

Arbitrary Normal and Tangential Loading Sequences for Circular Hertzian Contact

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Abstract: Stick-slip transitions of nonconforming elastic bodies (such as cylinders or spheres) are a commonly encountered problem in many areas of engineering interest, such as rotating machinery dynamics, tube/support interactions, and granular flows. The theoretically predicted contact zone division into sticking and slipping portions makes the problem difficult to model analytically, even for relatively simple geometries. The resulting distributions of surface shear stress are of particular interest. Although the solution for the simple static case – an initial normal load followed by a tangential load – is relatively straightforward, subsequent application (or removal) of normal and tangential loads serve to complicate the shape of the shear stress distribution so that it becomes not only difficult to solve for, but also difficult to envision. Only limited results of shear stress distributions obtained from experiments or alternative methods, such as finite elements, are available for verification of analytically-based algorithms.

In this paper, analytical surface shear stress distributions for elastically similar spheres under quasi-statically applied normal and tangential loads are compared to those obtained using Abaqus/Standard 6.9. The agreement is quite good, with both methods predicting seemingly paradoxical features such as shear stress free portions of the contact zone, as well as non-zero shear stress distributions that result in zero net tangential force. The finite element results for elastically dissimilar spheres, for which no analytically-based solution exists, show significant differences in the shear stress distributions from those seen for elastically similar materials, due to the additional radially symmetric shear stress caused by the elastic dissimilarity.

Keywords: Hertzian contact; surface stress; material dissimilarity; stick-slip friction.

1. Introduction

The smooth transition from sticking to sliding, or vice versa, between two bodies in contact is a commonly encountered problem in many fields of engineering. It is also a very difficult problem to model effectively. Even for the simple case of Coulomb friction, the classic Cattaneo-Mindlin solution for unidirectional static tangential loading less than the critical sliding value for Hertzian type contact between two non-conforming bodies predicts the presence of two zones of relative coincident point motion within the contact zone: an inner zone of zero relative motion (i.e., sticking of the coincident points on the two bodies) and an outer zone of non-zero relative motion (i.e., slipping of coincident points on the two bodies) (Johnson, 1985). The situation is termed partial-slip, or micro-slip, and is brought about to prevent the presence of the theoretically possible

infinite shear stress that would occur at the edges of the contact zone in the full sticking static case (Mindlin, 1949).

One important distinction is the load case considered by Mindlin. The partial-slip shear stress distribution, which is a combination of the static sticking shear stress distribution and the full sliding shear stress distribution, is only predicted for the load case in which the tangential load follows the normal load. A very different shear stress distribution is predicted if the normal and tangential loads are applied simultaneously (Walton, 1978).

In order to approximate arbitrary load cases, the normal and tangential loads can be applied in a quasi-static manner, with the hierarchical order of normal load changes preceding tangential load changes in each time step. Mindlin and Dersiewicz were the first to present the method for various combinations of applied loads (Mindlin, 1953). Their presentation showed the complex force-displacement relationships that must be adhered to in order to correctly model dynamic non-conforming contact problems with both normal and tangential loading. In the limit as the time step is decreased, this method approaches the result predicted by strictly simultaneous normal and tangential loading.

The shear stress distributions corresponding to these arbitrary loadings are considerably more difficult to model than the basic partial slip case. Jager presented an algorithm in which the appropriate partial-slip distributions are superimposed at each time step in order to provide the overall shear stress distribution (Jager, 1993). Although the algorithm is quite difficult to implement, with complicated rules for scenarios whereby shear stresses on given annuli can be overwritten based on changes to either the normal or tangential force, it does provide the solution for either prescribed forces or displacements.

Some limited confirmations of the assumed shear stress distributions used in developing the analytical methods for various load cases have been obtained. Johnson provided experimental evidence of fretting wear on the outer annulus of the contact zone for oscillating tangential loading of steel ball bearings (Johnson, 1955). This zone of fretting wear corresponds to the zone of relative slip of coincident points predicted by the Cattaneo-Mindlin partial-slip theory. Jager presented comparisons of surface stress distributions obtained from the previous analytical method to those obtained from Ansys 5.5 finite element software. For the load cases included, the agreement is nearly perfect (Jager, 2001).

Not only are the predicted shear stress distributions for arbitrary loads difficult to model mathematically, some are difficult to envision physically. For instance, the analytical method by Jager, which is based on the work by Mindlin and Dersiewicz (Mindlin, 1953), predicts a shear stress free outer annulus for the case of an initial normal load, followed by an initial tangential load, and followed by an increase in the normal load. It is a curious, and somewhat singular, case in the field of solid mechanics in which the stress distribution associated with a load does not act over the entire characteristic area available. For instance, it might be natural to assume that, once new contact annuli are available, the shear stress would spread to these annuli with no net change to the tangential force. Both experimental evidence and alternate simulation (say FEA, for instance) of this particular load case are absent from the literature. Also, the similar load case for tangential loading of elastically dissimilar bodies with Hertzian contact, for which there is no analytically based solution, is of interest.

The following discussion presents the results of an Abaqus/Standard 6.9 finite element simulation for the contact of two geometrically similar half spheres following an initial normal, initial

tangential, increased normal and removal of tangential load sequence. The load case mentioned earlier is included as an intermediate step in the load sequence. Also included are similar results for the case of elastically dissimilar bodies. Comparisons of the FEA results to those obtained from an implementation of the Jager algorithm indicate reasonable agreement between the two methods. The FEA results show that the shear stress distributions for elastically dissimilar materials are fundamentally different from elastically similar materials, even for the same overall loads.

2. Finite Element Model

The geometry, loads and boundary conditions for the finite element simulation are shown in Fig. 1 (a). Fig. 1 (b) shows the level of mesh refinement used for the actual contacting surfaces between the two hemispheres. The load sequence for both the elastically similar and elastically dissimilar cases is shown in Fig. 2. For the elastically similar case, the two hemispheres had a Young's modulus and Poisson's ratio of 206 GPa and 0.3, respectively. For the elastically dissimilar case, one hemisphere had values of 206 GPa and 0.3, while the other had values of 2 GPa and 0.35 for Young's modulus and Poisson's ratio, respectively. In both cases, both hemispheres had a radius of 20 mm. The assumption of linear elasticity may have been slightly violated for the case of dissimilar material, with a maximum strain of 6.3% at the time of maximum combined load (time = 1.5); for the similar materials, the maximum strain was 1.5% at maximum combined load.

