Comprehensive Multiaxial Fatigue Analysis with ABAQUS

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Abstract: Many engine components in large medium-speed diesel engines are subject to cyclic loading. In many cases constant amplitude loading in the high-cycle range is assumed, hence it is customary to design for an infinite lifetime. Many engine components have a complex geometry, and that is why, particularly in locations of stress concentrations, a multiaxial stress state will prevail. It will be of utmost importance, already in the design stage, to make a rigorous fatigue damage evaluation on the results from the finite element analysis. For this purpose an in-house post-processing program has been written in the C++ programming language. This program uses the ODB API interface and the functionality of the ABAQUS/VIEWER. Three different multiaxial fatigue criteria can be used for fatigue damage evaluation: the Findley criterion, the Dang Van criterion and the pseudo criterion by von Mises. Great effort has been put into speeding up the complicated Findley algorithm, and remarkable performance improvements have been made by implementing the down-hill simplex method for finding the critical plane. For the calculation of the safety factor, proper statistical treatment of the results from fatigue testing is done. The fatigue limit is corrected to match the population value with a certain confidence and allowed probability of failure. The size effect is taken into account by calculating a statistical size factor according to the theory of the weakest link. The iteration of the safety factor in the critical point is done either according to the multiaxial fatigue criterion or by translating the damage into an equivalent uniaxial stress case.

Keywords: Fatigue, Multiaxial fatigue, Fatigue damage and Statistical size factor

1. Introduction

Modern medium-speed diesel engines, like the one in Figure 1, contain a large quantity of components that are subject to cyclic loading. Many of these components have a complex geometry, which explains why a three dimensional stress state will prevail throughout the loading cycle. Since the cycle quantity, in many cases, runs into the billions, a rigorous multiaxial fatigue analysis on the results of the Finite Element Analysis (FEA) is required to ensure that any design will be safe and perform well.

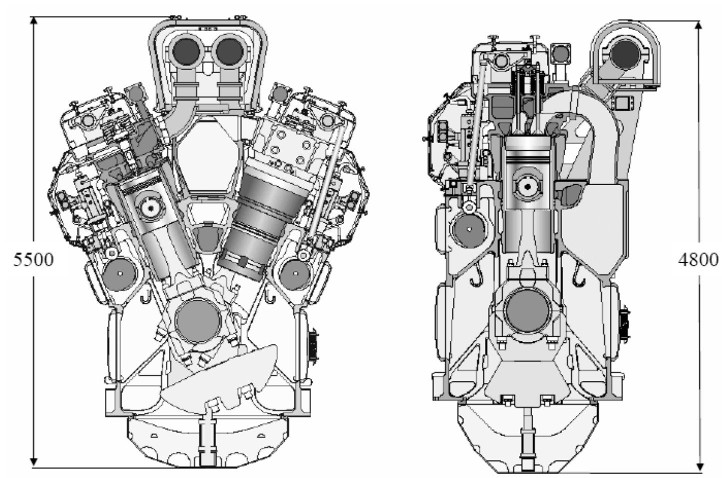

Figure 1. Type W46 V and in-line medium speed diesel engines.

Sophisticated multiaxial fatigue criteria, e.g. the Findley criterion, have existed for a long time and have been proven to predict fatigue crack nucleation for some metals with satisfying accuracy. Fatigue damage can be estimated on components based on FEA results, but a great challenge remains in how to map the simulation results against test data, i.e. a correct fatigue analysis requires the evaluation of the size effect and the proper statistical treatment of test data before the fatigue limit can be corrected to match component values. A sample fatigue limit needs to be corrected to match the population value for a given confidence level and allowed probability of failure. Furthermore, an evaluation of the size effect should be conducted by calculating the statistical size factor from the effective stress area based on the surface stress distribution of the component, because experience has shown that a fatigue crack will usually nucleate on the surface. The fatigue limit can then finally be corrected with the statistical size factor, surface roughness factor and life factor before the safety factor is evaluated at the critical point of the design.

The sophisticated methods and vast quantity of input needed to perform consistent fatigue analysis resulted in the decision, at the authors' company, to put effort into developing automated routines for I/O to the output database through the scripting interface in ABAQUS. The first software version was developed a few years ago and incorporates a main routine written in Python language. The software is run as a custom application from a tailored GUI in the ABAQUS viewer. The earlier version of the software uses the Findley damage criterion and the effective stress area has been evaluated with the help of membrane elements.

Even though this software works, it was seen that some performance improvements and increased functionality were needed. Therefore new software has been developed, for fatigue analysis according to Findley, Dang Van or pseudo criterion by von Mises. Great effort has been put into speeding up the Findley algorithm and remarkable performance improvements have been made by implementing the down-hill simplex method for finding the critical plane. Also the Dang Van algorithm has been implemented, by using a routine that calculates the smallest hyper sphere enclosing the stress history (Fischer, 2003). For effective stress area calculations, the program uses shape functions for face area calculations, rather than membrane elements. The main routine has been written in the C++ language and has been compiled as a stand-alone executable for both windows 32-bit and unix 64-bit platforms. The executable uses an ASCII-file as input-data. The input-file can be created in the ABAQUS/Viewer from a new custom GUI that has been written in Python language, and can be submitted as a 'job' to the executable. Alternatively, the input-file can be edited by hand and submitted to the executable directly from the command prompt.

2. Theory

The application of statistical methods forms the basis for probability-based fatigue analysis. The sample fatigue limit and sample standard deviation obtained from testing has to be corrected to match a population value with a certain confidence. The effective stress area has to be calculated according to the surface stress distribution of the component before the statistical size factor can be evaluated at the critical point. The critical point has to be found by applying the appropriate multiaxial fatigue criteria to the stress history of every calculation point. The safety factor should be calculated by applying the proper correction factors and according to the damage criterion, or by transforming the damage into an equivalent uniaxial stress case. Finally, the safety factor at the critical point should be compared with the required safety factor corresponding to the allowed probability of failure.

2.1 Mean fatigue limit and standard deviation

The statistical treatment of test data forms the starting point of the fatigue analysis. The fatigue limit and its standard deviation can be obtained from tabled values, e.g. with the help of the fatigue ratio, or else testing has to be conducted. If staircase testing is performed, reliable sample values can only be obtained with a sufficient quantity of test specimens and using stress increments close to the value of the sample standard deviation (Rabb, 1999).

