

Application of Abaqus for Practical Postbuckling Analyses of Cylindrical Shells under Axial Compression

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Abstract: For the buckling problem of circular cylindrical shells under axial compression, a number of experimental and theoretical studies have been made by many researchers. In the case of the very thin shell that exhibits elastic buckling, experimental results show that after the primary buckling, secondary buckling takes place accompanying successive reductions in the number of circumferential waves at every mode shift on systematic (one-by-one) basis. In this paper, we traced this successive buckling of circular cylindrical shells using the latest in general-purpose FEM technology. We carried out our studies with three approaches: the arc-length method (the modified Riks method); the static stabilizing method with the aid of (artificial) damping especially, for the local instability; and the explicit dynamic procedure. The studies accomplished the simulation of successive buckling following unstable paths, and showed agreement with the experimental results. As the example of practical application of this simulation method, a comparison with high-speed photography and applicability to viscoelastic buckling problem will be discussed.

Key words: Shell, Buckling, Riks method, Artificial damping, Explicit dynamic

1. Introduction

Utilizing the postbuckling analysis is an efficient way to find the ultimate strength in the final stage of deformed structures as well as for evaluating the effect due to initial imperfections. This study covers the case of a cylindrical shell structure that exhibits unique behavior largely different from the behavior of a column or a plate. This paper will present the findings from our attempt to derive solutions on the postbuckling strength for this cylindrical shell using the latest in general-purpose FEM technology. Elastic buckling of cylindrical shells under axial compression is regarded as a fundamental issue in the buckling of shell structures. It is probably fair to say that the foundations of shell stability theory were almost completely laid in the study of this subject [1]. In the field of actual products, the application of this buckling problem has been widened, beginning with the design of aircraft fuselages or liquid storage tanks and extending to cover new fields, such as impact energy absorption mechanisms. The classical theoretical solution to critical buckling stress can be expressed as in Eq. (1), which was derived already in 1910s.

$$\text{Classical elastic critical stress: } \sigma_{cl} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{r} = 0.605E \frac{t}{r} \quad (1)$$

Eq. (1) was originally derived using the energy method in which axisymmetric waves were assumed to form along the entire length of the cylinder (i.e. a corrugated appearance with waves only in the axial direction). However, it should be noted that the axisymmetric deformation assumed here can be substantiated only if the circumferential length of the cylinder varies as the deformation progresses. The form of buckling accompanied by changes in the length of the member is known as ‘extensional buckling’.

According to the modern stability theory, the classical result Eq. (1) also gives the critical stress with a mode shape that is sinusoidal both axially and circumferentially. At this critical stress, a very large number of different buckling modes or eigenmodes are all simultaneously critical (sometimes over 100 modes with loads within 1%) [1]. Many modes are possible in different non-axisymmetric patterns whose wavelengths in the axial and circumferential directions are related by the ‘Koiter circle’ [2], [3]. The non-axisymmetric buckling pattern typically observed in experiments is shown in Fig. 1 in the following section.

Buckling loads observed in experiments have always been found to be not only much lower than the loads predicted by Eq.1, but also in widely scattered distribution, even though most of the failure in these relatively thin shells was due to elastic instability. The length of the cylinder was found to have little effect on the buckling load, unless it was very short, and similarly the effect of end condition of the cylinder was small. It was empirically known that the actual measurement results were only 10-60% of the theoretically predicted values. This discrepancy made the researchers motivated to develop a large deflection theory for the shell with imperfect geometry.

It was not until the 1950s that it was fully understood that the interaction between unavoidable imperfections and the ill-natured postbuckling behavior was the reason for the large discrepancies between observed critical loads and the predictions of the classical theory. Karman and Tsien [4] and Donnell and Wan [5] were the first to calculate complete load-displacement curves of axially compressed cylinders with perfect and imperfect geometry, respectively, using nonlinear large-displacement formulations of Donnell’s shell theory. They showed that a buckled shell in the deep postcritical range can be in equilibrium under a much smaller load than the classical buckling load predicted by small deflection theory.