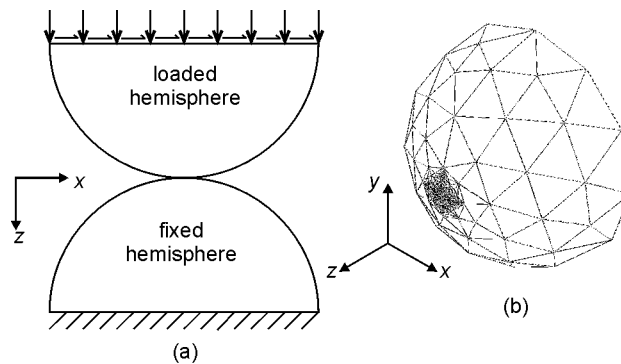


Figure 1. (a) Geometry and load; and (b) Finite element mesh.

The Abaqus/Standard model uses an implicit static numerical integration scheme so that the analysis can be considered as quasi-static with the loads being ramped linearly over the appropriate time period; the time step was chosen automatically by the software. The contact interaction used the surface-to-surface formulation with Lagrange multiplier contact enforcement in both the normal (z-axis) and tangential (x,y - plane) directions. Automatic contact stabilization was used in the simulations. The coefficient of Coulomb friction between the two bodies of 0.25 was used in both the elastically similar and elastically dissimilar cases. Each hemisphere was meshed using 2900 linear tetrahedral elements (C3D4); the two hemispheres used an identical mesh. The same mesh was used for both elastically similar and elastically dissimilar cases considered.

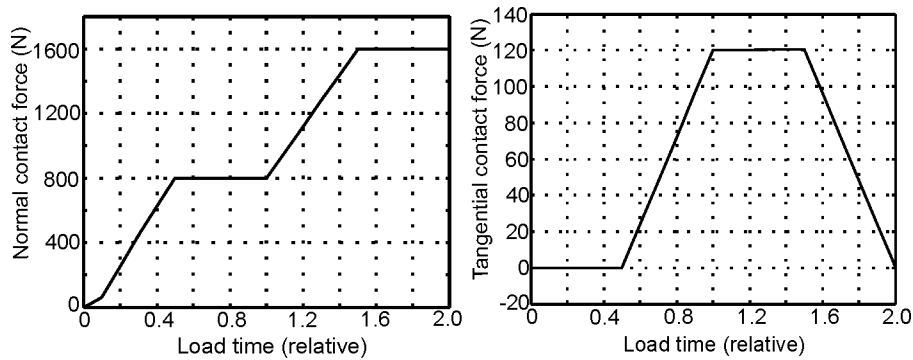


Figure 2. Load histories. (a) Normal load; and (b) X-direction tangential load.

3. Results

3.1 Elastically similar case

Figs. 3 (a) and (b) show the results for the 3 dimensional distributions of normal (CPRESS) and shear (SHEAR1) surface stress for the elastically similar case with time = 1.5. Although agreement of the results to the predicted Hertzian behaviour, in the case of the normal surface stress, and the predicted partial-slip behaviour, in the surface shear stress case, is difficult to ascertain quantitatively, we can see that the qualitative agreement in both cases is reasonable. Both stress distributions show some slight waviness that is not predicted theoretically as well as some small deviation from perfect symmetry. The surface shear stress does show the characteristically different zones of coincident point behaviour discussed earlier.

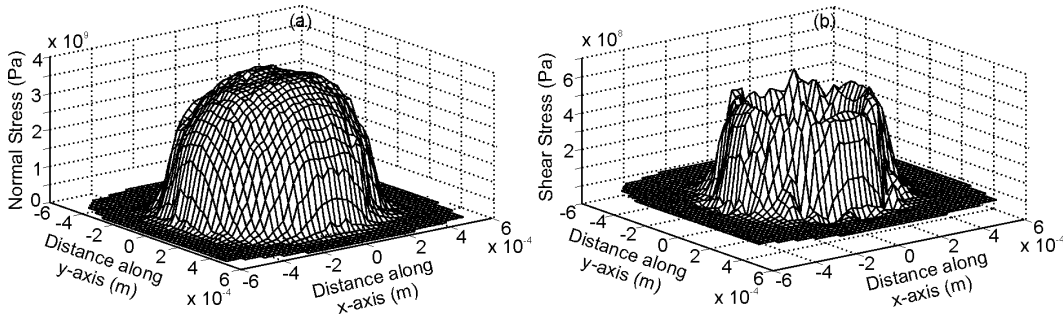


Figure 3. Typical three dimensional stress distributions (time = 1.5) for elastically similar bodies. (a) Normal Stress; and (b) X-direction shear stress.

In order to provide a better appreciation for the evolution of the normal and shear surface stress distributions, as well as some graphical comparison the finite element and theoretical results, the stress distributions along the x-axis of contact are plotted for each timestep (0.1) in Figs. 4 (a) through (t). Here the normal stress distribution is shown as the friction envelope obtained by multiplying the normal stress distribution with the Coulomb friction coefficient.

For the initial normal loading phase (time 0 to 0.5) shown in Figs. 4 (a) to (e), the finite element results indicate the expected increase in contact area and normal stress (Curve A). These results agree reasonably well with the analytical solution (Curve B) even though the finite element results show some slight waviness and a higher maximum normal stress. The finite element shear stress (Curve C) shows nonzero values, indicating that the result is susceptible to some numerical noise; the analytical solution (Curve D) predicts zero shear stress over the contact zone. The agreement in the contact area obtained from both methods is quite good.

