

Vibration Characteristics of Rolling Tires

Summary

Tires are the only load transfer mechanism between a vehicle's suspension and the road. Consequently, tire vibration has a significant impact on ride quality and vehicle interior noise.

To accurately predict the frequencies and mode shapes of a rolling tire, Abaqus allows pre-loading and gyroscopic effects to be included in a complex frequency extraction. These effects can also be included in a forced response analysis of the rolling tire to predict the spindle forces contributing to vehicle noise and vibration.

In this Technology Brief, we outline the analysis methodology for determining the vibration characteristics of stationary and steadily rolling tires.

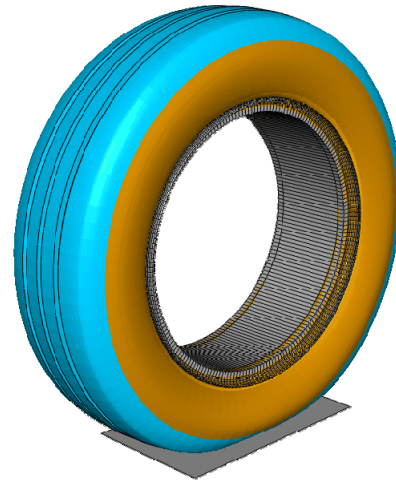
Background

The small amplitude vibrations of a tire on the road can be considered to be the linear superposition of small amplitude steady state vibrations onto a highly non-linear base state.

For stationary tires, the base state is the footprint configuration and contains the nonlinear effects arising from the contact between the tire and the road, the load-deflection behavior of the rubber compounds, and the reinforcement behavior. For steady rolling tires, the base state additionally includes gyroscopic effects.

It is common practice to employ a mixed Eulerian-Lagrangian scheme to compute the steady-state rolling base configuration of the tire. This methodology uses a reference frame that is attached to the axle of the tire but does not rotate with the tire; thus, the material of the tire is rotating through this reference frame. An observer in this frame sees the tire as points that are not moving, although the material constituting the tire is moving through those points.

The rotational velocity of a rolling tire causes its vibration characteristics to be considerably different from those of a stationary tire. To an observer in the aforementioned reference frame, the modes of a stationary tire appear as standing vibrations while the modes of a rolling tire appear as waves traveling in the circumferential direction. Some of these modes share the sense of the tire's rotation, while others rotate in the opposite sense. As the rotational velocity of the tire increases, the modes with the same sense of rotation as the tire increase in frequency



Key Abaqus Features and Benefits

- Symmetric Model Generation and Symmetric Results Transfer
- A mixed Eulerian-Lagrangian scheme to compute the steady-state rolling configuration of tires
- Complex eigenvalue extraction that accounts for pre-loading and gyroscopic effects
- Subspace-based steady state dynamics for relatively inexpensive forced response prediction

while the modes with the opposite sense of rotation decrease in frequency. The bifurcation in frequency arises from the gyroscopic effects.

Finite Element Analysis Approach

The tire model under consideration is shown in Figure 1. The tire is shown in a sectional view and is representative of a passenger car tire. The rim and road are modeled as analytical rigid surfaces. Contact is defined between these surfaces and the tire. A simple Mooney-Rivlin law is used to model the strain energy potential of the rubber materials. The viscoelastic nature of rubber is ignored in this simulation. A small amount of material damping is applied to the rubber in the form of Rayleigh damping. The plies and belts are modeled using rebar layers that are embedded in the surrounding rubber matrix. Linear

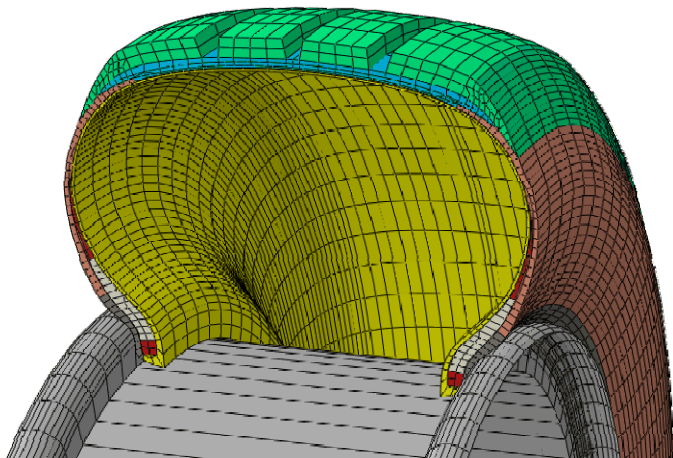


Figure 1: Tire and rim

elastic material properties are applied to the reinforcement fibers. The vibration characteristics are determined in an analysis sequence consisting of six steps.