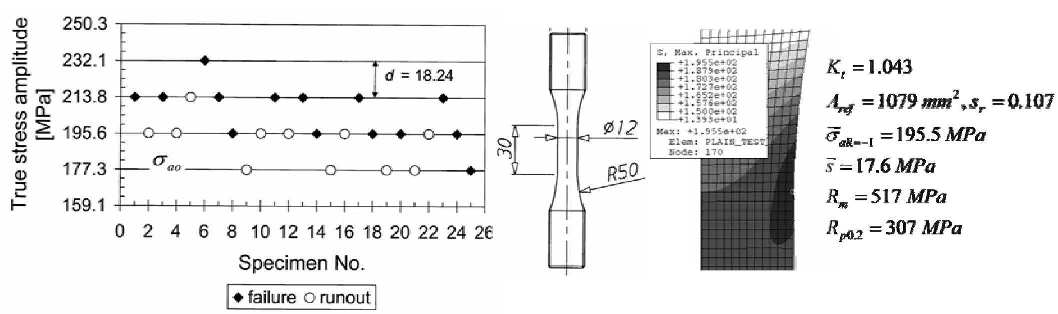

Figure 2. Staircase test on nodular cast iron grade 500-7/ISO 1083.

Based on the results of a staircase test, e.g. according to Figure 2, the values for the sample fatigue limit and sample standard deviation can be obtained by maximizing the following equation (Dixon et al., 1948):

$$P(n,m\mid\sigma_{a0}) = K\prod_{-\infty}^{\infty} p_i^{n_i} q_i^{m_1} \quad \text{where} \quad p_i = \frac{1}{s\sqrt{2\pi}}\int_{-\infty}^{\sigma_{ai}} e^{-\frac{(x-\overline{\sigma}_a)^2}{2s^2}}dx \quad \text{and} \quad q_i = 1 - p_i\ , \qquad (1)$$

where $\overline{\sigma}_a$ is the sample fatigue limit, s is the sample standard deviation, K is a constant neither dependent on the mean, nor on the standard deviation, n_i is failures on each level, m_i is runouts on each level, p_i is probability of failure on each level, q_i is probability of survival on each level and σ_{ai} is the stress amplitude.

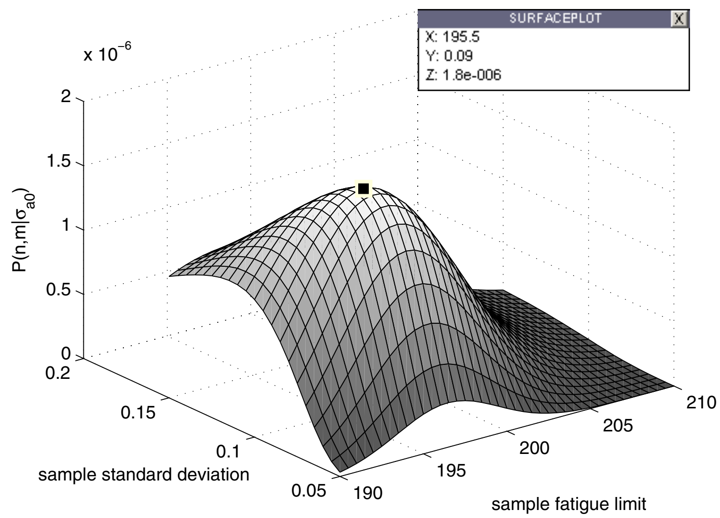

Figure 3. Maximum value according to the Dixon et al. algorithm.

As can be seen in Figure 3, the maximum likelihood is represented by the sample values $\overline{\sigma}_a = 195.5\ MPa$ and $\overline{s}_r = 0.09$. The fatigue limit obtained from the staircase test represents only a fraction of the population, which is why an appropriate confidence level needs to be applied on the sample value (Rabb, 2003). Theoretically, the population mean follows the Student's t distribution. For a given confidence level, the population mean can be evaluated according to Equation 2, when the upper integration limit t_2 is iterated from the probability integral:

$$\sigma_{aC\mu} = \overline{\sigma}_a - t_2\frac{s}{\sqrt{n}} \qquad \text{and} \qquad C_\mu = \int_{t_1\to\infty}^{t_2} f_{n-1}(t)dt\ , \qquad (2)$$

where C_μ is the applied confidence limit, $f_n(t)$ is the Student's t density function and n is number of specimen in the staircase test.

Likewise, confidence needs to be applied to the sample standard deviation. Theoretically, the sample value of the standard deviation is characterized by the chi-square distribution. For a given confidence level, the population standard deviation can be evaluated from Equation 3, when the lower integration limit h_1 is iterated from the probability integral:

$$s_{C\sigma} = \sqrt{\frac{(n-1)}{h_1}} \cdot s \qquad \text{and} \qquad C_\sigma = \int_{h_1}^{h_2 \to \infty} f_{n-1}(h)dh\,, \tag{3}$$

where C_σ is the applied confidence limit and $f_n(h)$ is the chi-square density function.

2.2 Statistical size factor

The statistical size effect is characterized by the scatter of defect sizes in the microstructure of the component material and takes expression in the form of a higher observed fatigue strength in notches. Engine components of medium-speed engines are typically large in size, see Figure 1, which explains why one has to rely on the fatigue testing of smaller specimens. It is therefore of the utmost importance to be able to transform the fatigue strength of the reference specimen to the fatigue strength of the component, e.g. as illustrated in Figure 4, in a reliable manner using the statistical size factor.

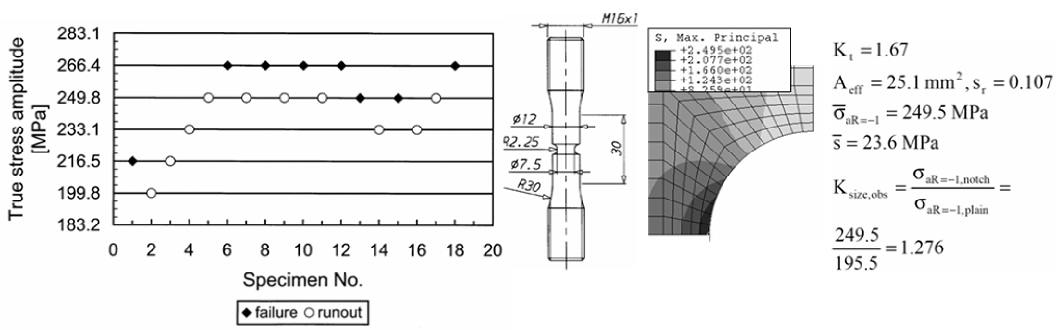

Figure 4. Staircase test on notched specimens of grade 500-7 with R=-1.

It has been observed that fatigue cracks usually nucleate on the surface of a specimen. Therefore the scatter in the fatigue limit is mainly caused by the scatter of the maximum defect size on the surface of the specimen. One can expect that when the surface area is increased, the likelihood of a larger defect increases and the fatigue limit decreases. In order to be able to calculate the statistical size factor of the component, one needs to make an estimate on this scatter which is reflected in the standard deviation of the fatigue limit. Therefore, by implementing the theory of the weakest link, where number of links is equal to the ratio between the effective stress areas of the component and the reference specimen, it will be possible to calculate the statistical size factor.