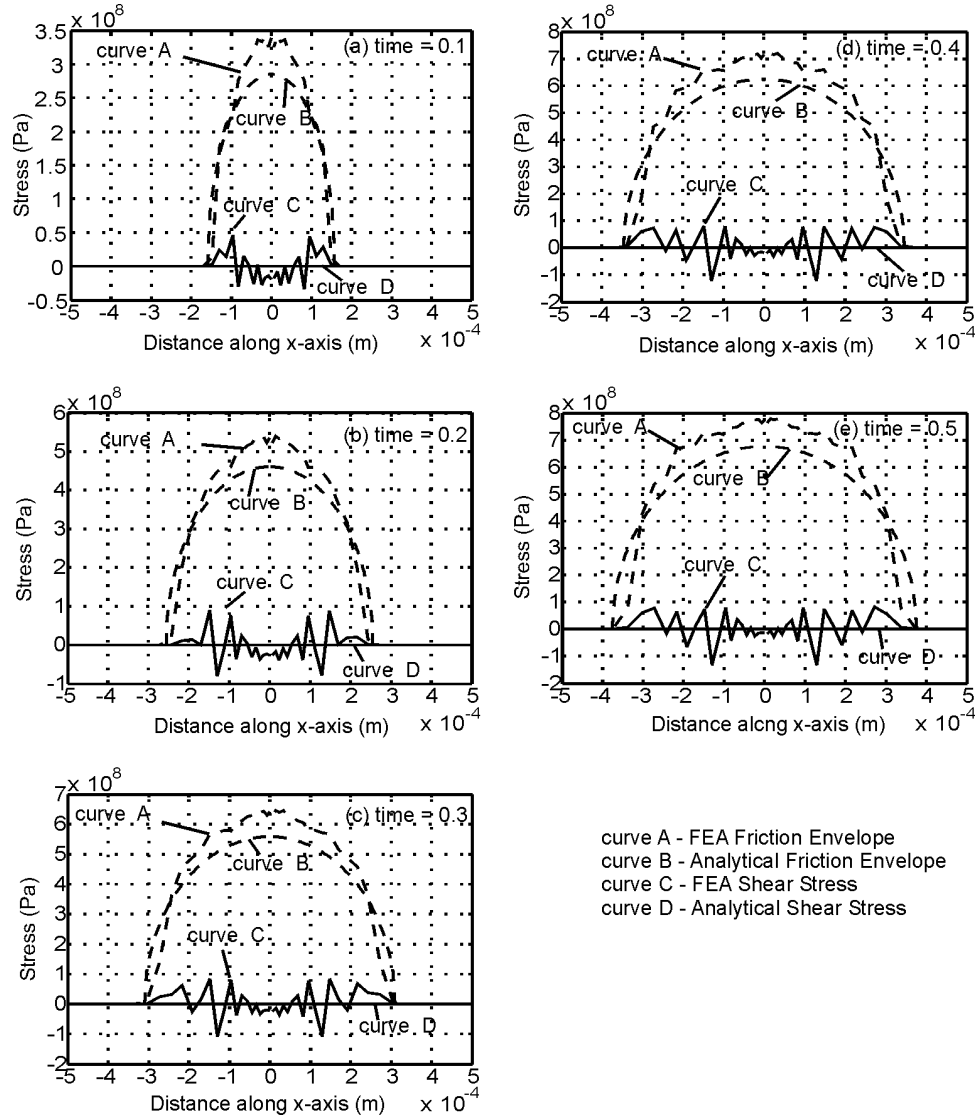


Figure 4 Stress distributions for elastically similar bodies

For the subsequent tangential loading phase (time 0.6 to 1.0) shown in Figs. 4 (f) through (j), the normal stress distribution from both the FEA (Curve A) and analytical (Curve B) simulations remained unchanged as there is no change in the normal force during this phase. The shear stress distribution obtained from the FEA simulation (Curve C) shows the characteristic partial slip distribution predicted by theory, even if there is some apparent numerical noise that contaminates the distribution. As expected, the partial slip boundary grows towards the center of contact as the tangential force is increased. The finite element results for the surface shear stress distribution show very reasonable agreement to the analytical results (Curve D).

