

Rolling - Off Simulation of Bevel Gears for Strength- and Durability Assessment

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Abstract: The present work will concentrate on the gear fatigue of automotive power transmission systems. Especially the strength of the bevel gears of the differential transmission has important influence on the durability of the whole transmission system. Therefore the bevel gears and pinions are made of hardened high strength steel. As an example the lecture shows a strength- and durability analysis of a differential gear. An accurate Finite Element Model is employed to provide a quantitative understanding of gear tooth contact behaviour under load. A rolling and sliding contact simulation between the bevel gears is performed to get the resulting stress distribution using the commercial Finite Element Analysis software Abaqus. The whole loading cycle for a tooth is considered to get an adequate stress – time history. Based on the stress history a damage analysis is performed. The damage simulation uses a moment classification obtained from measurements at a test bench. Therefore the total damage value results from the sum of the damage values of all moment classes. A significant influence on the damage comes from the hardening process. Besides the higher material strength behaviour of the hardened layer, another positive effect has to be taken into account. The hardening process induces a local volume increase which results in a residual stress distribution at the surface of the bevel gears and leads to a compressive preload at the tooth base. The results of the durability analysis show that this preload is significant to obtain damage predictions comparable to measurements.

Keywords: Contact Mechanics, Fatigue, Damage

1. Introduction

The fatigue of automotive power transmission has become an important factor for the quality of vehicles. The power transmission train consists of the components that transfer the power from the engine to the road surface. This includes transmission, driveshafts, differential and finally the drive wheels. Especially the strength of the bevel gears of the differential shows important influence on the durability of the whole transmission system.

A differential is a device, capable of transmitting torque and rotation through three shafts, almost always used in one of two ways. In one way, it receives one input and provides two outputs; this is found in most automobiles. In the other way, it combines two inputs to create an output that is the sum, difference, or average, of the inputs.

In automobile and other wheeled vehicles, the differential allows each of the drive wheels to rotate at different speeds, while supplying equal torque to each of them. This is necessary, for example, if the vehicle is turning corners. When cornering, the inner wheel travels a shorter distance than the outer wheel which leads to different speeds of the wheels. The differential compensates the different speed of the outer and inner wheel and avoids unpredictable handling, damage to tires and roads, and strain on (or possible failure of) the entire drive train.

The design of the differential is as the following: The ring gear is attached to a carrier, which holds what is sometimes called a spider, a cluster of four bevel gears in a rectangle, so each bevel gear meshes with two neighbors and rotates counterwise to the third that it faces and does not mesh with. Two of these spider gears are aligned on the same axis as the ring gear and drive the half shafts connected to the vehicle's driven wheels. These are called the side gears. The other two spider gears are aligned on a perpendicular axis which changes orientation with the ring gear's rotation. These two gears are just called intermediate gears. As the carrier rotates, the changing axis orientation of the intermediate gears imparts the motion of the ring gear to the motion of the side gears by pushing on them rather than turning against them (that is, the same teeth stay in contact), but because the spider gears are not restricted from turning against each other, within that motion the side gears can counter-rotate relative to the ring gear and to each other under the same force (in which case the same teeth do not stay in contact). The rotation of the ring gear is always the average of the rotations of the side gears. If for example the wheels are lifted off the ground with the engine off, and the drive shaft is held (preventing the ring gear from turning inside the differential), manually rotating one wheel causes the other to rotate in the opposite direction by the same amount. When the vehicle is traveling in a straight line, there will be no differential movement of the planetary system of gears.

The outline of the paper is as follows: Chapter 2 gives an overview of the whole simulation approach, containing a stress analysis of the bevel gears and a fatigue analysis. A detailed Finite Element model of the tooth contact of the bevel gears including the hardening procedure is shown in Chapter 3. The results are presented in Chapter 4.

2. The Simulation Approach

Bench tests of differentials show, that the tooth base of the spider gears is a weak spot. Therefore it is necessary to check the strength and durability of these bevel gears at an early state of industrial development. The bevel gears are made of hardened high strength steel. Beside the higher material strength behaviour of the hardened layer another positive effect has to be taken into account. The hardening process induces a local volume increase which results in a residual stress distribution at the surface of the bevel gears and leads to a compressive preload at the tooth base. This preload mitigates the damage caused by the applied moments.

