CONSIDERATIONS ABOUT NON-LINEARITIES ON ENGINEERING PLASTICS

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Abstract. *This work makes a revision of the main relative aspects of the nonlinear structural analysis in engineering plastic structures using the software ABAQUS Standard V6.5. Effects of temperature, mechanical effort, fluency and relaxation are considered. Mechanical properties of plastics were shown to be highly sensitivity with temperature. Then it´s important that these kind of effects will be considered for most plastics structural analysis. Another important aspect is relative to time effects due to viscoelastic nature of polymers. Thus, phenomena like creep and relaxation are important that also differentiates fundamentally the structural plastic analysis from the structural problems in metals. The work validates experimental results relative to creep and relaxation through a numerical study coupling thermo-mechanical and viscous material effects. Also is showed results of an application involving coupling thermo-mechanical analysis and creep.*

1. Introduction

Plastics have revolutionized some industries by significantly enhancing product features and product value. If the material properties of polymers are intelligently exploited, then multi-functional components can be produced that are commercially and functionally superior to previous designs. As a result, some industries continue to be a highly competitive and innovative business that provides outstanding value to the consumer. For these reasons the use of plastics to replace metal and other traditional materials is becoming a key strategy in many markets segments.

This work contemplates some important aspects concerned with the nonlinearities in plastics and that, in some cases, differs from the metal applications. Initially it is presented a discussion in the main relative aspects to the process and material of engineering plastics. Also are given a general formulation of equilibrium equations implemented in ABAQUS, which must be used with large strain and large rotation. Moreover, considering the sensitivity relative to the temperature and rate-dependence are introduced the thermo-elastic-plastic and viscoplastic constitutive equations using the structural analysis available in ABAQUS.

Finally, are presented results of numerical studies involving time effects that are compared with experimental data. An analytical result was used as reference for the application of the theory.

1.2 New Trends

Metal replacement by plastics is a trend that is expanding rapidly due to the multiple benefits provided by plastics compared to metal. All of them lead to significant productivity improvements and/or product differentiation. Today’s designs require increasingly complex geometries and materials and plastics solve many different types of problem. However, to match the plastic with the application needs very careful[1].

The properties of plastic materials can vary over a wide range through the addition of fillers/reinforcing materials. Most basic properties of plastics, however, form a marked contrast to those of metals. For example, some benefits of thermoplastics versus metals are: lighter than metal, electrically non-conductive, vibration dampening, low corrosion and chemical resistance. Indeed, plastics exhibit a manufacturing and design flexibility, formed into complex shapes and recyclable.

It’s important to know about the capabilities of engineering plastics and the ability to design components with them that add value and help reduce costs. Until some years ago, most engineers and designers used to regard all plastics as cheap replacements for metals, or even for wood, glass or leather. Designs in plastics were changed little or not at all from designs for such traditional materials. As a result, plastics parts often failed to live up to expectations. The public tended to consider all plastics together as low-quality materials and their general reputation suffered as a result[8]. In order to produce functional parts in plastic and at the same time save costs, radical design modification is generally necessary if the plastic is being used to replace metal.

Today’s engineers and designers distinguish between commodity plastics and high-performance engineering polymers. There is a broad awareness in industry, too, that the latter group, if correctly designed, can outperform traditional materials in many applications while allowing cost savings to be made.

Among the advanced technologies that are at engineers and designers disposal for designing and producing parts made of engineering polymers, an important tool is Computer-aided engineering (CAE). When used appropriately, CAE analysis can provide a wide range of advantages, including: improve performance and quality of final plastic product, optimum use of materials, weight savings, faster time to market and therefore a competitive edge and reduce development, production, and part costs[3].

The first step in designing a plastic part is to define properly and completely the environment in which the part will operate. Properties of plastic materials are substantially altered by temperature changes, chemicals and applied stress. Most plastics exhibit a change in their basic mechanical properties at room temperature or on exposure to short-term stress. Mechanical properties and performance are significantly affected by the solid-state structure formed during processing. Importantly, unlike many other common engineering materials, plastics exhibit mechanical properties that exhibit time dependent viscoelastic and aging behaviors, even at low temperatures. Otherwise, the creep rate is accelerated with increasing temperature on plastic materials.