2. Nomenclature

E	Young’s modulus
L	Cylinder length
m	Number of half-waves in the axial direction
n	Number of full-waves in the circumferential direction
P	Axial load ($P=2\pi r t \sigma$)
P_{cl}	Classical elastic critical load
P_{cr}	Critical load
r	Cylinder radius (mean)
T	Temperature

T_g	Glass transition temperature
t	Shell wall thickness
w_0	Amplitude of imperfection
Z	Batdorf parameter to characterize shell length
α	WLF shift factor for temperature-time reduction
δ	Axial shortening of cylindrical shell
ν	Poisson's ratio
σ	Axial stress
σ_{cl}	Classical elastic critical stress
σ_{cr}	Critical stress

3. Findings from Experimental Research

There are systematic findings obtained by a group led by Yamaki et al. for the elastic buckling of cylindrical shells (not limited to the case of axial compression). The product of their experiments and theoretical approaches are covered with literature [6] in the Reference list. Fig.1 shows their typical results. All the cylinder shells were constructed from Mylar (polyethylene terephthalate) film and finished with the careful cleanup of geometric imperfections. Each shell was fully fixed at both ends, and then compressed in the axial direction. The test shells provided are those with the same diameter but with different heights from each other. The Batdorf parameter Z is defined as follows,

$$Z = \sqrt{1 - \nu^2} \frac{L^2}{rt} \quad (2)$$

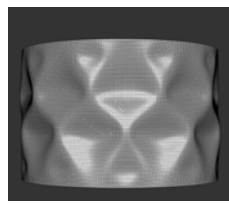
A typical pattern of the diamond buckling is shown in Fig.1(a). This photograph shows the resultant deformation of the test specimen with a length equivalent to $Z=500$ compressed up to the axial shortening $\delta=0.606\text{mm}$. The cylindrical shell deformed in a sinusoidal form of which wave was observed as having axial half-waves of $m=2$ and circumferential full-waves of $n=10$. The buckling mode with $m=2$ is considered a two-tier diamond pattern, and Esslinger observed the same deformation pattern using her high-speed photography [7], [8]. (In this observation, the test specimen used by Esslinger and Yamaki may be identical in terms of its cross-sectional shape and material.) Please note that this two-tier pattern is asymmetric with respect to the cross-section at the mid-length of the cylindrical shell. For the load-displacement curves plotted in Fig.1(b), the deformation path corresponding to this asymmetric buckling mode is drawn by solid lines. Within a region where the resultant load was lower than that on this deformation path, there is a path corresponding to the one-tier symmetric mode with $m=1$. Yamaki, however, points out that such symmetric mode would usually never appear unless some type of external interference (e.g., appropriate adjustment of the shell wall with fingertips) occurred.

The load-displacement curves in Fig. 1(b) represent successive occurrences of buckling modes in which the first peak corresponds to the primary buckling. While the classical buckling load P_{cl} can be calculated as 1290N from Eq. (1) for this test condition, the loading at the primary buckling as shown in Fig. 1(b) is about 900N, which is 30% lower than the calculated figure. From inspecting Fig. 2 shown later, the test cylinder is estimated to have had the initial imperfection equivalent to

10% of the shell thickness of 0.247mm. After the primary buckling, the deformed pattern of a cylinder progresses discontinuously while the number of waves in the circumferential direction decreases. The number of circumferential waves was first found to be $n=12$, and then was subsequently observed up to $n=9$, as shown in the figure. Such secondary buckling behavior occurs in the region where deformation has been well progressed, and therefore this behavior is considered not to have been influenced by the initial imperfection. The primary purpose of this study is to simulate this unstable but imperfection-insensitive secondary buckling behavior of geometrically perfect cylindrical shells.

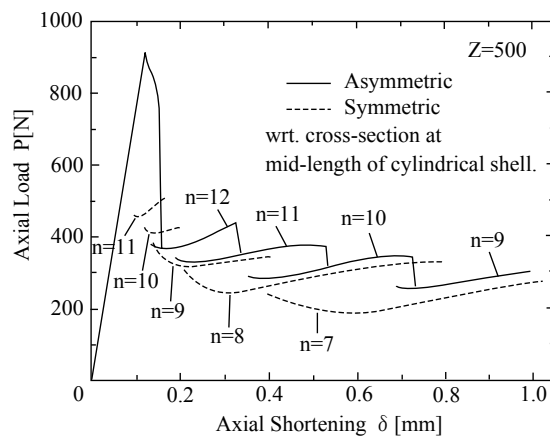


Experiment (Yamaki)



FEM (This study)

$Z=500$ ($r=100\text{mm}$, $t=0.247\text{mm}$, $L=113.9\text{mm}$)
 $E=5560\text{ MPa}$, $\nu=0.3$



(a) Deformation of test cylindrical shell ($\delta=0.606\text{mm}$)

(b) Load-displacement curves

Figure 1. Experimental study for postbuckling of elastic cylindrical shell under axial compression, Yamaki et al. [6].