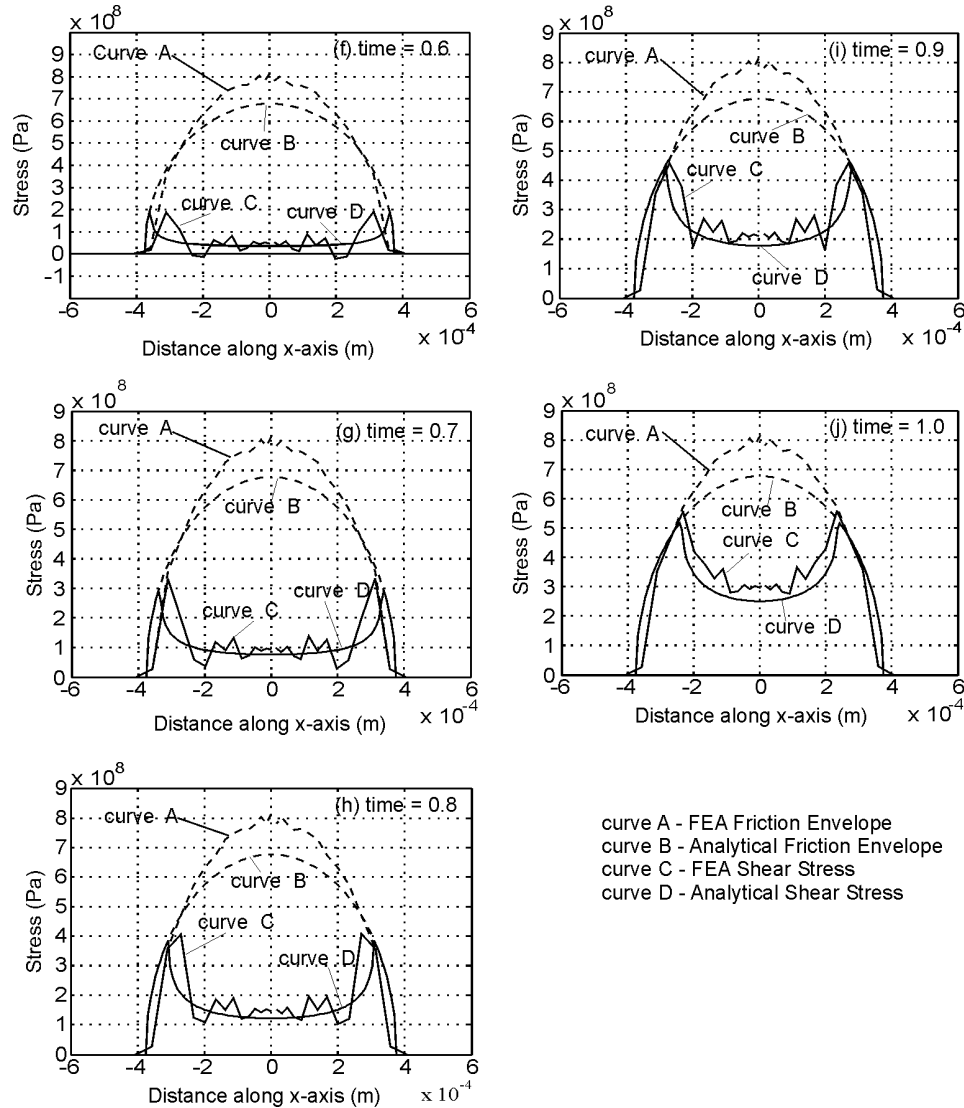


Figure 4 Stress distributions for elastically similar bodies – cont'd

Figures 4 (k) through (o) show the evolution of surface stress distributions for the third loading phase (time 1.0 to 1.5) in which the normal load is further increased. As can be seen, the contact area grows as the normal stress increases for both the finite element (Curve A) and analytical (Curve B) results, with the agreement between the two being quite reasonable with the exception of the apparent noise of the finite element result.

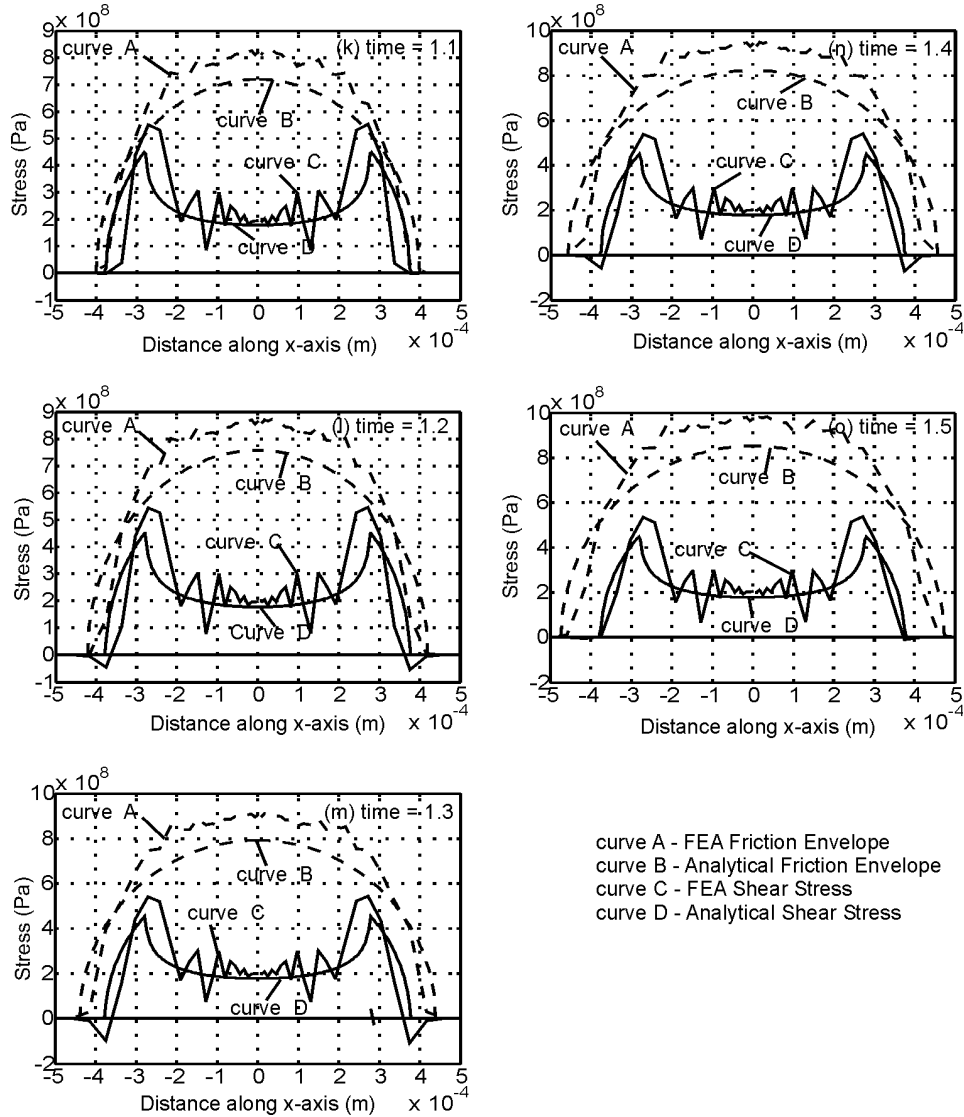


Figure 4 Stress distributions for elastically similar bodies – cont'd

During this loading phase, the tangential load remains unchanged, and an interesting feature of the Mindlin-Dereswicz theory is that the newly laid contact area (i.e., the outer annulus) is free of

shear stress. The finite element results (Curve C), although exhibiting some apparent negative shear stress in this area during the loading, appears to confirm this behaviour. It would seem that these mathematical models do not predict spreading of the shear stress to act over the entire contact zone, although important physical confirmation is still lacking. Overall, the finite element results show good agreement to the analytical results (Curve D).