The damage computation depends significantly on the load time history respectively the resulting stresses, which are obtained by Finite Element stress analyses. The load time history comes from measurements at a test bench. An example of a moment – time series measured at a test bench is shown at Figure 1.

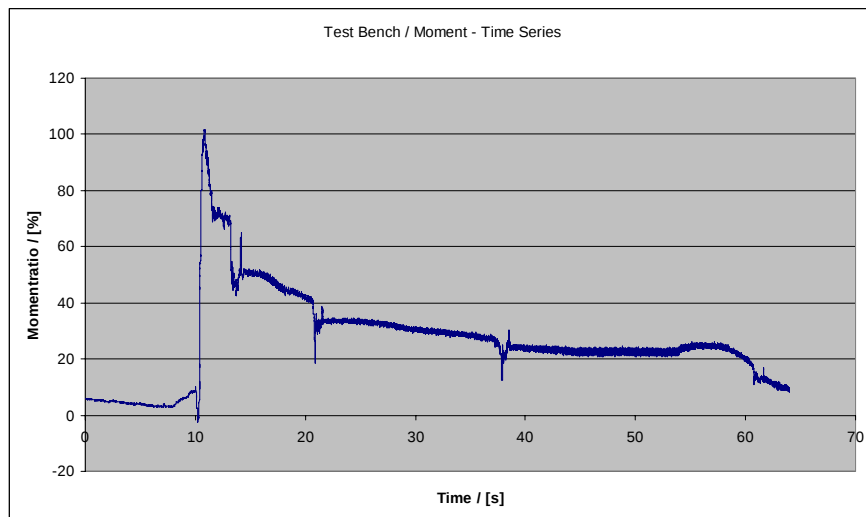


Figure 1. Moment – time series measured at test bench (example).

Because of complexity of the measured data it is necessary to classify the data. For this purpose several intervals of moments are defined where each interval is called a “class”. The moment classification transforms the measured time history of varying moments into an array of moment classes and the corresponding exposure time. Combining this data with the number of revolutions per second it can be used to find the corresponding load cycles per tooth for each moment class. A sketch of the classification procedure can be found in Figure 2.

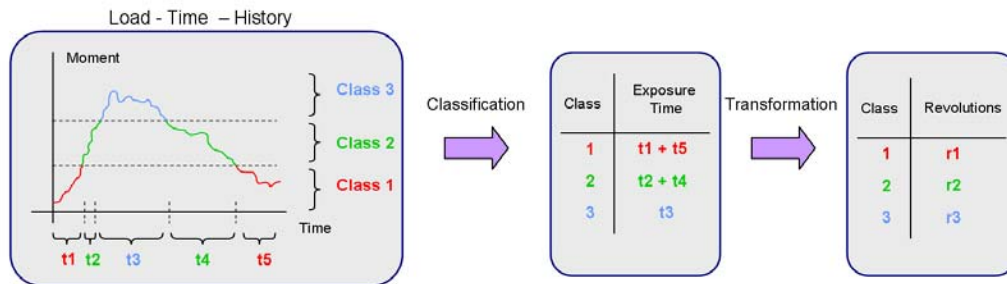


Figure 2. Classification of the Load – Time – History.

These moment classes - respectively the mean moment per class - are used for generating the stress – time – history by performing a Finite Element rolling – off simulation for each moment class.

2.1 The Finite Element Stress Analysis

For each moment class a Finite Element analysis, executed with Abaqus STANDARD V6.8 (Abaqus, 2008), delivers the stress time history required for the subsequent fatigue analysis. The Finite Element analyses are nonlinear rolling – off analyses incorporating the contact between the side bevel gears and the intermediate bevel gears. The rotation angle for the bevel gears are chosen big enough that at about tree teeth perform a whole loading cycle, consisting of getting in contact, rolling – off and getting out of contact. Since the bevel gears are connected via a contact formulation the Finite Element simulations are nonlinear and extremely computational expensive. To save computation time (and data complexity) the stress computation has to be simplified. This is done by performing only one contact simulation at a specially chosen reference moment. The reference moment is about 2/3 of the maximum moment. The stress time history for the different moment classes are approximated by linear scaling of the stress data of the reference moment simulation. The resulting error due to scaling is acceptable small which was checked in an additional analysis for the maximum moment.