4. Postbuckling Analysis Using General-Purpose FEM

A new trend is ongoing such that the stability analysis of shell structures with very sophisticated approaches are being attempted to switch to the analysis using some general-purpose FEM codes instead. As a typical case, Croll and his group [12] reported using a general purpose FEM code in lieu of using the reduced-stiffness method that has regularly contributed for obtaining remarkable achievement. Now we proceed with the study to reproduce the Yamaki's experimental results using a general-purpose finite element code Abaqus [13].

4.1 Practical postbuckling analysis and imperfections

As mentioned previously, it is a critical point that the buckling loads observed in experiments have always been found to be not only much lower than the loads derived from the classical theories, but also in widely scattered distribution. Besides these buckling loads, involvement of the difficulty with predicting their buckling modes complicates the problem even further. Typically, the European standard for steel shell structures [14] regulates that, when geometric nonlinear analysis is applied for the structures with explicit representation of initial imperfection, if any specific pattern of the imperfection cannot be identified as presenting a high risk of structural damage, a possible range including risky imperfection dimensions should be explored. The code also recommends that the imperfection should be specified in terms of the buckling modes

obtained from linear elastic bifurcation analysis, unless undesirable imperfection geometry cannot be identified individually. It should be noted that the buckling modes resulting from the linear elastic bifurcation analysis were not necessarily consistent with the buckling modes occurring in real cases [15]. For remarks on conducting the actual design, refer to the guidelines found in literature by Rotter [1].

From the above-mentioned point of view, this study mainly aimed to prove that the analysis for the cylindrical shell structure with perfect geometry could be performed through the region extending to deep postbuckling range using the general-purpose FEM code. Due to some restraints imposed on the analysis, a linear buckling mode was employed as the initial imperfection so that it could trigger steering deformation patterns to the bifurcation path. However, we recognize it as a method that is adopted simply as an analytic tactic. Fig. 2 shows the relation between the buckling stress and the amplitude of imperfection, which was organized by Rotter [1].

When the amplitude of imperfection is about 0.01 of the shell thickness, the buckling response of the cylindrical shell under axial compression is estimated as being very close to the result obtainable from the shell with perfect geometry. The linear buckling modes were so scaled that they could be mapped on the initial perfect geometry in order to generate perturbed mesh. The range of perturbation as 0.01 times the shell thickness was used in this study.

4.2 Arc-length method

Geometrically nonlinear static problems sometimes involve buckling or collapse behavior, where the load-displacement response shows a negative stiffness and the structure must release strain energy to remain in equilibrium. Several approaches are possible for modeling such behavior. Among them, path tracing based upon arc-length method (modified Riks method in Abaqus) is the most fundamental procedure. This method is used for cases where the loading is proportional; that is, where the load magnitudes are governed by a single scalar parameter. The arc-length method works well in snap-through problems—those in which the equilibrium path in load-displacement space is smooth and does not branch. Generally, you do not need take any special precautions in problems that do not exhibit branching (bifurcation).

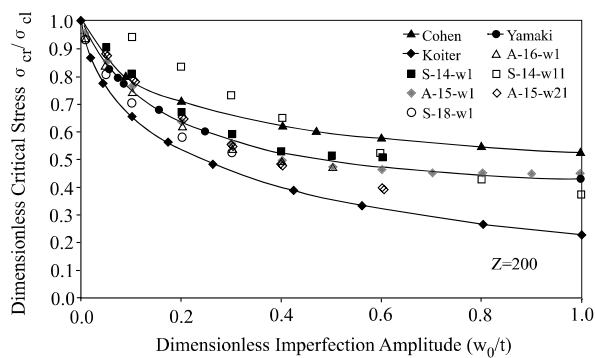


Figure 2. Sensitivity of bifurcation load to amplitude of non-axisymmetric imperfections, Rotter [1].

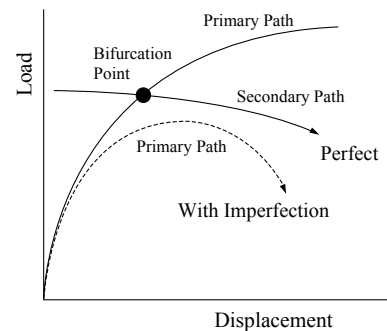


Figure 3. Smoothing bifurcation discontinuity by introducing imperfections.