The effective stress area can be calculated based on the surface stress distribution in the vicinity of the notch. Then for a discrete distribution, for every sub element in a descending order with respect to the stress amplitude $\sigma_{a,i}$, an effective stress area $A_{eff,i}$ can be calculated by reducing the size of the original area A_i to a value corresponding to the survival probability P_s of the critical point:

$$A_{eff} = \sum_{i=1}^{n} \frac{\lg(P_s(\sigma_{a,i}))}{\lg(0.5)} \cdot A_i \quad \text{where} \quad P_s(\sigma_{a,i}) = \frac{1}{\sqrt{2\pi}} \int_{\lambda_i}^{\infty} e^{\frac{x^2}{2}} dx \quad \text{and} \quad \lambda_i = \frac{1}{s_r} \left(\frac{\sigma_{a,i}}{\sigma_{a,\max}} - 1 \right), \tag{4}$$

where $\sigma_{a,\max}$ is assumed to correspond to a survival probability of 0.5 and λ_i is the normalized variable at a survival probability of $P_{s,i}$

The number of links can be calculated from the ratio of the effective stress areas:

$$n = A_{eff} \,/\, A_{ref} \quad \text{when} \quad A_{eff} > A_{ref} \quad \text{or} \quad n = A_{ref} \,/\, A_{eff} \quad \text{when} \quad A_{eff} < A_{ref} \tag{5}$$

where A_{eff} is the effective stress area of the component and A_{ref} is the effective stress area of the reference specimen.

To obtain the expectation value of the fatigue limit of the larger stress area the fatigue limit of the smaller area has to be reduced to the following survival probability:

$$P_s = \sqrt[n]{0.5} = \frac{1}{\sqrt{2\pi}} \int_{\lambda}^{\infty} e^{-\frac{x^2}{2}} dx \tag{6}$$

For notches with extreme stress gradients the number of links can be very high and the value of the normalized variable λ very low. Accordingly, the log normal distribution with base b will give a better approximation on the statistical size factor than the normal distribution. By using the quantity of logarithmic standard deviations as the correction factor and for small values on λ :

$$\lg_b \sigma_{a,plain} = \lg_b \sigma_{a,notch} + \lambda s_{\lg_b} \Rightarrow K_{size} = b^{-\lambda s_{\lg_b}} \quad \text{and} \quad s_{\lg_b} \approx -\lg_b(1 - s_{rC_\sigma}) \tag{7}$$

A conservative estimate of the statistical size factor will require a conservative estimate of the population standard deviation, meaning that the appropriate confidence level needs to be applied to the sample standard deviation. This is consistent with applying a confidence limit in the upper range, e.g. C =90%, on the sample standard deviation if the component has a larger effective stress area than the reference specimen. Likewise, a confidence limit in the lower range, e.g. C =10% should be applied to the sample standard deviation, if the component has a smaller effective stress area than the reference specimen.

2.3 Haigh diagram

From the basic technical material data available in standards and on the fatigue limit from two stress states it is possible to construct a complete Haigh diagram.

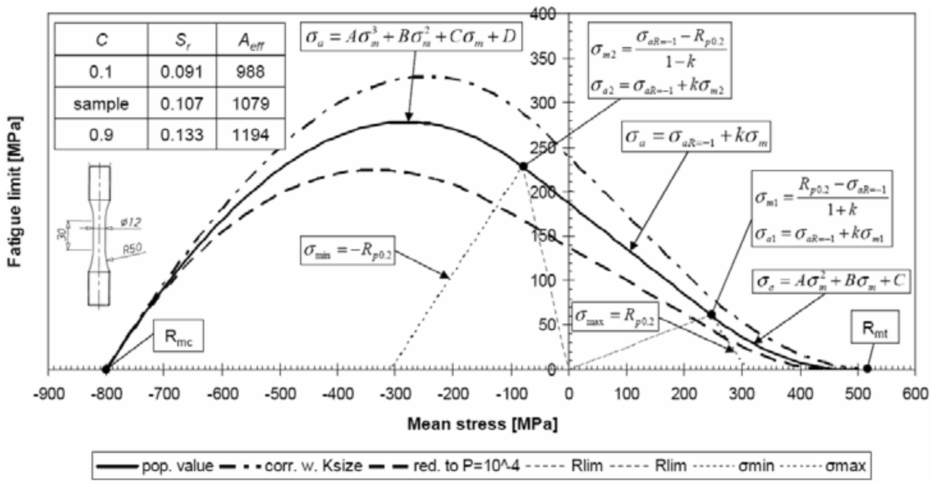

Figure 5. Haigh diagram for test specimen in Figure 2.

In Figure 5 for grade ISO 500-7/ISO 1083, the tested fatigue limit at fully reversed tension-compression loading $\sigma_{aR=-1} = 186.9\ MPa$ and the tested fatigue limit for pulsating loading $\sigma_{aR=0} = 123.7\ MPa$ have been used. In practice, there is no need to estimate the fatigue limit whenever the max stress is negative, fatigue cracks can nucleate in this stress state, but not grow. More tests need to be conducted for determining the fatigue limit in the plastic domain for positive mean stresses, since some uncertainty prevails about the shape of the Haigh diagram in this region. Until now, the fatigue limit for nodular cast irons in this domain has been constructed with a parabola that levels out at the nominal ultimate tensile strength, an approximation that needs confirmation from future fatigue tests.

In the same figure, the corrected Haigh diagrams have been constructed. The semi-dashed line corresponds to the base diagram that has been corrected with the statistical size factor $K_{size} = 1.276$ according to the effective stress area in Figure 4, the dashed line corresponds to the Haigh diagram that has been reduced from the base diagram to the allowed failure probability $P = 10^{-4}$ according to Equation 27. The correction should always be made in the direction of the amplitude.

2.4 Damage criteria

Various damage criteria for multiaxial fatigue analysis have been proposed over the years. In this work, only stress-based models have been used: the Findley criterion, the Dang Van criterion and the pseudo-criterion by von Mises. The Findley and Dang Van criteria are critical plane criteria, i.e. these are based on observations that fatigue cracks nucleate and grow on planes with a particular direction. The von Mises criterion is an effective stress criterion that uses the octahedral shear stress as a measure on fatigue damage.

2.4.1 Findley damage criterion

The Findley criterion has been used because by experience this criterion gives the best prediction on fatigue failure for steels, and a satisfying failure prediction for cast irons (Rabb, 2005). The Findley criterion considers the sum of the shear stress amplitude $\Delta\tau/2$ and a fraction of the normal stress σ_n on the critical plane as the damage parameter (Findley, 1959):

$$(\frac{\Delta\tau}{2} + k_f \sigma_n)_{\max} \leq f \ , \tag{8}$$

where k_f is a constant and f is the shear fatigue limit.