2.2 The Damage Simulation

The damage simulation is performed with the Fatigue Software FEMFAT V4.7c – Module TransMAX (FEMFAT, 2008). Based on the scaled stress time history of the reference moment, obtained from a nonlinear Finite Element contact simulation, a transient and multi axial damage simulation is executed for each moment class. In FEMFAT the scaled stress time history is rainflow – classified delivering mean –, amplitude stresses, and the number of repeats. Based on the results of the rainflow method and using material properties and influence factors like surface roughness, stress gradient, and statistical influence etc. the fatigue analysis finally delivers the damage of the bevel gears (to be exact only for the teeth which see a full load cycle) for one revolution for each moment class. The total damage is evaluated in a two step way. In the first step for each moment class the damage value for one revolution is multiplied by the number of

revolutions per class delivering the damage per class. In the second step the damages per class are summed up to get the total damage for the hole load – time – history.

The fatigue material parameters used in FEMFAT are well known for the common groups of base materials (FKM Guideline, 1994). More complicated is the setting of the fatigue parameters for the hardening layer. A feasible approximation for the fatigue parameters of the hardening layer can be derived by scaling the parameters of the base material. Incorporating the hardness of the layer, which is generally known, an approximation of the fatigue parameters derived from analytical formulas can be found in the thesis of Mr. Hertter (Hertter, 2003). Since linear elastic material behaviour is used in the Finite Element analysis, local plastic behaviour is considered in the fatigue analysis by Neuber's stress rearrangement.

A draft of the whole analysis procedure is shown in Figure 3 (reference moment of this chart corresponds to class 2):

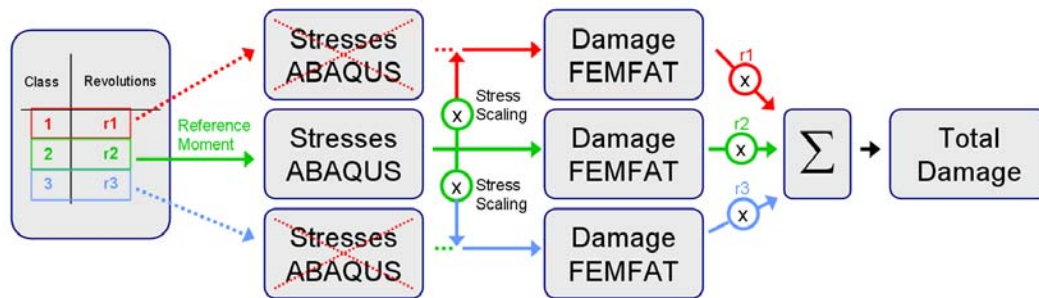


Figure 3. Flow – Chart of the whole Analysis Procedure.

3. The Finite Element Stress Simulation

In order to achieve precise and reliable fatigue results all relevant loading conditions have to be considered for the stress analysis. To obtain a trustful stress time history all stiffness relevant parts of the differential have to be included into the Finite Element - analysis. An overview of the Finite Element - model is given in the next section.

3.1 The Finite Element Model

The Finite Element model consists of the following parts:

Ring gear: The ring gear is connected to the differential cage and serves as input for the applied loads. Especially we load the system with gear tooth forces.

Differential cage: The differential cage transfers the moment applied at the ring gear via a bolt to the bevel gears.

Bevel gears: The intermediate – and side bevel gears are bedded into the differential cage and build the connection to the wheel shafts.

Ball bearing: The ball bearings are used for supporting the differential cage. The outer bearing ring is fixed in space.

A cut view of the whole Finite Element model is shown in Figure 4.