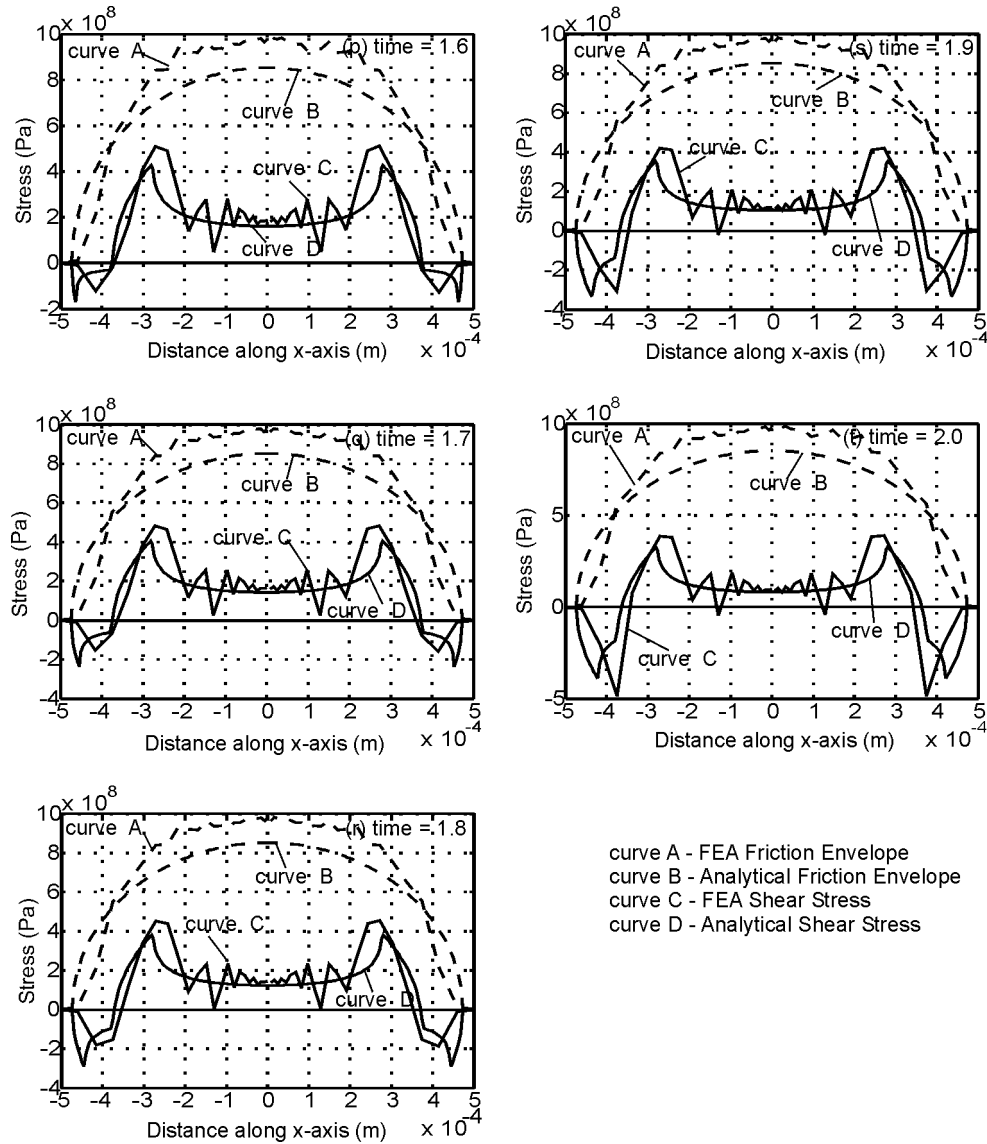


Figure 4 Stress distributions for elastically similar bodies – cont'd

In the final loading phase (time 1.5 to 2.0), the tangential load is gradually removed while the normal load remains unchanged. The surface stress distributions for this phase of loading are shown in Figs. 4 (p) through (t). Since there is no change in the normal load, the normal stress distribution does not change. As opposed to simply removing the existing shear stress over the surface, the analytical solution (Curve D) shows the somewhat more complex behaviour in which the total shear stress distribution becomes a superposition of the existing shear stress due to the initial tangential load (i.e., that which had been present at time 1.0 and acts over the initial contact area) and the new (negative) shear stress distribution which is acting over the entire contact area. By the end of this load phase, the tangential force has reduced to zero but there is non-zero shear stress acting over the entire contact zone.

3.2 Elastically dissimilar case

As mentioned in section 2, the elastically dissimilar case is essentially the same problem as that used in the elastically similar case, with the exception of the different material constants used for one of the hemispheres. For the most part, the normal stress distribution results for the elastically dissimilar case show the same basic features as the elastically similar case, namely, Hertzian type distributions with some apparent numerical noise. Owing to the effective combination of elastic constants of the two materials, the contact area and maximum normal stress will be different for the elastically dissimilar case compared to the elastically similar case for the same normal load.

Figures 5 (a) and (b) show typical (time = 1.5) 3 dimensional distributions of normal and shear stress, respectively, obtained from the finite element simulation. Here we see that the finite element solution gives somewhat unsmooth distributions as opposed to the typically smooth distributions assumed by analytical solutions. These “bumps” on the distributions are again believed to be numerical noise associated with the solution. The shear stress distribution has a significantly different characteristic shape than the corresponding distribution at the same time for the elastically similar case seen in Fig. 3 (b). Since the differences in the shear stress distributions between the two cases is the feature of interest, the remaining discussion focuses on these and very little discussion of the normal stress is included.

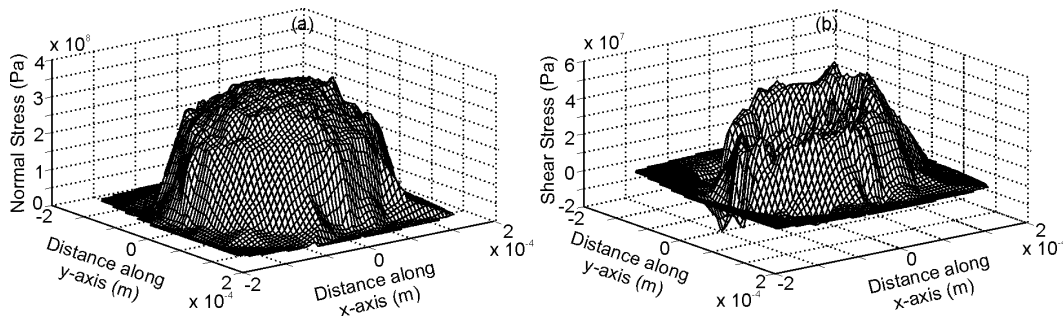


Figure 5. Typical three dimensional stress distributions (time = 1.5) for elastically dissimilar bodies. (a) Normal Stress; and (b) X-direction shear stress.

The stress distributions along the centerline of the x-axis (i.e., in the direction of tangential loading) at each timestep is shown in Figs. 6 (a) through (t). The normal stress distribution is once again included as the friction envelope in these figures. Comparison of the finite element results to

a continuum model solution is not available, as there is no analytical solution for the shear stress distributions when there is arbitrary variation of loads in both the normal and tangential directions.