Step 1: Rim mounting and inflation analysis

Rim mounting and inflation are carried out using a 2-D axisymmetric model of the tire cross-section, which saves significant analysis time. Leveraging the symmetry of the model and the loading conditions, only one half of the tire is modeled. Axisymmetric elements with twist are used to capture the out-of-plane deformation introduced by the belts.

Step 2: Footprint loading

The symmetric model generation capability is used to revolve the 2-D axisymmetric cross-section into a partial three-dimensional model. Uniform mesh density around the circumference is used in this study. The results from the end of the inflation step are transferred to the partial three-dimensional model to act as the base state for the ensuing footprint loading simulation. The road is modeled as an analytical rigid surface, and the vehicle load contribution is applied as a concentrated load on the road surface reference point.

With a second application of symmetric model generation to reflect the half-symmetric model, a full three-dimensional model is created. The results from the end of the footprint loading of the partial model are transferred to the full three-dimensional model to achieve the final static footprint configuration.

Step 3: Computation of the steady state rolling configuration

The static footprint configuration acts as the base state for the ensuing steady state transport analysis at the desired velocity. The steady state transport analysis takes the effect of inertial forces and frictional forces into account. The coefficient of friction between the road and the tire is set to 1.0. Free-rolling configurations corresponding to 15

km/h and 50 km/h are computed.

Step 4: Frequency Extraction

An eigenvalue analysis is performed after both the static footprint and steady state rolling steps to compute the natural frequencies and mode shapes of the tire in its stationary and rolling configurations. Note that the natural frequency extraction does not account for gyroscopic effects or damping. Although a small amount of damping is applied to the system, it is not expected to alter the behavior of the tire significantly. Contact conditions from the base state are preserved in both the normal and tangential directions. Points that are slipping (equivalent shear stress greater than critical shear stress) are free to move tangential to the contact surface, whereas the points that stick are kept fixed.

Step 5: Complex Eigenvalue Extraction

The modes of a rolling tire, computed in the reference frame associated with the Eulerian-Lagrangian scheme, are complex due to the contributions from the gyroscopic terms. The complex frequency extraction procedure uses a subspace projection method based on the eigenmodes extracted in the frequency extraction step (*Step 4*).

Step 6: Steady State Dynamics

A subspace-based steady state dynamics procedure is used to compute the frequency response of the tire to an applied road excitation. The subspace projection method uses the eigenmodes extracted in the frequency extraction step (*Step 4*). The forced response analysis is performed for both stationary and rolling tires.

Results and Discussion

As mentioned earlier, the modes of stationary tires appear to an observer as standing vibrations. These standing vibrations can be thought of as a superposition of two circumferentially counter-rotating waves with the same wave speed. However, for a rotating tire, the wave with the same sense of rotation as the tire appears to have a higher wave speed, while the wave rotating against the tire appears to have a lower wave speed.

Consider the radial mode of circumferential order two. For the stationary tire, a pair of such modes appears at 98.45 Hz and 100.97 Hz. Note that for a tire with no footprint load, these modes occur at the same frequency.

However, for the tire traveling at 15 km/h, as shown in Figure 2, these modes appear at 97 Hz and 101.8 Hz. When the tire is traveling at 50 km/h, these modes appear at 93.08 Hz and 106.23 Hz. The mode that has the same sense of rotation as the tire increases in frequency and the mode that has the opposite sense of rotation decreases in frequency.

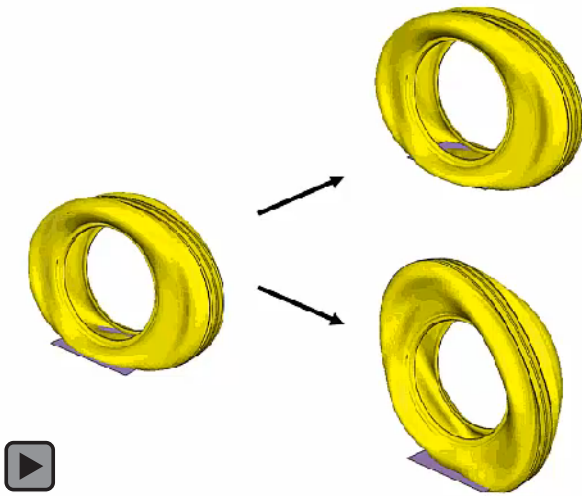


Figure 2: Radial mode of circumferential order two, 15 km/h. Standing wave motion (left) is superposition of counter rotating waves (right).
Click to view animation

Similarly, for the stationary tire, the radial modes of circumferential order three are at 124.77 Hz and 126.61 Hz. At 15 km/h, these modes shift to 121.05 Hz and 129.47 Hz, while at 50 km/h, they shift to 111.95 Hz and 134.48 Hz. The deformed shapes at 15 km/h are shown in Figure 3.