2. Equilibrium

2.1 Virtual Velocity Principle

Considering a body of volume V submitted at an instant t by a set of external forces f, the differential form of equilibrium is given by : (1)

$$\nabla\boldsymbol{\sigma} + \mathbf{f} = \mathbf{0}$$

Each point of the body satisfies this relation. If it´s applied an arbitrary field of velocities $\delta\mathbf{v}$ satisfying the boundary conditions, for the equilibrium configuration we get an incremental work at arbitrary rate. If the gradient of velocities is,

$$\mathbf{L} = \frac{\partial \mathbf{v}}{\partial \mathbf{x}} \quad \text{and} \quad \delta\mathbf{L} = \delta\mathbf{D} + \delta\mathbf{W} \tag{2}$$

Where $\mathbf{D} = \text{sym}\mathbf{L} = \frac{1}{2}\left(\mathbf{L} + \mathbf{L}^{T}\right)$ and $\mathbf{W} = \text{skw}\mathbf{L} = \frac{1}{2}\left(\mathbf{L} - \mathbf{L}^{T}\right)$ (3)

Results on the Virtual Velocity Principle:

$$\int_{V} \boldsymbol{\sigma} : \delta\mathbf{L}\,dV = \int_{S} \mathbf{f}^{S}.\delta\mathbf{v}\,dS + \int_{V} \mathbf{f}^{B}.\delta\mathbf{v}\,dV \tag{4}$$

Where V e S are the volume and surface in the current configuration, f^{B} are the body forces and f^{S} are the surface forces.

3. Constitutive Law

3.1 Rate-independent thermo-elastic-plastic constitutive law

Assuming which the total deformation rate can split on elastic, inelastic and thermal part as follow:

$$\mathbf{D} = \mathbf{D}^{e} + \mathbf{D}^{p} + \mathbf{D}^{t} \tag{5}$$

where the elastic portion of the constitutive relation is $\mathbf{D}^{e} = \mathbf{C}^{e^{-1}} : \boldsymbol{\sigma}^{\nabla}$ (6)

and $\boldsymbol{\sigma}^{\nabla} = \mathbf{C}^{e} : \left(\mathbf{D} - \mathbf{D}^{p} - \mathbf{D}^{t}\right)$ (7)

$\boldsymbol{C}^{e}$ is the elastic tensor relating the elastic part of the deformation rate with the Jaumann rate of the Cauchy Stress, σ^{∇}.

Plastic deformations are assumed to comply with normality rule, so that the plastic flow rule is given by, $\mathbf{D}^{p} = \dot{\lambda}\frac{\partial F}{\partial \boldsymbol{\sigma}}$ (8)

The Cauchy Stress can be divided in deviatoric and hydrostatic part,

$$\boldsymbol{\sigma} = \mathbf{S} - p\mathbf{I} \tag{9}$$

$$\text{where} \quad p = -\frac{1}{3}\text{tr}(\boldsymbol{\sigma}) = -\frac{1}{3}\left(\sigma_{11} + \sigma_{22} + \sigma_{33}\right) \tag{10}$$

For the Von Mises theory the equivalent stress is $q = \sqrt{\frac{3}{2}\mathbf{S} : \mathbf{S}}$ (11)

This expression depends on the plastic potential F, and for the associative case is $f = F$,

$$\text{where} \quad f = q - S_{y} = 0 \tag{12}$$

is the yielding function and S_{y} the yielding stress.

3.2 Rate dependent constitutive law

Polymers exhibit time-dependent deformation even at room temperature and is rapidly accelerated by small increases in temperature. The time effects are studied under viscoelasticity and viscoplasticity areas. In the viscoelasticity response the elastic portion is recoverable (instantaneous) and the viscous portion is non-recoverable and occurs during the time[3]. The main difference between an elastic-plastic constitutive law, showed in the previous section, and the viscoplasticity is the time. For the elastic-plasticity phenomenon time doesn't have a physical meaning, however, for the viscoplasticity the constitutive law is based on real time.

3.2.1 Visco-Plasticity Constitutive Law

Creep is a phenomenon of strain accumulation through a span of time due a constant applied stress. The creep strain rate can be writing as a function of stress, strain, temperature, and time[3].

$$\dot{\varepsilon}_{cr} = f_1(\sigma) f_2(\varepsilon) f_3(t) f_4(T) \tag{13}$$

Creep and visco-plasticity are the same phenomena from a material standpoint. In engineering usage, creep is generally used to describe a thermally-activated process with a low strain rate. There are three stages of creep[4] for uniaxial test at constant stress and temperature. In the primary stage, the strain rate decreases with the time. This event occurs over a short period. The secondary stage has a nearly constant strain rate associated and in the tertiary stage, the strain rate increases rapidly until failure. For engineering analysis, the primary and secondary stages are usually of greatest interest. The strain rate associated with primary creep is usually much greater and is a shorter period than those associated with secondary creep. Tertiary creep is usually associated with the onset of failure (necking, damage) and is not modeled in ABAQUS[5].