The shear fatigue limit f and the constant k_f can be evaluated when the fatigue strength at two uniaxial stress states are known, e.g. for fully reversed tension-compression loading $\sigma_{aR=-1}$ and for pulsating loading $\sigma_{aR=0}$:

$$\frac{\sigma_{aR=0}}{\sigma_{aR=-1}} = \frac{k + \sqrt{(1+k^2)}}{2k + \sqrt{(1+(2k)^2)}} \quad \text{and} \quad f = \frac{\sigma_{aR=-1}}{2}(k + \sqrt{1+k^2}) \tag{9}$$

The shear stress amplitude and the normal stress have to be evaluated by calculating the stress matrix in a rotated cartesian coordinate system. The transformation from the original to the new cartesian coordinate system can be done with spherical coordinates when the base vectors are calculated according to the following:

$$\begin{cases} \vec{n}_{x'} = \sin\varphi c \cos\theta\vec{i} + \sin\varphi\sin\theta\vec{j} + \cos\varphi\vec{k} \\ \vec{n}_{y'} = -\sin\theta\vec{i} + \cos\theta\vec{j} \\ \vec{n}_{z'} = -\cos\varphi\cos\theta\vec{i} - \cos\varphi\sin\theta\vec{j} + \sin\varphi\vec{k} \end{cases} \Rightarrow Q = \begin{bmatrix} \sin\varphi\cos\theta & -\sin\theta & -\cos\varphi\cos\theta \\ \sin\varphi\sin\theta & \cos\theta & -\cos\varphi\sin\theta \\ \cos\varphi & 0 & \sin\varphi \end{bmatrix} \tag{10}$$

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \Rightarrow [\sigma'] = Q^T[\sigma]Q \tag{11}$$

The most general type of loading is non proportional, meaning that the principal stress axis is allowed to rotate during the loading cycle. In practice, such a stress history very often needs to be described with more than two time steps. Every time step will correspond to one value of the shear stress resultant ($\tau_{a..c}$ in Figure 6) on the critical plane. The limits of the shear stress range on the critical plane can be determined by the minimum circumscribed circle or by the longest chord (see Figure 6). The normal stress σ_n in the damage parameter in Equation 8 is determined by the largest value found on the critical plane in the stress history.

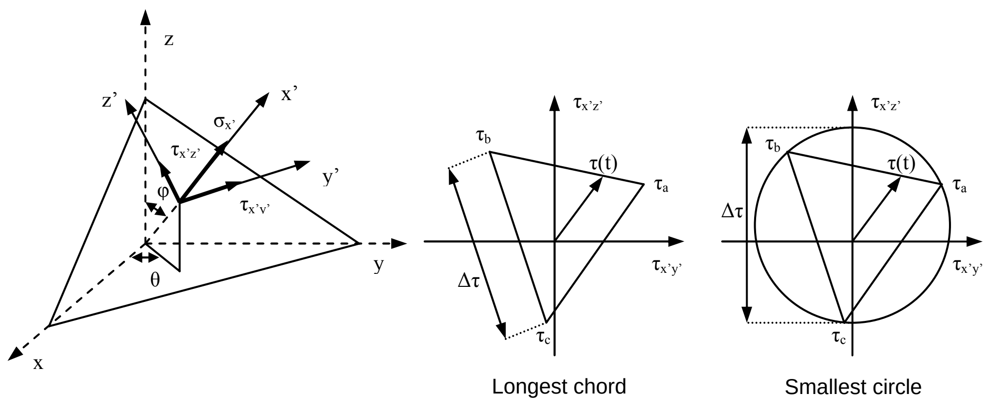

Figure 6. Stress components on an arbitrary plane and definition of the shear stress amplitude.

When the necessary correction factors are considered, the safety factor at the critical point can be evaluated according to the Findley criterion:

$$S_{F,crit} = \frac{K_N K_R f}{K_{size} D} = \frac{K_N K_R f}{K_{size}\left(\frac{\Delta\tau}{2} + k\sigma_n\right)} \qquad \text{when} \qquad A_{eff} \geq A_{ref} \tag{12}$$

$$S_{F,crit} = K_{size} K_N K_R / D \qquad \text{when} \qquad A_{ref} \leq A_{eff} \tag{13}$$

Note! For steel and cast iron, $K_N = 1$ at the surface. The staircase test is typically made on polished specimens. If the surface roughness of the component does not correspond to this finish, a conservative reduction factor K_R can be calculated from the ten point height of irregularities R_z of the profile (Gudehus et al., 1995):

$$K_R = 1 - 0.22(\lg R_z)^{0.64} \cdot \lg R_m + 0.45(\lg R_z)^{0.53} \tag{14}$$

The life factor K_N also needs to be used as a reduction factor if the number of cycles is in the giga-cycle range. It has been recommended to decrease the fatigue limit for steel and cast iron in the interior of the material by approximately 5 % per decade of cycles (Kaufmann et al., 2002):

$$K_N = 1 - \left[\lg N - \lg(2 \cdot 10^6)\right] \cdot 0.05 \qquad \text{when} \qquad 2 \cdot 10^6 \leq N \leq 10^{10} \tag{15}$$

In many cases it is preferable to transform the damage parameter into an equivalent uniaxial stress case (Rabb, 2005):

$$\sigma_{x,\max} = \sigma_n / \cos^2 \alpha \qquad \text{and} \qquad \sigma_{x,\min} = \sigma_{x,\max} - 2\Delta\tau / \sin(2\alpha) \tag{16}$$

$$\tan(\alpha) = \sqrt{\frac{\Delta\tau}{4k\sigma_n + \Delta\tau}} \qquad \text{when} \qquad 4k\sigma_n + \Delta\tau > 0 \tag{17}$$

When an equivalent uniaxial mean stress and stress amplitude has been calculated from the equations 16 and 17, the safety factor can be directly evaluated from the corresponding fatigue limit from the corrected Haigh diagram in Figure 5.

2.4.2 Dang Van damage criterion

The Dang Van criterion has been chosen because it allows a very fast calculation of the damage parameter on large stress result files. The Dang Van criterion is based on observations that fatigue crack nucleation takes place on the microscopic level as a consequence of slip band formation. The criterion involves the calculation of stresses on both the macroscopic and microscopic levels. A critical step in the model is the calculation of the microscopic stress tensor from the residual stress tensor. In the Dang Van criterion the sum of the instantaneous shear stress $\tau(t)$ and a fraction of the instantaneous hydrostatic stress $\sigma_h(t)$ are used for the damage parameter (Dang Van et al., 1989):

$$\tau(t) + a\sigma_h(t) \le b\ , \tag{18}$$

where a and b are constants.