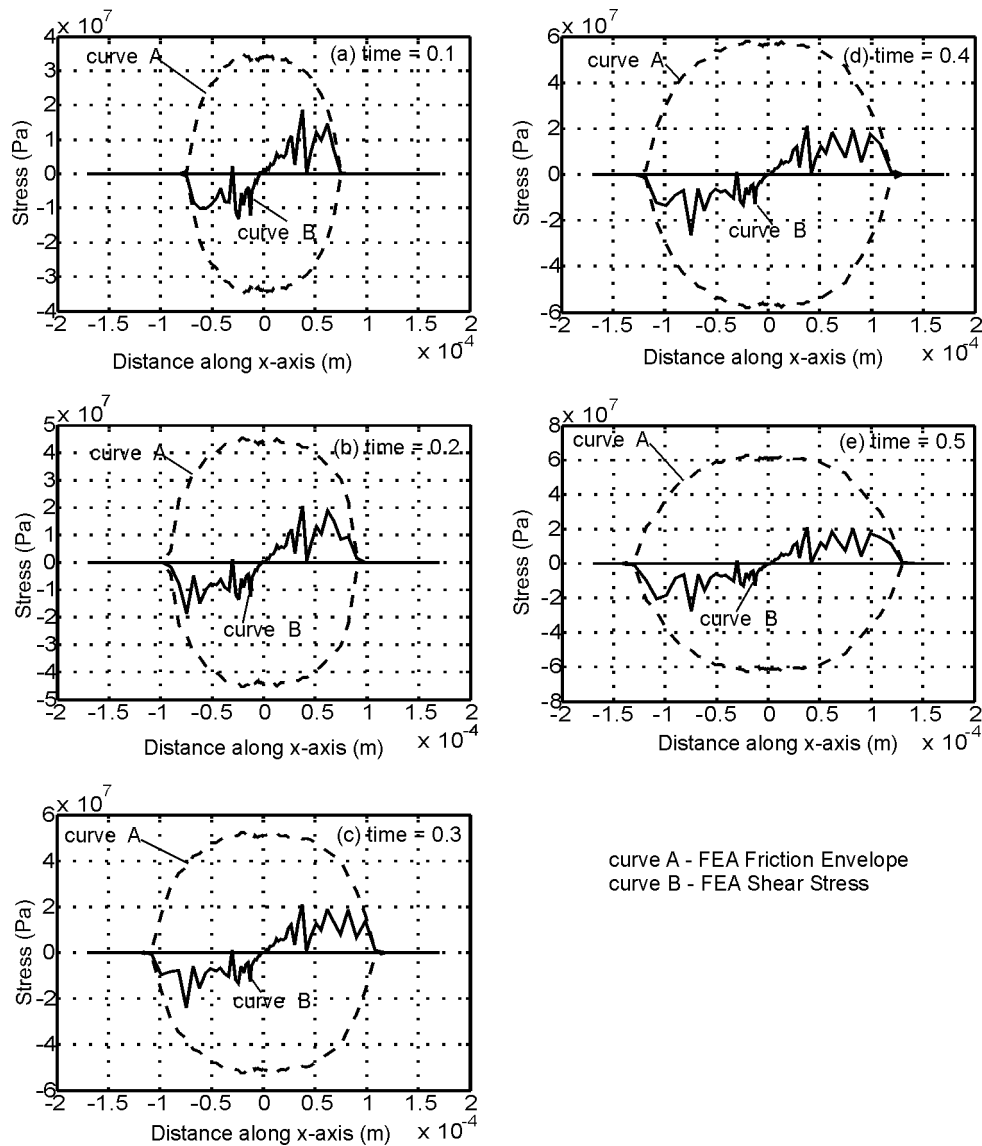


Figure 6 Stress distributions for elastically similar bodies

During the initial normal phase (time 0 to 0.5) shown in Figs. 6 (a) to (e), we can see that the finite element solution gives an antisymmetric shear stress distribution. Even though there is significant noise superimposed on these distributions, the general shape of the shear stress distributions

matches that predicted for static normal loading of bodies with dissimilar elastic constants (Johnson, 1985). The shear stress acts radially inward on the more rigid body and radially outward on the more flexible one. In the outer annuli of contact, the shear stress lies on the friction envelope (positive or negative) which indicates that the coincident contact points in this area are in a state of local slip. Summation of this stress distribution over the contact area gives a zero net tangential force.