The additive strain decomposition when calculating elastic, plastic, and creep strain is:

$$\dot{\varepsilon} = \dot{\varepsilon}_{el} + \dot{\varepsilon}_{pl} + \dot{\varepsilon}_{cr} \tag{14}$$

The elastic, creep, and plastic strains are all evaluated on the current stress state, but they are calculated independently. Assuming isotropic behavior, the von Mises equation is used to compute the effective stress, and the equivalent strain is used in the creep strain rate equation.

Creep, like plasticity, is an irreversible strain which is based on deviatoric behavior. Thus, the material is assumed to be incompressible under creep flow. Hence, creep does not require a higher stress value for more creep strain to occur. Creep strains are assumed to develop at all non-zero stress values. Creep models use an equivalent deviatoric stress q to define the inelastic deformation that occurs in the material.

The time hardening power law is the simple creep law, but it is valid only at lower magnitudes of stress. This is most suitable when the state of stress in the material remains essentially constant and it assumes that the creep strain rate depends only upon the time from the beginning of the creep process.

The time hardening power law has the following form[5]:

$$\dot{\bar{\varepsilon}}^{cr} = A.q^n.t^m \tag{15}$$

where $\dot{\bar{\varepsilon}}^{cr}$ is the uniaxial equivalent creep strain rate; q is the equivalent deviatoric stress; *t* is the total time (versus step time); and *A*, *n*, *m* are user-defined parameters.

Visco-plastic Constitutive Model

1. Elasto-plastic split strain rate

$\mathbf{D} = \mathbf{D}^e + \mathbf{D}^{cr}$

2. Stress Rate

$\boldsymbol{\sigma}^{\nabla} = \mathbf{C}^e : \mathbf{D}^e = \mathbf{C}^e : (\mathbf{D} - \mathbf{D}^{cr})$

3. Plastic flow Rule

4. Yielding Function

$f = q - S_y \qquad q = \sqrt{\frac{3}{2}\mathbf{S} : \mathbf{S}}$

5. Loading and Unloading Condition

$f \geq 0$

6. Plastic Multiplier

$\dot{\lambda} = A.q^m.t^n$

$\mathbf{D}^{cr} = \dot{\lambda}\frac{\partial F}{\partial \boldsymbol{\sigma}}$	*7. Stress Rate* $\boldsymbol{\sigma}^{\nabla} = \hat{C}^{ep} : \mathbf{D}$

Box 1: Viscoplastic Constitutive Law[6]

3.2.2 Relaxation

Stress relaxation test measures the stress (force) response as function of time while strain (displacement) is held constant on the specimen.

Isotropic linear visco-elasticity is described by the rate dependence of deviatoric,

$$S(t) = S_0(t) + \frac{1}{G_0}\int_0^{\infty} \frac{dG(\tau)}{d\tau} . S_0(t-\tau)d\tau \tag{16}$$

Abaqus uses the relaxation modulus $G(\tau)$ expanded in a Prony series:

$$G(\tau) = G_0\left(1 - \sum_{i=1}^{N} \bar{g}_i^p . \left(1 - e^{-\tau / \tau_i^G}\right)\right) \tag{17}$$

4. Numerical Simulations

4.1 Validation – Creep

This sub-section shows the results obtained by numerical simulation considering the effect of creep in a block with dimensions: 14,7x10x4 mm. The block is constituted in Zytel® 73G30 NC010, which is a 30% glass fiber reinforced polyamide 6. Figure 1 shows the evolution of the creep modulus with the time and stress.

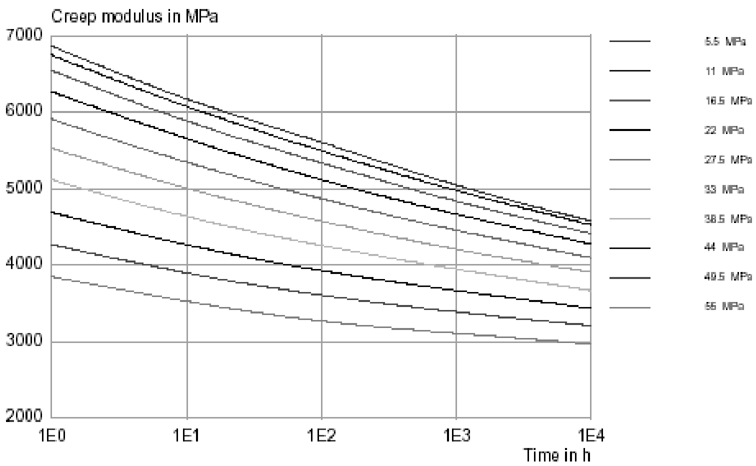


Figure 1: Creep Modulus – Zytel® 73G30 NC010

From eq. 15,

$\dot{\varepsilon}^{cr} = A.\sigma^m.t^n$

$\dot{\varepsilon}^{cr}$ = creep strain rate

σ = Cauchy stress (true)