The constants a and b can be determined when the fatigue strength at two stress states, e.g. for fully reversed tension-compression $\sigma_{aR=-1}$ and for pulsating loading $\sigma_{aR=0}$, are known:

$$a = \frac{3(\sigma_{aR=-1} - \sigma_{aR=0})}{2(2\sigma_{aR=0} - \sigma_{aR=-1})} \qquad \text{and} \qquad b = \frac{\sigma_{aR=-1}\sigma_{aR=0}}{2(2\sigma_{aR=0} - \sigma_{aR=-1})} \tag{19}$$

The microscopic stress tensor is calculated from the macroscopic stress tensor $\sum_{ij}(t)$ and the deviatoric part of the stabilized residual stress tensor $dev\rho_{ij}$:

$$\sigma_{ij}(t) = \sum_{ij}(t) + dev\rho_{ij} \tag{20}$$

The deviatoric part of the stabilized residual stress tensor can be calculated when the hydrostatic part is known:

$$\rho_h = \frac{\rho_{11} + \rho_{22} + \rho_{33}}{3} \tag{21}$$

$$\rho_{ij} = \rho_{ij,h} + dev\rho_{ij} = \begin{bmatrix} \rho_h & 0 & 0 \\ 0 & \rho_h & 0 \\ 0 & 0 & \rho_h \end{bmatrix} + \begin{bmatrix} \rho_{11} - \rho_h & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} - \rho_h & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} - \rho_h \end{bmatrix} \tag{22}$$

In the general case, for non proportional loading, the stabilized residual stress tensor ρ_{ij} is the centre point of a six- or nine-dimensional hyper sphere enclosing the whole loading path describing the process of elastic shakedown (Socie et al., 2002). Experience has shown that for two equal stress histories, the centre point of a six-dimensional hyper sphere might deviate from the centre point of a nine-dimensional hyper sphere, which explains why some care has to be taken when choosing whether the symmetric stress components are to be included in the stress history or not.

The instantaneous shear stress to use in the damage criterion is the maximum shear stress found in the stress history. The instantaneous shear stress can be calculated from the microscopic principal stresses according to the Tresca maximum shear stress theory:

$$\tau_a(t) = \frac{\sigma_I(t) - \sigma_{III}(t)}{2} \tag{23}$$

The instantaneous microscopic hydrostatic stress used in the damage criterion is determined by the maximum value of the hydrostatic stress found in the stress history of the microscopic or macroscopic stress tensor:

$$\sigma_h(t) = \frac{\sigma_{11}(t) + \sigma_{22}(t) + \sigma_{33}(t)}{3} \tag{24}$$

The safety factor can be calculated in the same manner as for the Findley criterion according to Equation 12 and Equation 13. Alternatively, the Dang Van damage can be transformed into an equivalent uniaxial stress case, the safety factor can be calculated from the equivalent stress amplitude and the fatigue limit can be calculated from the corrected Haigh diagram in Figure 5 (Rabb, 2005):

$$\sigma_{\max} = 3\sigma_h(t) \quad \text{and} \quad \sigma_{\min} = \sigma_{\max} - 4\tau_a(t) \tag{25}$$

Unfortunately, the Dang Van criterion fails to predict fatigue crack nucleation for cast irons (Rabb, 2005), and has some major drawbacks in comparison to the Findley criterion. The local normal stress acting on cracks in transgranular slip bands has proven to have a considerable effect on fatigue crack nucleation. This term is evaluated and used in the Findley criterion as a part of the damage parameter. In the Dang Van criterion, the hydrostatic stress is considered instead; this term is easier to calculate but cannot quantify the damage with the same accuracy as the normal stress (Fouvry, 1996). Furthermore, the critical plane, according to Dang Van’s criterion, will always bisect the principal stress axis at 45-degree angles, which is not the case in the Findley criterion, where the direction of the critical plane explains the anisotropy in the fatigue limit.

2.4.3 Signed von Mises pseudo criterion

The signed von Mises pseudo criterion has been used to avoid singularities when $\alpha \to 90^{\circ}$ in the equivalent uniaxial stress case of the Findley criterion and because comparability to older methods is needed. In this criterion, an equivalent stress amplitude and mean stress is calculated for every stress component before the von Mises stress is calculated.

2.5 Failure probability and safety factor

The defects in the microstructure in a metal are randomly distributed, which is the reason why the fatigue strength of an engine component should be treated as a stochastic variable. By good approximation, the fatigue strength can be described by the normal or log normal distribution.

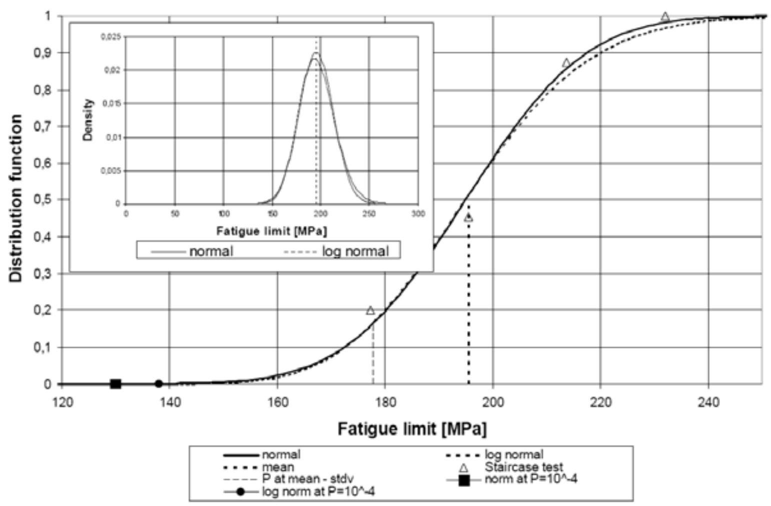

Figure 7. Normal and log normal distribution and density functions based on the outcome from the staircase test in Figure 2.

The normal distribution allows negative values on the fatigue limit. In reality this can of course never be the case. The log normal distribution generally gives a better approximation and allows no values on the fatigue limit below zero. Still it seems, according to Figure 7, that the outcome of the staircase test is better fitted to the normal distribution for higher failure probabilities. Based on the outcome of only one staircase test, it is hard to judge if this is the general case. However, the log normal distribution does provide a better approximation for lower failure probabilities, in the region that is typically of most interest. The Weibull distribution would provide the best approximation on the actual fatigue limit, but the three parameters needed to describe it are hard to evaluate based on the outcome of a staircase test, and as long as the standard deviation is small, the gain is very limited in comparison to the normal and log normal distribution.