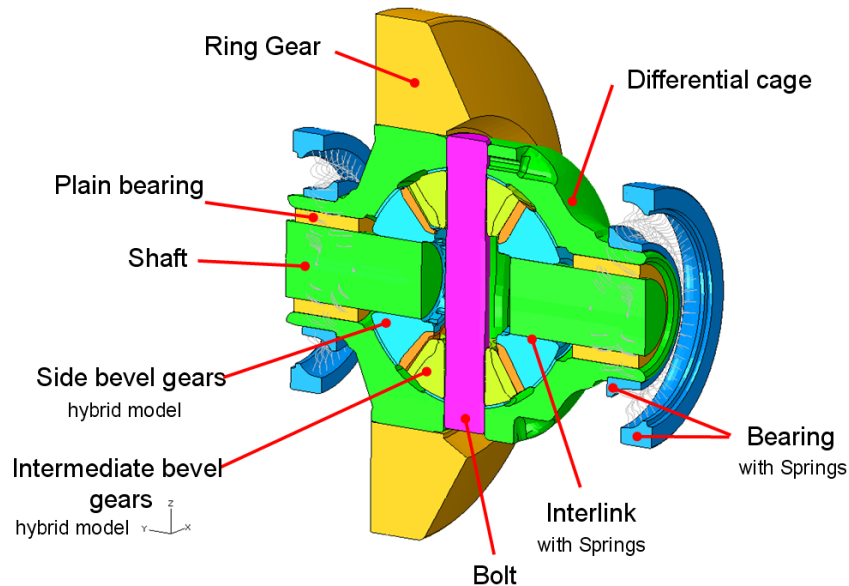


Figure 4. Finite Element model of the differential, cut view.

The bevel gears transfer the input torque applied at the ring gear to the wheel shafts. For the Finite Element simulation we are using a hard and finite sliding contact formulation with friction. The contact definition reads as:

```

**I1_friction
**
*SURFACE INTERACTION, NAME=I1_friction
*FRICTION
    0.1,
**
**CONTACT_FL1
**
*CONTACT PAIR, INTERACTION=I1_friction
S30;SET_slave_GEAR1, S31;SET_master_GEAR1
**

```

Please note that an “adjust” – operation is not allowed for the tooth to tooth – contact definitions because this would destroy the shape of the slave surface. Therefore the setup of the bevel gears is done in a special way. To guarantee that the simulation starts with a defined (closed) contact situation, the contact flanks of the bevel gears are modeled with a small spatial penetration at the corresponding contact zones. This procedural method leads to a stable launch of the simulation without using an artificial damping method for the Abaqus STANDARD simulation. During the

first step of the simulation we remove the penetration by using a “shrink” – command at the contact definition. The corresponding lines in the step definition section of the Abaqus input file look like:

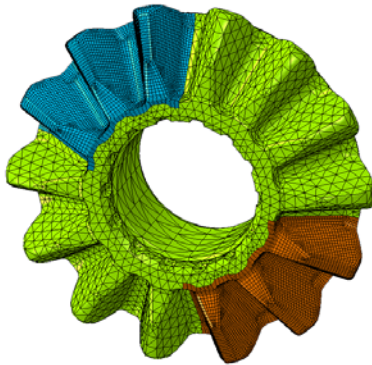
```

**
** INTERFERENCE
**
*CONTACT INTERFERENCE, SHRINK
S30;SET_slave_GEAR1, S31;SET_master_GEAR1
S32;SET_slave_GEAR2, S33;SET_master_GEAR2
S34;SET_slave_GEAR3, S35;SET_master_GEAR3
S36;SET_slave_GEAR4, S37;SET_master_GEAR4

```

For an adequate and stable Finite Element representation of the contact situation it is necessary to use a fine mesh at the areas where contact occurs. Otherwise a fine mesh increases the computation time dramatically. As a compromise the bevel gears are modeled using two different mesh densities and element types (a hybrid mesh). The body of the gear is meshed using a coarse second order tetrahedron mesh. The contact zone and hardening layer is meshed fine using first order hexahedron elements. The hybrid mesh of the bevel gears is shown in Figure 5. The hardening layer of about 1,5 millimeter is modeled with four elements over the thickness. Hexahedron elements have the advantage that the element length can be simple controlled by the meshing tool to build the four-layer-model. Additionally the hexahedron layers at the contact zone are used for the mapping of the induced hardening stresses. Further details to the hardening process are explained in Chapter 3.4. A tie contact formulation connects the body of the gear with the hardening layers.

Bevel side gear:



Intermediate gear:

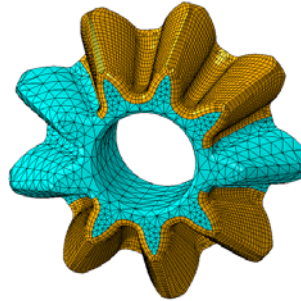


Figure 5. Hybrid mesh of the bevel gears.