A, n, m are material constants $$\bar{\varepsilon}^{cr} = \frac{A}{m+1}\sigma^n t^{(m+1)} \qquad (18)$$

From the graphic showed on figure 1 the parameters can be obtained,

$$n = \mathbf{ln}\left(\varepsilon^{cr1} / \varepsilon^{cr2} \right) / \mathbf{ln}\left(\sigma_1 / \sigma_2 \right) \quad \text{constant time} \qquad (19)$$

$$m = \left[\mathbf{ln}\left(\varepsilon^{cr1} / \varepsilon^{cr2} \right) / \mathbf{ln}\left(t_1 / t_2 \right) \right] - 1 \quad \text{constant stress} \qquad (20)$$

$$A = (m+1)\varepsilon^{cr1} / \left(\sigma^n t^{(m+1)} \right) \qquad (21)$$

From the curves at 5 MPa and 22 MPa (chosen due to proximity to the interesting region), the parameters were determined:

A = 6,53 e -6
m = 1,0577
n = -0,947

In figure 2 is showed the finite element model of the structure. The analysis was divided in 2 steps: loadstep 1: the block is submitted to traction of 5.5 MPa , loadstep 2: analysis of creep during 10^4 hours.

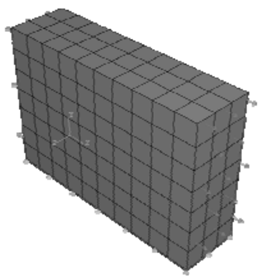


Figure 2: Model

The figures 3,4 e5 show the results of creep analysis. The phenomenon of creep considers the maintenance of the constant tension (figure 3) while the creep strain (figure 6) increases with the time.

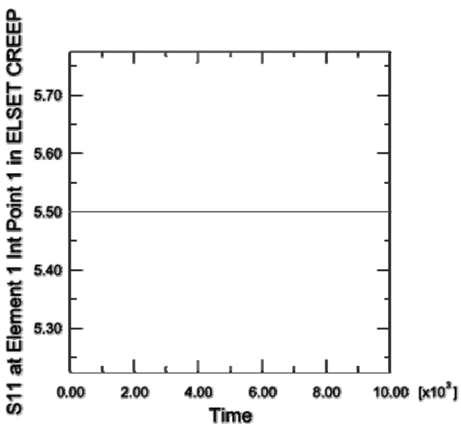


Figure 3. Stress (MPa)

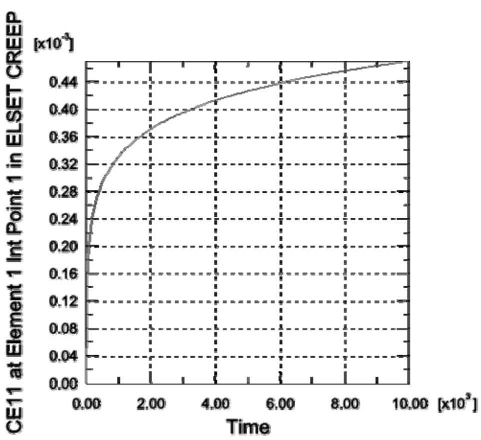


Figura 4. Creep Strain

The creep modulus resulted of the experiments achieved by Dupont Laboratories to obtain the constitutive law of Zytel® 73G30 NC010 is plotted in figure 7 versus the results of the creep analysis performed in Abaqus.

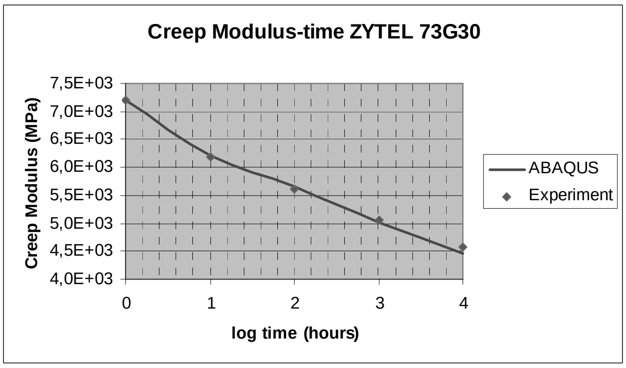


Figure 5. Experimental x Numerical

4.2 Validation – Relaxation

Some tests of long-term stress relaxation was performed by the DuPont Corporate Center for Analytical Sciences (CCAS) to determine the stress decay up to 10 years ($10^{4,94} \cong 10^{5}$ hours) of the plastic Hytrel® 5526, a thermoplastic polyester elastomer produced by Dupont.