It makes no sense to mention the safety factor without giving the probability of failure. The fatigue strength should be reduced to match the allowed probability of failure and the safety factor calculated accordingly. The normalized variable λ can be iterated from the probability integral:

$$\Phi(\lambda) = P(x \le \lambda) = \int_{-\infty}^{\lambda} f(x)dx \quad \text{and} \quad f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \tag{26}$$

For small values on λ the normal and log normal distribution should give approximately the same failure probability. When assuming the log normal distribution, the logarithm of the fatigue limit, according to the given failure probability, is obtained by reducing the population fatigue limit with a reduction factor corresponding to a quantity λ of population standard deviations (Rabb, 1999):

$$S_{F,req} = b^{-\lambda_{allow} s_{\log b}} \tag{27}$$

In the same manner, the probability of failure for the component can be evaluated from the safety factor at the critical point:

$$P_{crit} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\lambda_{crit}} e^{-\frac{x^2}{2}} dx \quad \text{and} \quad \lambda_{crit} = -\frac{\lg S_{F,crit}}{s_{\lg b}} \tag{28}$$

Therefore, a safe design requires that the following condition is fulfilled at the critical point:

$$P_{crit} \le P_{allow} \Leftrightarrow S_{F,crit} \ge S_{F,req} \tag{29}$$

3. Application

The application has been created for fatigue analysis on stress results from the output database (ODB) with the help of documentation found in the manuals (ABAQUS Scripting User’s Manual and ABAQUS GUI Toolkit User’s Manual). With the application, fatigue analysis can be conducted on domains according to surface, element or node sets. The implementation of a routine for the calculation of the effective stress area has been emphasized and can be conducted by using the surface set as the calculation domain. Major effort has been put into the handling of these sets on both instance and assembly level so that the stress history can be collected for all calculation points according to analysis procedure and performance targets. Meanwhile, the implementation of mentioned damage criteria require routines that favor computing intensive analysis. This is particularly true for the calculation of the damage parameter for every calculation point in the model, since a lot of effort has been put into optimization routines. Effort has also been put into developing a user-friendly interface.

3.1 Program structure

The program has been divided into two main parts, the custom application with a tailored Graphical User Interface (GUI) that collects the needed input-data for the fatigue analysis, and a main routine that handles the computing intensive fatigue analysis and needed I/O to the output database (ODB).

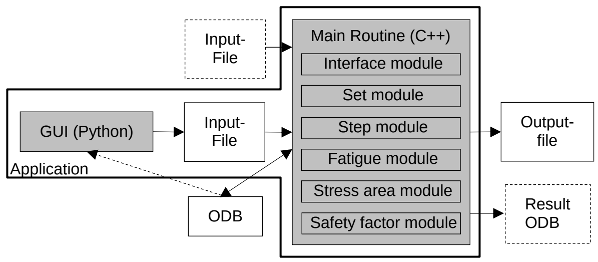

Figure 8. Program flow-chart.

The script that initiates the custom application that creates the GUI has been written in Python object-oriented scripting language by using the ABAQUS GUI toolkit. The ABAQUS GUI toolkit is an extension of the FOX GUI toolkit that provides all the necessary classes for the creation of objects and the displaying of widgets according to required functionality and design. The main requirements set on the GUI by the application are to read the ODB that is open in the Viewer, display the necessary information for the user and collect the necessary input for the writing of the input-file that is used by the main routine.

The main routine has been written in C++ object-oriented programming language. The ODB API interface provides the necessary classes and objects for I/O to the ODB. The C++ interface has been used because of the high-performance requirements. The main routine has been divided into modules which perform tasks in accordance with the analysis procedure (see Figure 8).

- ‘Interface Module’ – reads data into the buffer from the input-file and creates necessary element sets (the usage of the member getSubset() in the FieldOuput object takes only odbSet objects of element set type as an argument, since element sets have to be created on node and surface sets).
- ‘Set Module’ – creates domain objects based on lists provided by an interface object that contains lists on element, node and face labels required for the creation of custom domain objects.
- ‘Step Module’ – provides every calculation point in the domain object with its stress history.
- ‘Fatigue Module’ – applies necessary fatigue criteria on the domain objects.
- ‘Stress Area Module’ – applies stress area calculations on every domain object that has been created on a surface set.

- ‘Safety Factor Module’ – calculates a safety factor in the critical point that corresponds to the reduction factor.

At the end of the run, field data is collected from result objects and written back to the ODB and, if needed, to a new result ODB. Critical point results are written to an output-file in ASCII-format.

3.1.1 Optimization routines

Most of the programming challenges relate to performance issues. This is particularly true for the complicated and computing intensive Findley algorithm. The rotation of the stress matrix requires that the damage criterion is evaluated at every increment. The number of time steps is also an important performance limitation because the normal stress has to be calculated at every time step. The calculation of the shear stress amplitude is even more computing intensive, as the whole stress history has to be considered. In the Dang Van criterion, performance issues are mainly related to the calculation of the smallest hyper sphere enclosing the stress history.

The maximum value of the Findley damage parameter is found by implementing the down-hill simplex method (Nelder et al., 1965). The down-hill simplex method requires a rough estimate on the location of the extreme value. A rough estimate on the damage parameter is found by calculating the stress matrix in a rotated coordinated system according to the discretization in Figure 9 within the limits of a half sphere (Weber et al., 1999). The half sphere is discretized in a manner that surface elements will have a similar or an identical surface area. In this way a 35% reduction in calculation time can be obtained with retained accuracy.

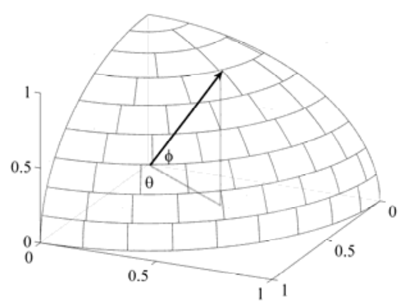

Figure 9. Discretization of coordinates with 18 increments for rotation of the stress matrix on one fourth of a half sphere.

The location of the estimated damage value is used as a starting point for the simplex routine (a simplex in 2D is a triangle with three function values). The simplex is then moved to the minimum value through a series of different transformations; reflection, expansion or contraction. All these transformations strive to move and contract the simplex around the minimum. The iteration is stopped when the simplex reaches a critical size with function values according to given tolerances. Another bottleneck with reference to calculation performance in the Findley algorithm has earlier been the calculation of the longest chord (see Figure 6). The previously used routine evaluated the longest chord by calculating the distance between every point in the stress history. In the new routine the number of calculated distances is reduced. In a case with 180 time steps this leads to a tenfold reduction in calculation time. For the calculation of the smallest enclosing circle, a method described in Weber et al. has been used (see Figure 10).