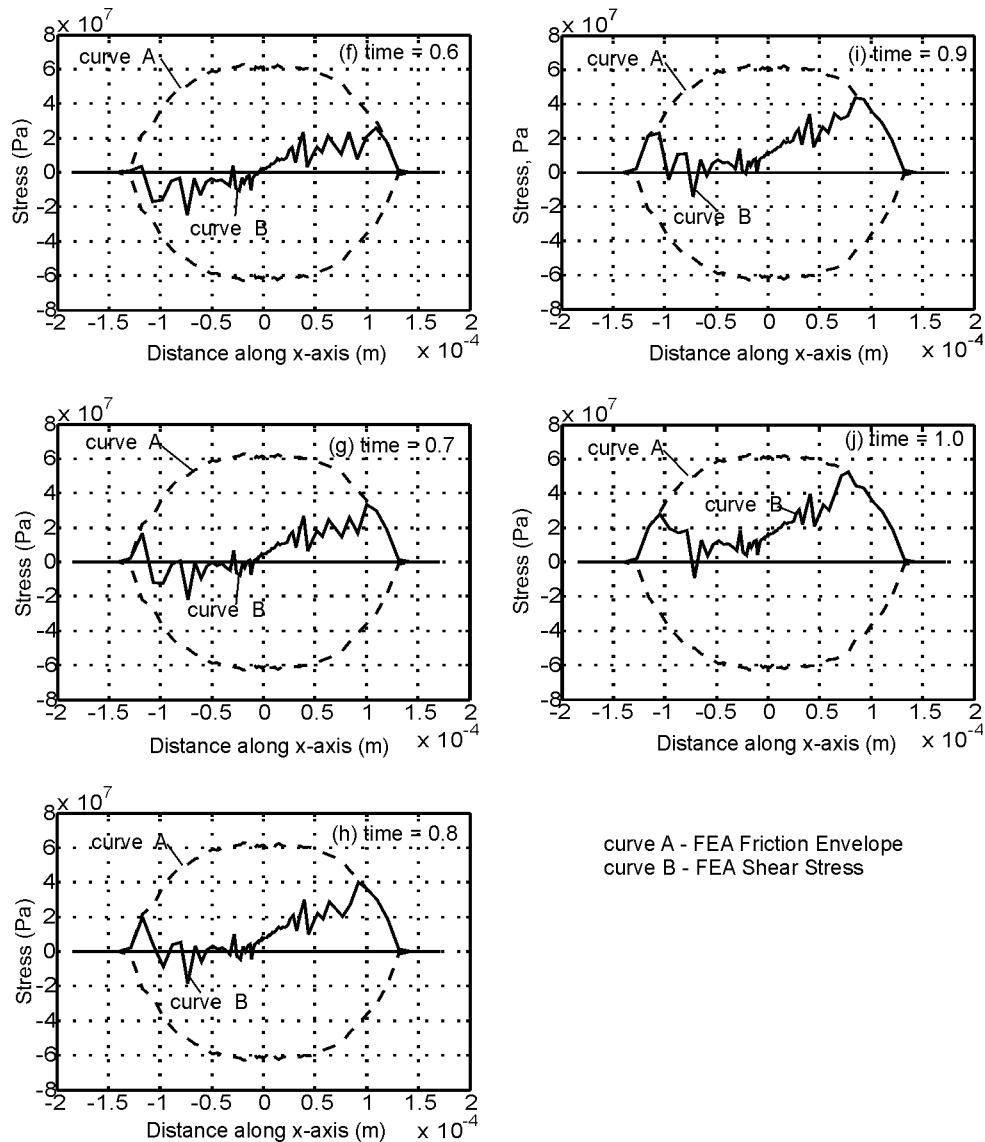


Figure 6 Stress distributions for elastically similar bodies – cont'd

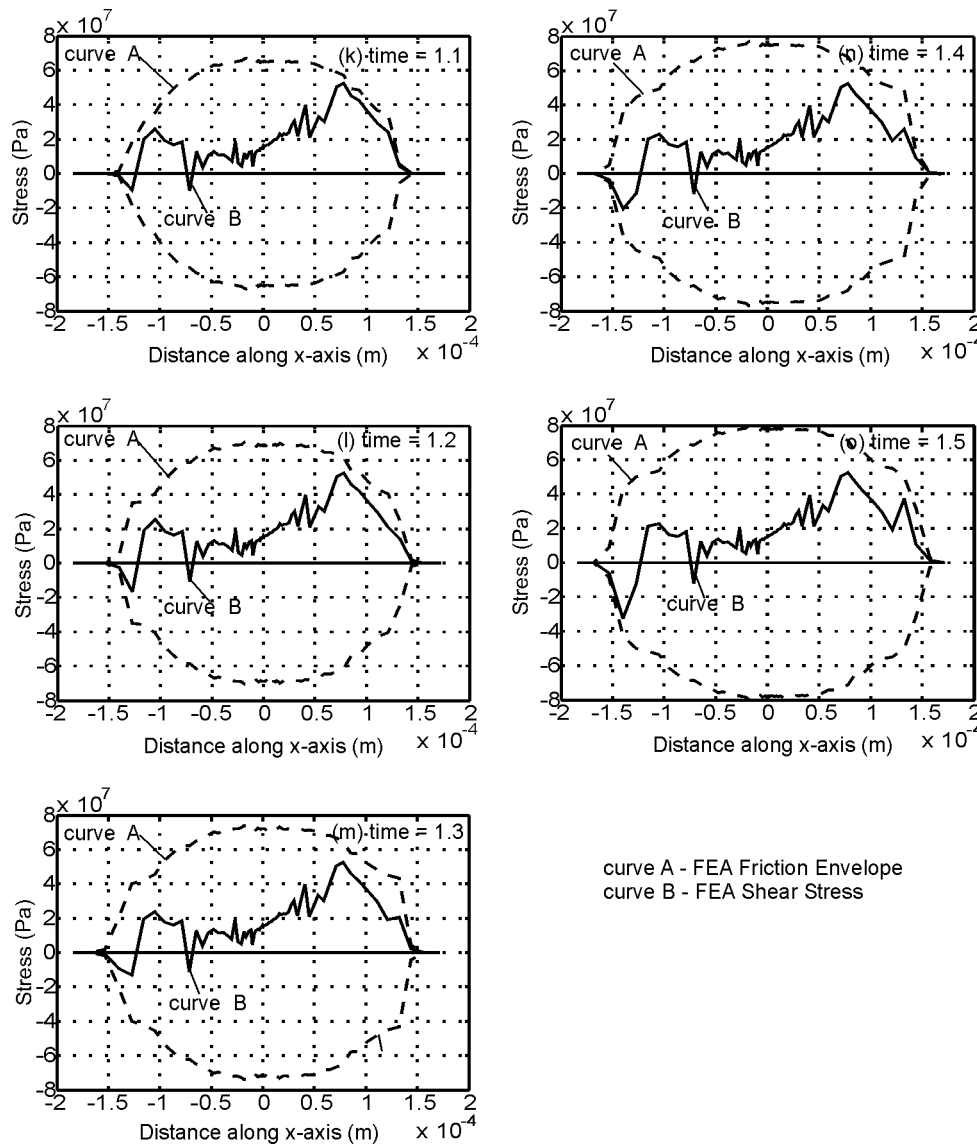


Figure 6 Stress distributions for elastically similar bodies – cont'd

Figures 6 (f) through (j) show the stress distribution results for the x-direction tangential loading phase (time 0.6 to 1.0). As the tangential load is increased, we can see that the radial symmetry of the distribution is lost and the shear stress distribution moves further above the zero line while retaining, to some extent, the radially symmetric distribution associated with application of the initial normal load. As such, one can envision these distributions as arising from a superposition of the radially symmetric distribution of Fig. 6 (e) and the axisymmetric distributions of Fig. 4 (f) to (j). This superposition is not perfect; the radially symmetric distributions are not retained due to

limitations of the shear stress by the friction envelope. As such, the radius value at which local slip occurs is smaller on the positive x-axis than on the negative x-axis. In other words, the percentage of the contact zone experiencing local slip is higher on the positive x-axis, which corresponds to the direction of loading.