The simulation reproduce the boundary conditions and applied at constant strain of 3,6%. The model has the same dimensions as that the section 4.1. The Poisson modulus is 0,5. The analysis was divided in two steps. On the first one was solved a static analysis of the displacement necessary to cause the strain. On the next step is solved the relaxation analysis. The terms of Prony series are defined by a rule of thumb. Typically is recommended that number of Prony series as decades of time data. To perform this analysis was necessary six Prony terms (10E5 hours).

The phenomenon of relaxation considers the maintenance of the constant deformation (figure 6) while the tensions go decaying in the time (figure 7).

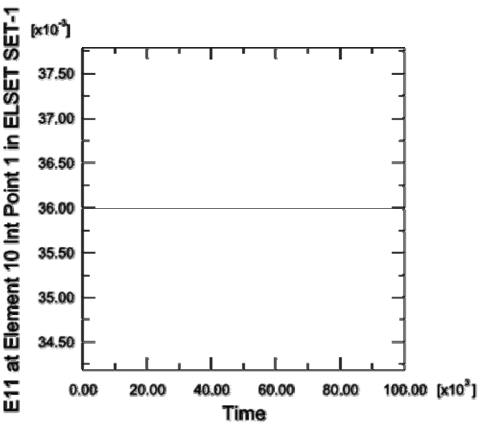


Figure 6. Total Strain

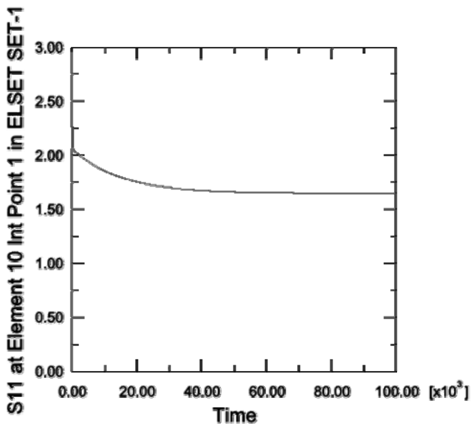


Figure 7. Stress (MPa)

The relaxation modulus resulted from the experiments achieved by CCAS laboratories to obtain the constitutive law of Hytrel® 5526 is plotted in figure 12 versus the results of the relaxation analysis performed in ABAQUS.

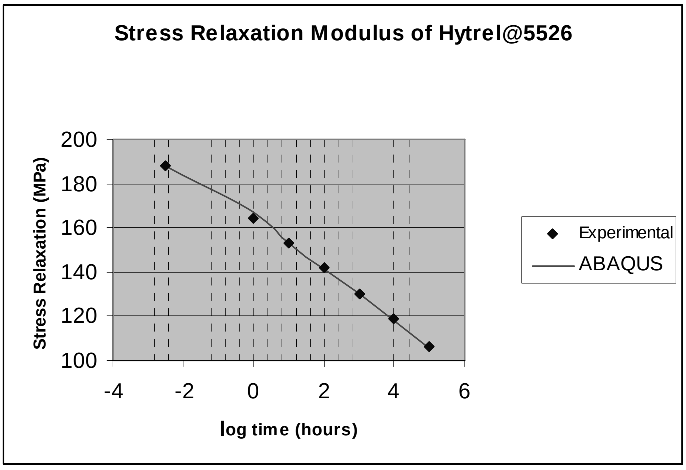


Figure 8: Experimental x Numerical

4.3 Application - Pressure Vessel

A plastic pressure vessel manufactured with Zytel® 73G30 NC010 (30% glass fiber reinforced polyamide 6) is submitted to an internal pressure of 1 MPa and 60°C. The dimensions of the vessel are showed below[10]. Once the problem presents geometry and loading symmetry, was utilized an axissymmetric finite element model.