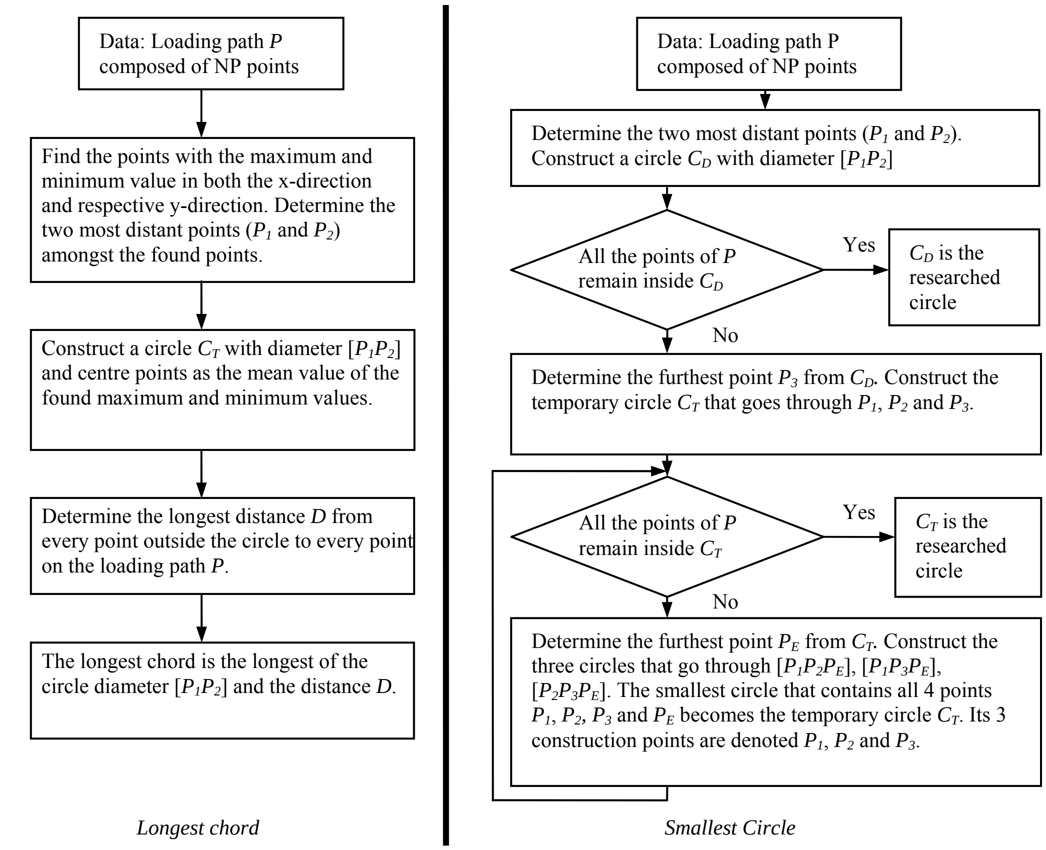

Figure 10. Flow-chart for 'longest chord' and 'smallest circle' algorithms.

3.2 Graphical User Interface (GUI)

The GUI has been written in the Python language and is initiated from the command prompt as a custom application. The custom application loads the Viewer module including the toolset required for setting up the fatigue analysis.

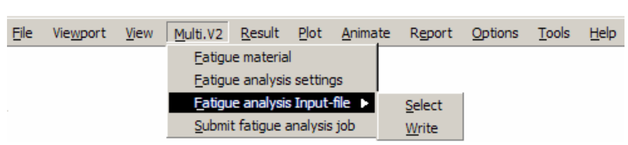

Figure 11. Fatigue analysis toolset.

The application uses form modes to display dialog boxes for interaction with the user. Three form objects are created including all the members needed for writing the input file. Every member in the form object relates to a widget that collects input in a dialog box (DB). Three dialog boxes can be displayed from the toolset:

- 'Fatigue material DB' – collects input related to nominal technical material parameters according to the reference specimen.
- 'Fatigue analysis settings DB' – collects input corresponding to analyzable sets, general analysis settings and calculation parameters.
- 'File DB' – selects filename of input-file

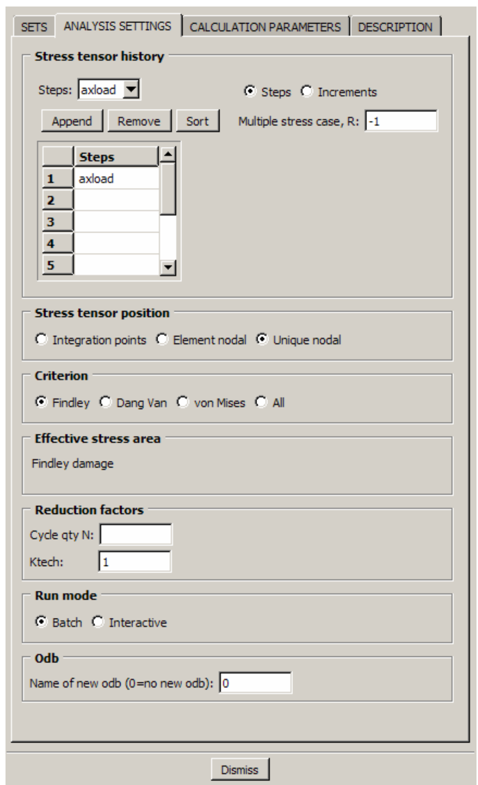

Figure 12. Analysis settings DB.

Dialog boxes are loaded from the forms with default values for some parameters (this significantly reduces the needed time for setting up the analysis). Dialog boxes are kept up to date by enabling and disabling widgets according to the input given by the user. The dialog boxes can be opened and widgets edited by the user in any order. The command for the writing of the input-file is simply executed from the fatigue analysis toolset. The input-file for the main routine is written by implementing the write() member on the Python library built-in ‘file’ object by usage, where the argument corresponds to a value from the form members:

```
datafile = open(self.fileForm.nameKw.getValue(), 'w')
...
datafile.write(self.getString(self.matForm.matNameKw.getValue()))
```

The input-file is submitted to the main program upon execution from the fatigue analysis toolset. The implementation of the routine for the execution of the main program is done with the help of the Python built-in operating system interface. The program is executed in a new process by implementing the spawnv() member on the Python library built-in ‘os’ object:

```
os.spawnv(os.P_WAIT, ‘multi2Main.exe’, [‘0’,self.fileForm.nameKw.getValue()])
```

As an alternative, the input-file can be submitted to the main routine directly from the command line:

```
abaqus multi2Main.exe ‘myInputFile.txt’
```

When the analysis has finished successfully, the ODB can be re-opened in the viewer for post-processing of field output.