Note that the difference in frequency between the bifurcated modes is approximately twice the circumferential order times the rotating frequency. Therefore, the difference increases with speed as well as the circumferential mode order.

Road asperities act as a source of excitation for tire vibrations. The vibration of the tire produces spindle forces

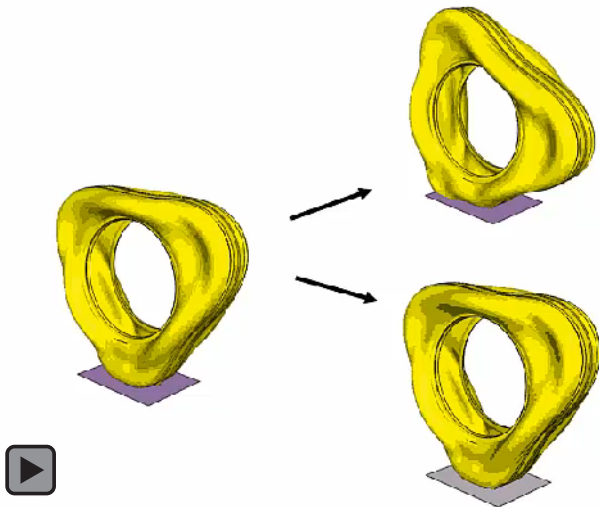


Figure 3: Radial mode of circumferential order three, 15 km/h. Standing wave motion (left) is superposition of counter rotating waves (right).
Click to view animation

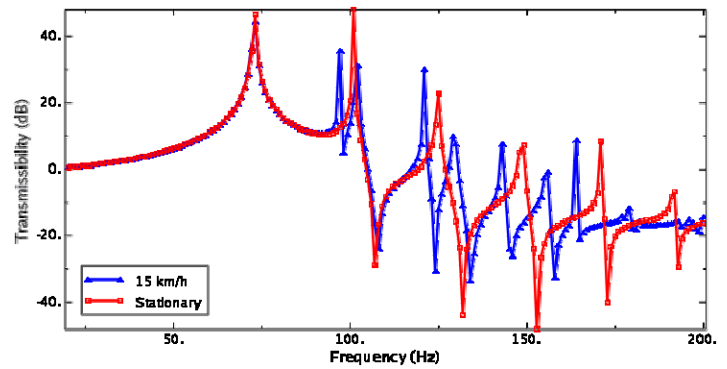


Figure 4: Comparison of vertical transmissibility: stationary and 15 km/h.

that act as the primary source of excitation for the vehicle interior cavity. To predict the spindle forces, a unit load is applied to the road surface reference node and a steady-state dynamics frequency sweep from 0 to 200 Hz is performed for both the stationary and the rolling tire.

Figure 4 compares the vertical transmissibility for the tire when stationary and rolling at 15 km/h. Transmissibility is the ratio of the amplitude of the force transmitted to the vehicle to the amplitude of the applied excitation force. In this case, it is calculated as the ratio of the amplitude of the reaction forces at the hub to the amplitude of the applied excitation force, and is plotted in decibels ($20\log_{10}$).

The graph clearly illustrates the bifurcation in frequencies for various radial modes. Figure 5 includes results at 50 km/h and focuses on the frequency range near the circumferential mode of order two. For a stationary tire, the peak corresponding to this mode appears around 101 Hz. At 15 km/h, the bifurcated peaks appear at 97 Hz and 102 Hz. When the speed is increased to 50 km/h, the bifurcated peaks shift to around 93 and 106 Hz.