Mechanical properties
Material = Zytel® 73G30 NC010 (PA66+30% glass fiber)
Tensile Modulus = 3.873 MPa
Poisson = 0,35

Loads
Internal Pressure = 1 MPa
Temperature = 60 °C

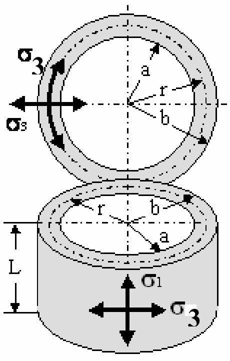
Dimensions
Length = 100 mm
Inside radius = 40 mm
Wall thickness = 5 mm

It was request the strain after a period of 5 years. This analysis was made through 3 loadsteps:

1. Static Analysis
2. Coupled analysis
3. Creep Analysis.

4.3.1 Thermo-mechanical results

The results of static analysis were compared with the analytical response given by Timoshenko[7]. The circumferential stress formulae for a pressure vessel with open ends is shown in figure 9:



$$\sigma_3 = \frac{p.a^2.\left(b^2 + r^2\right)}{r^2.\left(b^2 - a^2\right)} \qquad (\text{MPa})$$

Figure 9: Analytical Model

The result of static analysis is showed in figure 10. The resulted stress is 7,8 MPa. Once the analytical response is 8,0 MPa, and considering that they have geometric differences, the results agree with precision. The result of coupled analysis is represented in figure 11 and shows a stress of 22,2 MPa.

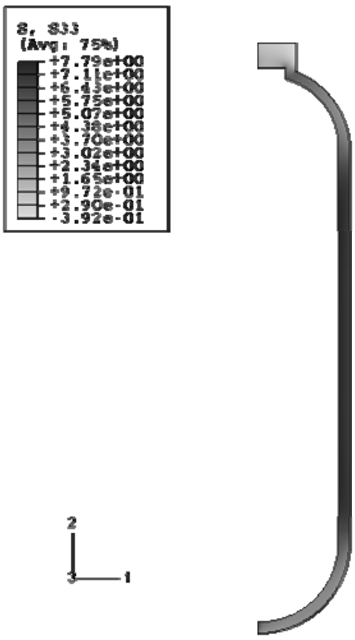


Figure 10. Results of Static Analysis

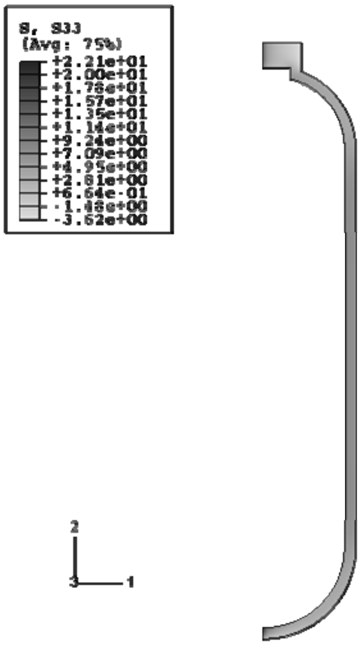


Figure 11. Results Coupled Analysis

4.3.2 Results of Creep

The creep results show that there is consistency of results. The creep strain increase with the time (figure 12), while the stresses maintain approximately constant (figure 13). Note that Abaqus doesn't solve a creep problem from a coupled displacement-temperature analysis[5].

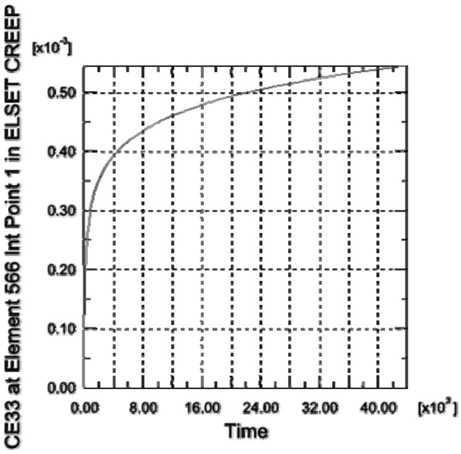


Figure 12. Creep Strain

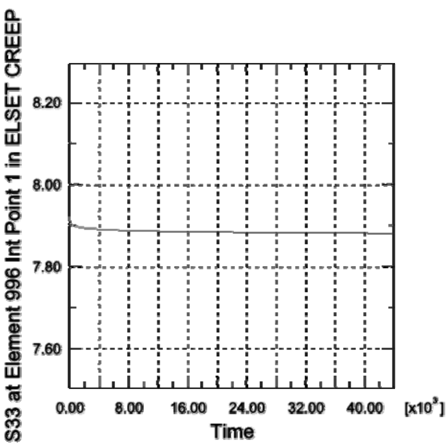
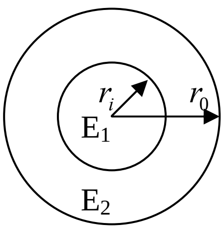