3.3 Post-processing of fatigue analysis results

Many of the parameters (27 all in all) associated with the fatigue analysis are written back to the ODB as field data, e.g. direction of the critical plane and stress amplitude. These results can be post-processed in the viewer as contour or vector plots. The Haigh diagram is written to the ODB as a history plot, including the uniaxial stress case of calculations points in the analyzed domain. The application calculates the safety factor at the critical point iteratively according to the statistical approach, i.e. the safety factor is the reduction needed to make the Haigh diagram cross the given stress state.

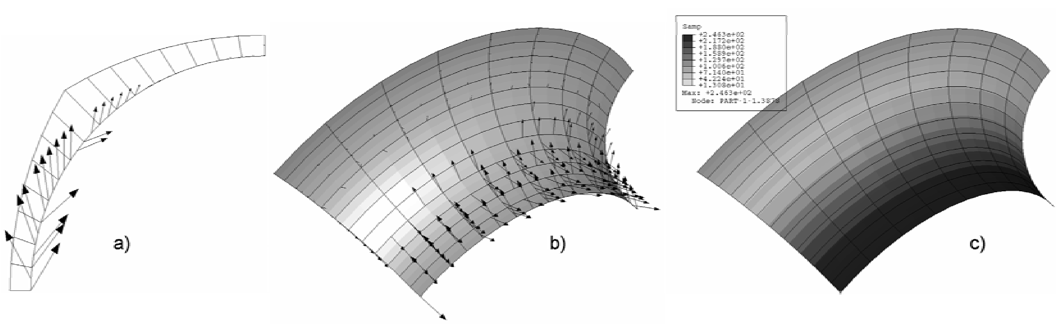

Figure 13. a) Findley critical plane (axisymmetric model) b) Findley critical plane (3D-model) b) Findley equivalent uniaxial stress case.

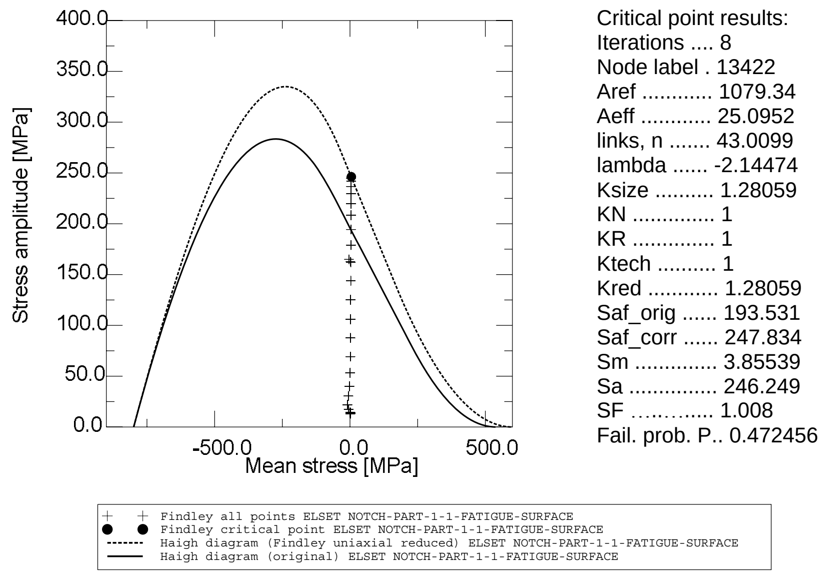

Figure 14. Haigh diagram and Findley equivalent uniaxial stress case as history output and critical point results.

Figure 13 and Figure 14 show the results of a case study on the notched specimen. This case is more of a verification of the program than a real analysis problem. It should be emphasized that most real analysis problems are typically more complex than this example. The fatigue calculations where made on two models, on one with 3D and one with axisymmetric surface domain (see Figure 13), with equivalent outcome. The fatigue test was modeled with two load steps, to gain an extreme stress amplitude in the results corresponding to the value of the fatigue limit in fully reversed tension compression loading. The damage was calculated from NODAL stresses. The sample standard deviation $s_{rC50} = 0.108$ of nodular cast iron was used in the effective stress area and statistical size factor calculations. According to the theory, at the critical point when the fatigue limit has been corrected with the statistical size factor, the expected safety factor is 1.0 and the expected failure probability is 0.5.

4. Conclusion

The 2.0 version of the fatigue analysis tool is a very different application as in comparison to the 1.0 version. Even though the 2.0 version was recently released it performs very well, and has already enjoyed successful implementation on analysis problems at the company. Already at the beginning, the development process involved many stakeholders, something which along the way contributed to the quality of the work and in the end led to a versatile tool that match the requirements of a modern probability based multiaxial fatigue analysis application. As a part of the aftermath some key-facts that have contributed to the quality of the work should be addressed:

- C++ is an excellent choice for programming language when performance is an issue. Moreover the classes and objects provided by the C++ ODB API interface can be implemented in an efficient and convenient way.

- To compile the calculation intensive part to a stand-alone executable seems to have been the correct choice. The advantage of the modularization is e.g. that the GUI can be developed independently from the rest of the program. Moreover the input-file provides a convenient interface between the GUI and the calculation part of the program.

- The implementation of surface sets and shape functions can in many ways be considered as optimal for effective stress area calculations. Surface sets are easy to define in a pre-processor or in the ABAQUS input-file. In many cases a surface set is pre-defined in potentially interesting calculation domains for fatigue analysis, e.g. as a part of contact definitions on contact faces.

- Implementation of optimization routines, e.g. down-hill simplex, for several calculation intensive operations has significantly decreased the execution time of particularly the Findley damage criterion.

The functionality of the 2.0 version is quite differing as in comparison to the 1.0 version. It was already in the beginning stated that the interface should be created so that it allows versatile fatigue analysis settings that could be applied to a wide range of analysis problems. Some of the key-features that provide the application with its flexibility can be outlined as:

- Fatigue analysis can be conducted according to several multiaxial fatigue criteria, namely the Findley, the Dang Van and the pseudo criterion by signed von Mises, three very versatile criteria when it comes to applicability.

- Fatigue analysis can be conducted on both full three-dimensional and axisymmetric models.

- Fatigue analysis can be conducted on surface, element and node sets, as calculation domains (it is typically useful to restrict the calculation domain to the particular location of interest, because it significantly decreases the execution time of the program).

- The stress history for calculation points in the analysis domain can be collected from integration points, element nodes and global nodes. The NODAL (stresses from global nodes) choice is preferred when the most conservative estimate is to be made on the statistical size factor. NODAL stresses are not available through the ODB API interface as that; hence averaging can be conducted by the application upon analysis execution. Moreover the NODAL choice allows vector presentation of field values, e.g. direction vector of critical plane, at nodes.

- The GUI has been grouped into dialog boxes and tabs that collect input data in a logical manner. The GUI provides a convenient introduction of the application to the beginner, while the direct editing of the input-file allows the more experienced user to make a fast setup of the fatigue analysis.

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