The Finite Element model of the ball bearing consists of the inner – and outer bearing ring and springs connecting the two rings. The setup of the springs is done in a special way. To build up a rotatable bearing model we have to divide the spring connection between the inner – and outer bearing ring. Especially we use two spiders made of springs. The first one, using springs which are

stiff due to compression, connects the inner bearing ring with a point at the rotation axis. The second spider, made with springs transferring only tensile forces, goes from the point at the rotation axis to the outer bearing ring. The axial and radial nonlinear displacement curves, obtained from the manufacturer, are used for the calibration of the a-priori unknown nonlinear spring stiffness. We use several iteration loops readjusting the spring stiffness to obtain an adequate approximation of the displacement curves for the ball bearing.

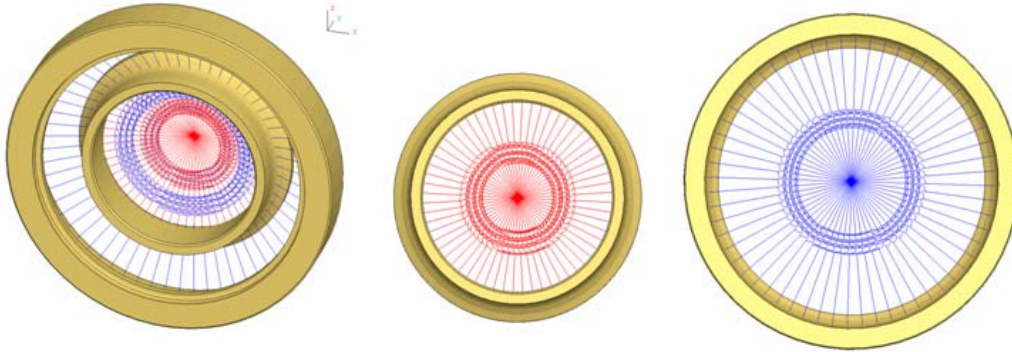


Figure 6. Assembly of the rotatable ball bearing (left), inner bearing ring with spider (middle), outer bearing ring with spider (right).

3.2 The Contact Pairs

The different parts of the differential are interacting via contact definitions. An illustration of the used contacts can be found in Figure 7 (right side). In the following we give a list of the considered contacts:

TIE CONTACTS:

Differential cage / ring gear

Differential cage / plain bearing

Differential cage / bolt

Differential cage / inner bearing ring

Gear body / hardening layer

CONTACT:

Differential cage / bevel side gear: The differential cage interacts with the bevel side gear using a finite sliding contact with friction. To improve the convergence a soft contact is applied by a pressure – overclosure option. This kind of contact allows small penetrations. The corresponding lines at the input deck can be found below (I2_soft_contact_friction).

Differential cage / intermediate gear: We use the same contact specifications as between the differential cage and the bevel side gear.

Plain bearing / shaft: The initial bearing gap is 20 μm . To start the simulation with a stable contact situation a special pressure – overclosure relationship is defined. At the initial gap of 20 μm already 0.3 N/mm² contact pressure is acting. In this way the shaft is stably bedded in the bearing at the beginning of the simulation. We use a finite sliding contact without friction (I3_soft_contact).

Bolt / intermediate gear: Hard finite sliding contact with friction.

Bevel gear / intermediate gear: Hard finite sliding contact with friction.

An Abaqus soft contact can be specified in the input deck with the following lines:

```
**
** I2_soft_contact_friction
**
*SURFACE INTERACTION, NAME=I2_soft_contact_friction
*SURFACE BEHAVIOR, PRESSURE-OVERCLOSURE=TABULAR
      0.,          0.
    100.,         0.01
    200.,         0.04
   1000.,         0.1
*FRICTION
      0.1,
**
** I2_soft_contact_lubrication
**
*SURFACE INTERACTION, NAME= I3_soft_contact
*SURFACE BEHAVIOR, PRESSURE-OVERCLOSURE=TABULAR
      0.,          -0.2
     0.3,         -0.02
      1,         -0.002
   100.,          0
*FRICTION
      0.0,
```

The following figure illustrates the whole Finite Element model and the contact surfaces:

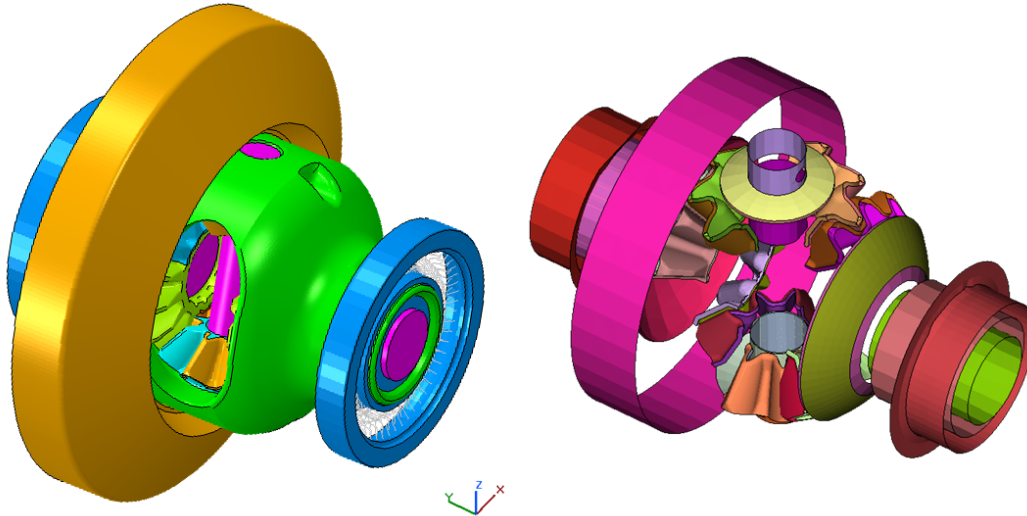


Figure 7. The whole Finite Element model (left). Contact surfaces (right).

3.3 The Analysis Steps

The rolling – off simulation is divided into two steps. The first step is used to apply the ring gear load for the reference moment with fixed shafts. The input torque is split by the differential cage and we observe the half input torque at the fixed wheel shafts. The second analysis step simulates the rotation of the wheel shaft. As mentioned in the introduction the intermediate gears only rotate if the drive wheels rotate with different speeds. For this purpose one wheel shaft is fixed and the second one rotates about an angle of $103,2^\circ$. This angle is enough to ensure that three teeth perform a full load cycle. The whole rotation is divided into 48 increments with an increment size of $2,15^\circ$.

Since the rotation of the differential cage is quite big and the change of the position of the tooth force on the ring gear is not considered we implemented a “FOLLOWER” – load. The according line in the input deck reads as:

```

**
** CLOAD
**
*CLOAD, OP=NEW, FOLLOWER

```

Figure 9 shows the initial – and final position of the rolling-off simulation. One observes that the bevel side gear and the bevel intermediate gears are rotated. Additionally the bolt position has

changed only the half angle of the side gear rotation. The rotation angle of the bevel intermediate gears - with respect to the bolt axis - results from the transmission ratio.

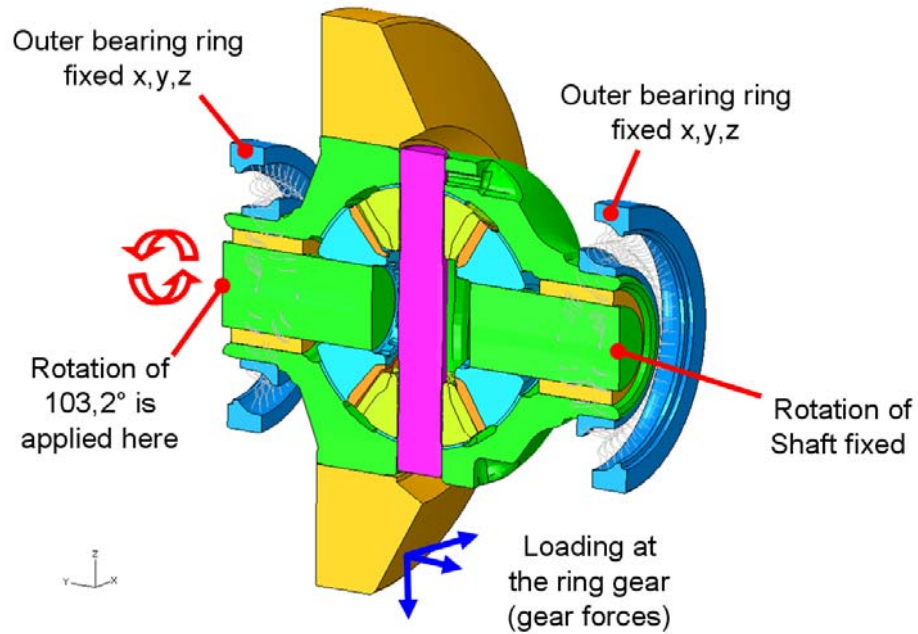


Figure 8. Applied boundary conditions.