Figure 13. Stress

4.3.3 Leak Analysis

On this simulation it was applied an intern pressure of 5 BAR in order to verify if would occur leak at the edge of the vessel where an end cap part is built by interference. The analysis was shared into 3 steps: an interference of 0,1 mm (δ), pressure of 5 BAR and creep during 50.000 hours. It was determined a first aproximation for the contact pressure value at the edge between the inside diameter of the vessel and the outside diameter of the end cap by using Timoshenko equation:



$$p = \frac{\delta}{\frac{r}{E_2}\left[\frac{r_0^2 - r^2}{r_0^2 - r^2} + \nu_2\right] + \frac{r}{E_1}\left[\frac{r^2 + r_i^2}{r^2 - r_i^2} - \nu_2\right]}$$

$$r = \frac{r_i + r_0}{2}$$

The analitic result is 6.23MPa. In the simulations contact pressures around 9MPa appeared. The difference exists due to modeling aproximation. The tensions that appears because of the form of the roller body do not allow a perfect assembly between the end cap and the vessel. The tension and pressure contact results are presented for the three steps.

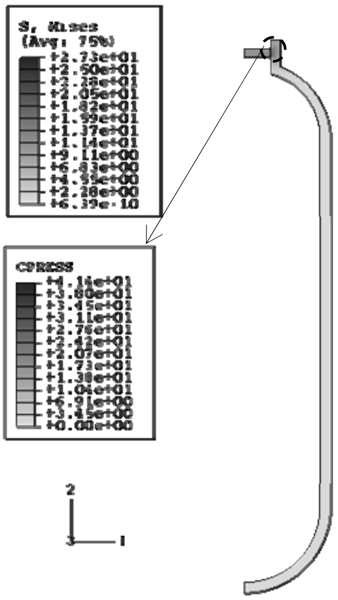


Figure 14. Results Step1

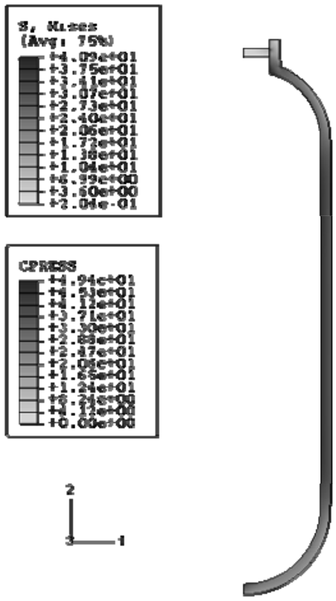


Figure 15. Results Step2

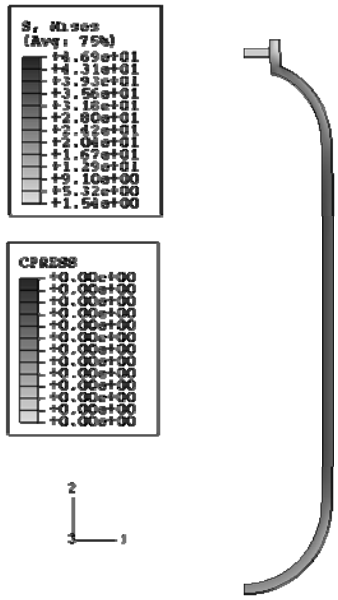


Figure 16. Results Step3

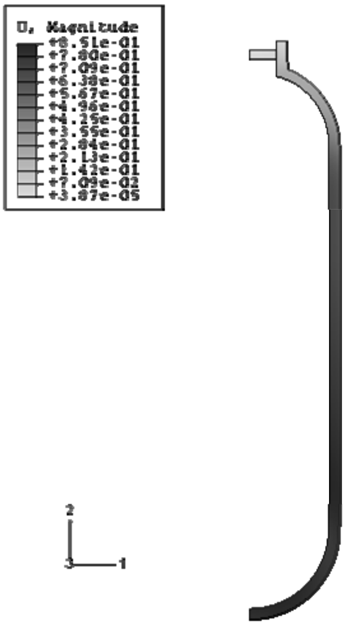
It can be observed on the figure 16 that after the Creep analysis occurs a gap on the vessel (Cpress=0).

4.3.4 Comparative results between Zytel® 73G30 vessel and Zytel® HTN53G50

It was carried out a comparative study between Zytel® 73G30 (Poliamide 66 reinforced with 30% glass fibers) and Zytel® HTN53G50 (Super poliamide reinforced with 50% glass fibers) to verify the difference of the effect of the phenomenon of creep between the materials. The analyzed results are the displacements of node 573.

4.3.4.1 Zytel® 73G30

Below it was showed the displacement results whose vessel is submited after the application of 5 BAR during 5 years.