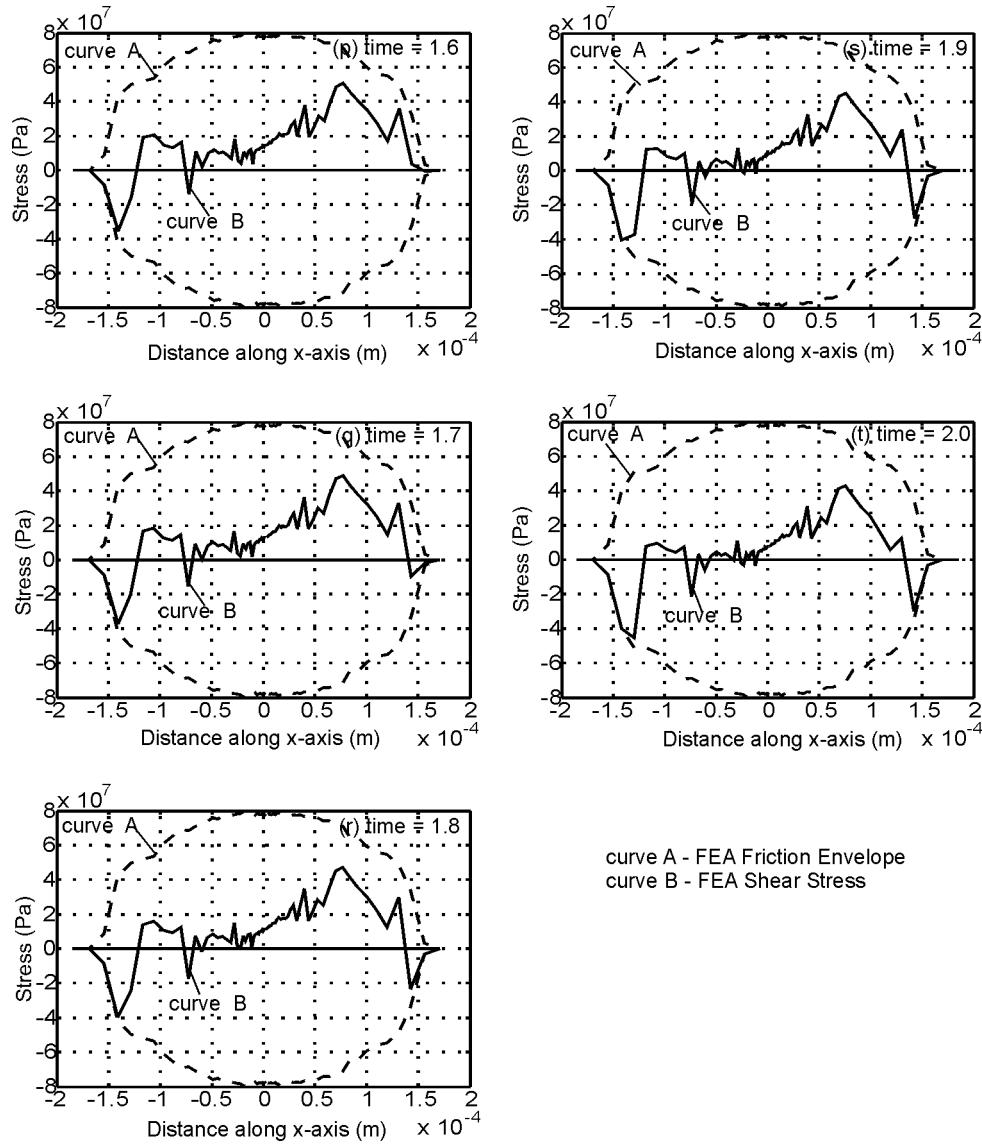


Figure 6 Stress distributions for elastically similar bodies – cont'd

In the third phase (time 1.1 to 1.5), the tangential load is held constant while the normal load is increased. The stress distributions of Figs. 6 (k) to (o) show that the friction envelope moves away from the shear stress so that coincident points that were experiencing local slip are now sticking. Although the shear stress is quite complex, it appears that there is a further superposition of additional radially symmetric shear stress on the distribution that existed at the beginning of this load phase.

In the final load phase (time 1.6 to 2.0), the tangential force is gradually removed. As can be seen in Figs. 6 (p) to (t), the shear stress distribution is, once again, everywhere non-zero. The characteristic shape is even more complex, but there once again appears to be a superposition of a unidirectional shear stress, this time negative, in addition to the shear stress distribution that existed at the beginning of the final load phase. For large magnitude x values, the larger values of shear stress reach the friction envelope, indicating larger areas of slip in these parts of the contact zone. Unfortunately there is no analytical solution available for comparison, but there does appear to be some contamination of the shear stress solution by numerical noise on all of the results given in Figs. 6 (a) through (t). Even with the noise, at time = 2.0 the tangential shear stresses give a net tangential force of essentially zero.

4. Conclusion

Several interesting features of contact loading are shown in the results. Firstly, it is evident that the present finite element method of solution is susceptible to apparent numerical noise. The analytical solution methods are based on assumed shapes of the stress distributions, whereas the finite element method uses a mixed geometric/force constraint approach applied at local nodes to obtain these stress values.

Even with this numerical noise issue, it appears that the finite element method predicts the same essential characteristics for the evolution of shear stress distribution for elastically similar circular contact as both the normal and tangential loads between the two bodies are varied. Through the simple load history examined, it has been shown that, when the normal load is increased following a normal and tangential loading phase, both FEA and continuum models predict that the new area of contact will be free from shear stress. Also, both methods show that an everywhere non-zero shear stress distribution is predicted when the tangential load is subsequently removed. Although the agreement of these independent numerical models is encouraging, important physical confirmation of these behaviours through experiments is still lacking.

Finally, it has been shown that the shear stress distribution results for contact of elastically dissimilar bodies is much more complex than those found for elastically similar bodies. The finite element results for this case appear to show that the same basic principle of superposition of new shear stress distributions with those existing at the beginning of the load phases applies. However, the resulting distributions take on much different characteristic shapes due to the presence of the radially symmetric shear stress that accompanies normal loading of the dissimilar bodies. Unfortunately, the correctness of the finite element results can not be confirmed because an analytically based solution has yet to be formulated.

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