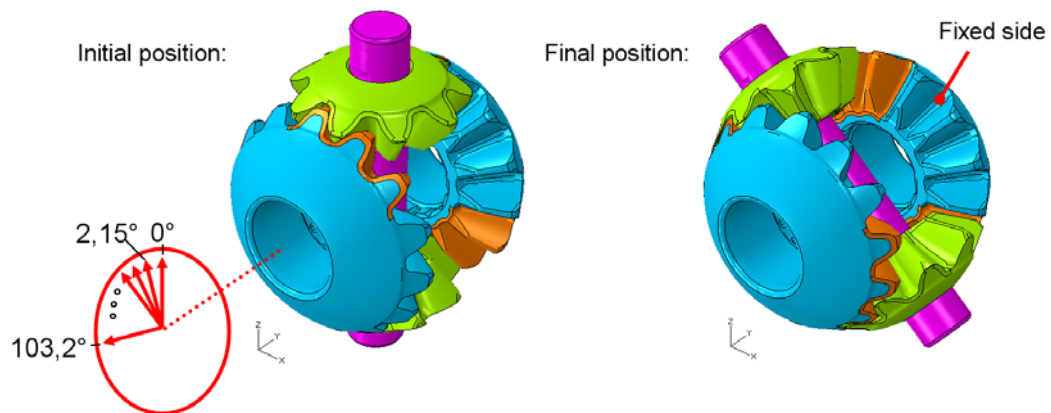


Figure 9. Initial – and Final position of the rolling-off process of the differential bevel gears.

3.4 Hardening Stresses

The hardening process influences the material in two ways. First the hardening increases the strength data by crystalline transforming effects. Additionally the hardening process induces stresses in the hardening layer due to the increase of the volume. The idea, how to implement the hardening stress in the FE – simulation, is based on the volume increase. We use a spatially varying temperature field in the hardening layer (linearly decreasing from the surface) for the implementation of the volume increase, which leads to the hardening stresses. The necessary temperature is derived from the relation

$$V = V_0 (1 + \alpha \times \Delta T)^3,$$

where V is the hot volume, V_0 is the reference volume, α denotes the linear temperature (hardening) expansion coefficient and ΔT describes the artificial temperature difference in the hardening layer.

The induced hardening stresses (Von Mises stresses) for the intermediate- and side bevel gear are shown at Figure 10. One can observe that the highest values of the hardening stresses can be found at the tooth base, which correlates very well to observations.

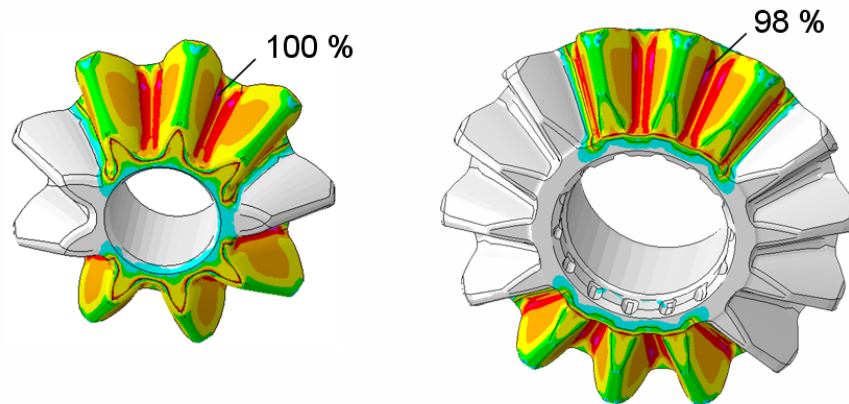


Figure 10. Von Mises stress (% of max. value) induced by the hardening process.