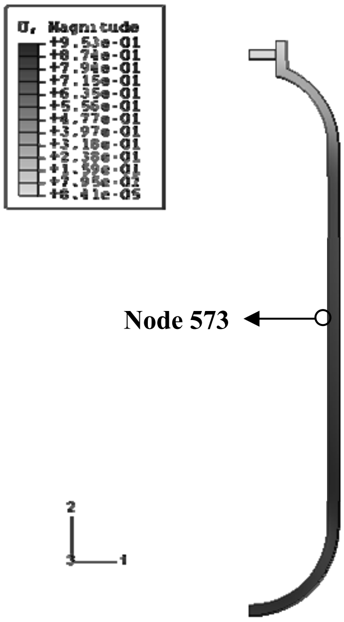


Figure 17. Displacement Step 2 Figure 18. Displacement Step 3

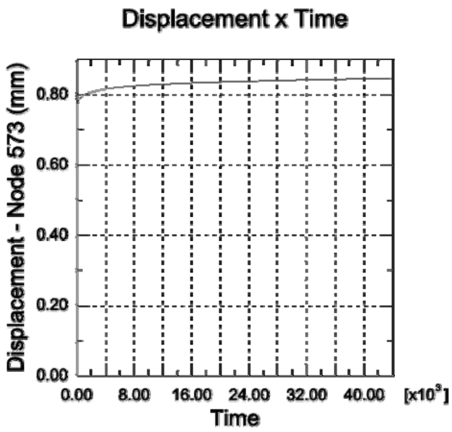


Figure 19. Displament of Node 573 during the creep step

The displacement of the node 573 during the period of 5 years is 0,1087mm.

4.3.4.2 Zytel® HTN53G50 Results

Below it´s showed the displacement results whose vessel is submitted after the application of 5 BAR during 5 years.

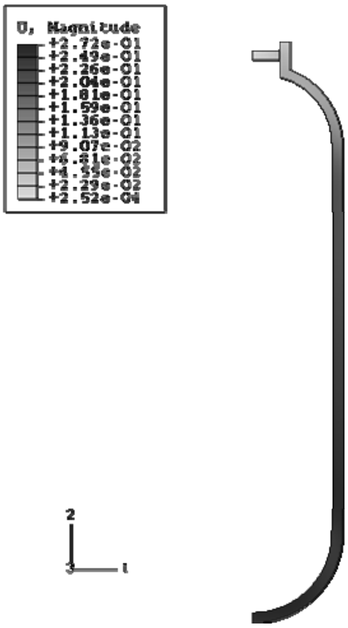


Figure 20. Displacement Step 2

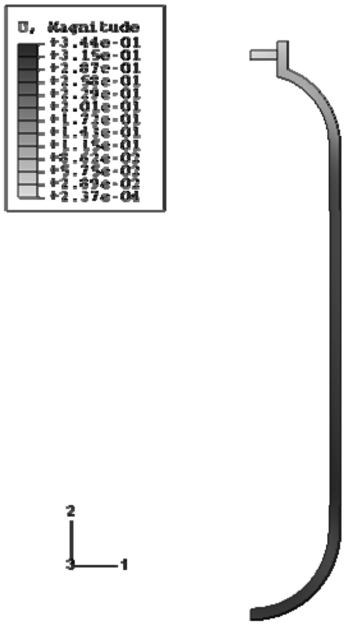


Figure 21. Displacement Step 3

The displacement results have shown a better rigidity of the PPA material (Zytel® HTN53G50). The displacement of the node 573 during the period of 5 years is 0,0792 mm.

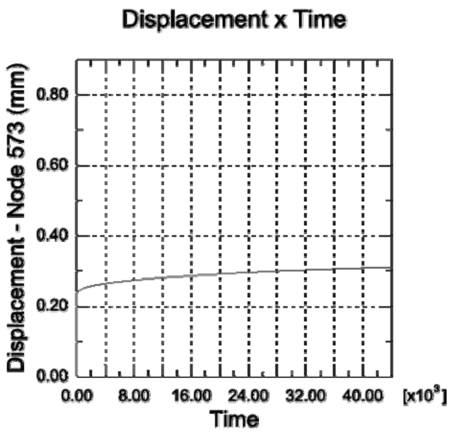


Figure 22. Displament of Node 573 during the creep step

5. Conclusions

This work made a synthesis of the main relative aspects of the nonlinear analysis of plastic structures. It was presented the basic formulation of balance and the constitutive equations for elastic-plasticity problems and rate dependent. Numerical simulations by ABAQUS had been carried out and were compared with the experimental constitutive laws. The results demonstrate that the transposition of the experimental curves for software had been carried out with precision. This paper proves the high performance of ABAQUS solving structural problems involving rate independent and rate dependent constitutive law of plastic materials.

6. References

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