For a stationary tire in contact with the road, a vertical excitation applied to the road produces a spindle force only in the vertical direction. However, for a rolling tire, a vertical excitation produces spindle forces both in the ver-

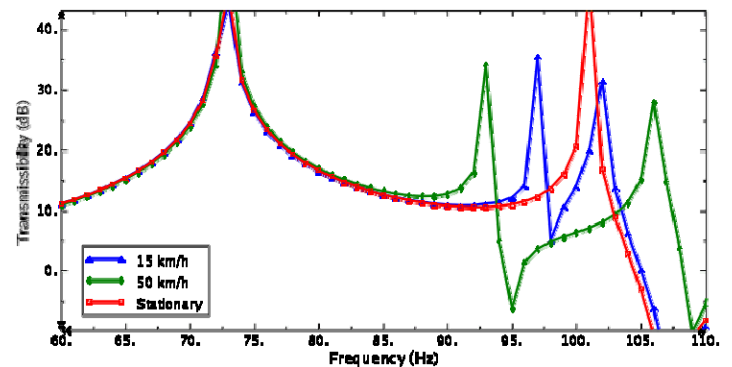


Figure 5: Comparison of vertical transmissibility near circumferential mode of order two: stationary, 15 km/h, and 50 km/h.

tical and fore/aft directions. In Figure 6, the fore/aft transmissibility for the rolling and stationary cases is compared. It is seen that for a stationary tire, the vertical excitation produces virtually no fore/aft spindle force. However, the magnitude of the fore/aft force is considerable for a rolling tire.

Conclusions

An observer in a non-rotating reference frame attached to the axle of a tire observes the modes of a stationary tire as standing vibrations. However, the modes of a rolling tire appear as counter-rotating waves. The wave rotating with the tire appears to have a higher wave speed than the wave rotating against the tire. This phenomenon manifests itself in the frequency domain as a bifurcation of eigenmodes. The amount of bifurcation is a function of the tire speed and has a significant effect on the transmissibility of the tire. The nonlinear and steady state dynamics capabilities of Abaqus/Standard easily capture this

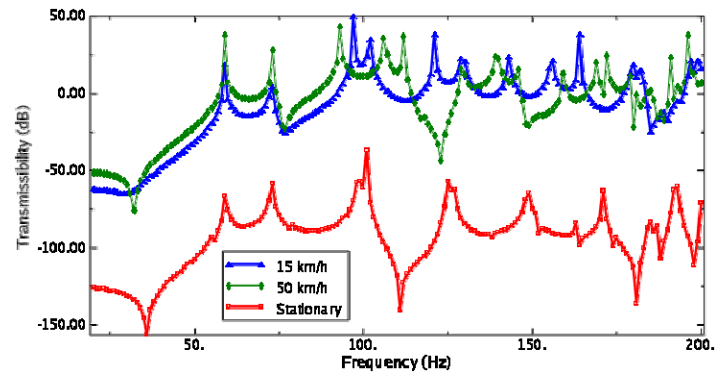


Figure 6: Comparison of Fore/Aft transmissibility: stationary, 15 km/h, and 50 km/h.

effect, allowing for a more accurate understanding of how the interaction between a rolling tire and the road contribute to vehicle noise and vibration.

Abaqus References

For additional information on the Abaqus capabilities referred to in this brief, please see the following Abaqus Version 6.13 documentation references:

- Analysis User's Guide
 - "Direct-solution steady-state dynamic analysis," Section 6.3.4
 - "Natural frequency extraction," Section 6.3.5
 - "Mode-based steady-state dynamic analysis," Section 6.3.8
 - "Subspace-based steady-state dynamic analysis," Section 6.3.9
 - "Acoustic, shock, and coupled acoustic-structural analysis," Section 6.10.1
- Example Problems Guide
 - "Steady-state dynamic analysis of a vehicle body-in-white model," Section 2.2.6
 - "Coupled acoustic-structural analysis of a pick-up truck," Section 9.1.6

About SIMULIA

SIMULIA is the Dassault Systèmes brand that delivers a scalable portfolio of Realistic Simulation solutions including the Abaqus product suite for Unified Finite Element Analysis, multiphysics solutions for insight into challenging engineering problems, and lifecycle management solutions for managing simulation data, processes, and intellectual property. By building on established technology, respected quality, and superior customer service, SIMULIA makes realistic simulation an integral business practice that improves product performance, reduces physical prototypes, and drives innovation. Headquartered in Providence, RI, USA, with R&D centers in Providence and in Vélizy, France, SIMULIA provides sales, services, and support through a global network of over 30 regional offices and distributors. For more information, visit www.simulia.com

The 3DS logo, SIMULIA, Abaqus and the Abaqus logo are trademarks or registered trademarks of Dassault Systèmes or its subsidiaries, which include Abaqus, Inc. Other company, product and service names may be trademarks or service marks of others.

Copyright Dassault Systèmes, 2007