In the damage simulation the hardening stresses are superposed to the stress history of the rolling-off simulation.

4. Results

This chapter shows the results of the fatigue analyses for two examples. The first example shows the damage values including the residual stresses of the hardening process for the fatigue analysis.

In the second example the hardening stresses are neglected, all other parameters of the fatigue computation stay unchanged. One observes, that the influence of the hardening stresses leads to increased fatigue life times. The damage values found at the tooth base of the bevel gears are about a factor of two lower if the hardening stresses are considered. The reason for the decreased damage is the compressive hardening stress at the tooth base. The compressive preload shifts the mean stresses to the left side of the Haigh diagram (10, Haigh), where we observe an increased resistance to alternating loads.

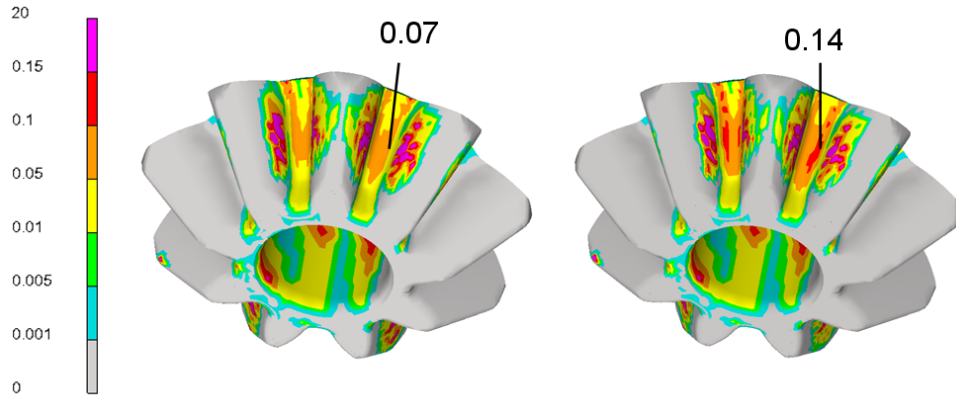


Figure 11. Damage values for the intermediate gear, with hardening stress (left), without hardening stress (right).

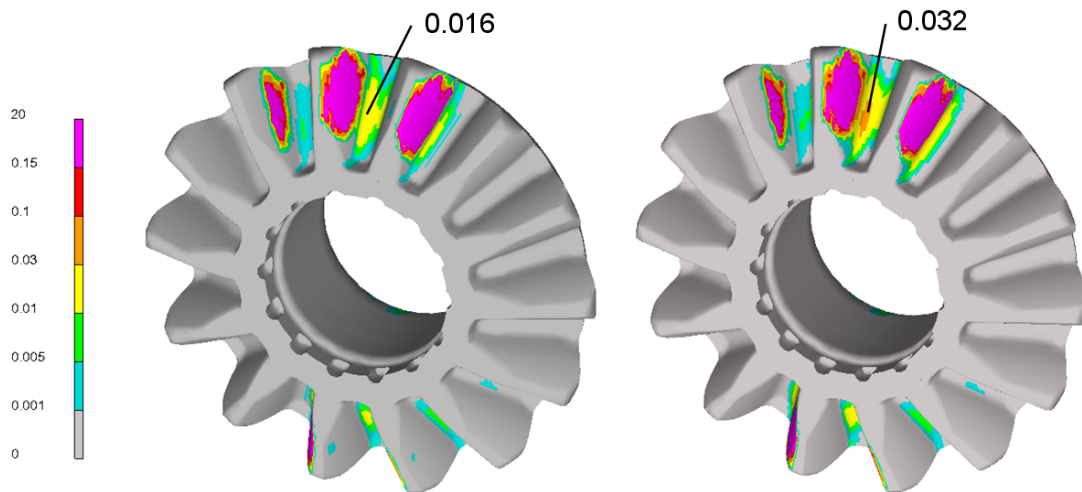


Figure 12. Damage values for the side gear, with hardening stress (left), without hardening stress (right side).

5. Conclusions

A reliable stress based fatigue life prediction method has been presented for bevel gears of a differential. The method considers the hardening process incorporating the induced hardening stresses. It has been shown, that the inclusion of the hardening stresses is essential for a trustable damage result.

6. References

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