate manner by imposing a fixed linear combination of compressive and shear stresses outside the kink band. The effect of initial misalignments is very distinct on the peak stress where the critical stress at 10° misalignment is reduced to roughly 1/4 the critical stress for compression in the fiber direction at a fixed band of imperfections. At a misalignment of 15° and above the peak in the stress versus end shortening response vanishes and the composite structure fails by another mechanism.

The model is expected to have some immediate applications such as kink band formation in plates with holes or other, more complex, structural components. Also, the model can be used to study competing compressive failure mechanisms such as buckling and kink band formation in fiber composite based structures.

6. Reference

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