The arc-length method can also be used to solve postbuckling problems with bifurcation. However, the exact branch-switching problem cannot be analyzed directly due to the discontinuous response at the bifurcation point. To analyze a branch-switching problem, it must be turned into a problem with continuous response instead of bifurcation. This effect can be accomplished by introducing an initial imperfection into a perfect geometry so that there is some response in the buckling mode before the critical load is reached. From the buckling theory aspect, this operation is equivalent to converting the bifurcation point to the limit point, as shown in Fig.3. That is, the path after bifurcation, which is naturally the secondary path, is changed to a smooth primary path. Imperfections are usually formed with perturbations in the geometry of structures. Imperfection represented with a linear buckling mode can be advantageously applied to the practical analysis as described in the above.

4.3 Artificial damping method

The arc-length method realizes the stable analysis process under the global load control. If the analysis process traces unstable paths under the global load-displacement response with negative stiffness, the arc-length method is effectively usable. However, if the instability is localized (e.g., surface wrinkling, material instability, or local buckling), there will be a local transfer of strain energy from one part of the model to neighboring parts, and global solution methods may not work. This class of problems has to be solved either dynamically or with the aid of (artificial) damping. Buckling of a real, thin-walled shell is typically a local phenomenon that may be triggered by a small, local disturbance. It has for instance been observed by high-speed photography [7], [8] that when an isotropic cylindrical shell is compressed axially, single relatively small buckles form at the beginning of the buckling process.

Abaqus provides an automatic mechanism for stabilizing unstable quasi-static problems through the automatic addition of volume-proportional damping to the model. Viscous forces of the form are added to the global equilibrium equations. Where is an artificial mass matrix calculated with unity density, c is a damping factor, $\dot{u}$ is the vector of nodal velocities, Δt is the increment of time (which may or may not have a physical meaning in the context of the problem being solved), F is the total applied load, and F_{int} is the structure's internal force. When local instability occurs, the deformation rate of that portion begins to increase, and consequently, locally released strain energy is dissipated due to the appended damping effect. The ratio of the dissipated energy to the strain energy is called the energy ratio (Dissipated Energy Fraction), and it has a default value of 2.0E-04 in Abaqus. The damping coefficient should be appropriate for the purpose (i.e., not too large); the damping coefficient applied in our analysis was lowered to 1/1000 of the default.

4.4 Explicit dynamic analysis

Nonlinearities are usually more simply accounted for in dynamic situations than in static situations because the inertia terms provide stability to the system; thus, the method is successful in all but the most extreme cases. Especially the explicit integration method is more efficient than the implicit integration method for solving extremely discontinuous events or processes. The explicit dynamics procedure performs a large number of small time increments efficiently. An explicit central-difference time integration rule is used; each increment is relatively inexpensive because there is no solution for a set of simultaneous equations. Many of the advantages of the explicit procedure also apply to slower (quasi-static) processes. The postbuckling behavior of axially compressed cylindrical shells may be represented by dynamic mode jumping. From the viewpoint

of the history in the traditional research work, it is reasonable for us to consider that the essential nature of postbuckling behavior can be regarded as a static problem, even though it may be possible to raise some side issues such as occurrence of overshooting in deformation due to the inertia effect. In this study, the explicit dynamic analysis method was applied. However, the reason for doing so was simply to rely on the robustness of the analysis process, not to conduct the simulation for the dynamic effect. A sufficiently slow axial velocity of 1mm/sec was applied to compress the edge of the cylindrical shell in the analysis.

5. Numerical Examples

5.1 Analysis model

We carried out some analysis work in order to trace the findings by Yamaki et al., as summarized in Fig. 1. The analysis model is shown in Fig. 4. The object of the analysis is a model with $Z=500$. This model was fully constrained at its top and bottom ends and then compressed in the axial direction. As it is assumed some asymmetric deformation patterns are likely to occur, the entire body of the cylinder was modeled. Since any buckling mode could conceivably be sensitive to initial imperfections, all numerical data for the coordinates were carefully determined to maintain consistency in precision, so that an ideal geometry of the cylindrical shape could be generated. The model was divided into 80 meshes in the axial direction and 400 meshes circumferentially.

Making a mesh of this size was necessary to map 20 waves as the maximum number of the circumferential waves to be analyzed in this study. The shell element S4R implemented in Abaqus is used for the analysis. As this element has a great advantage such that it can be used to model both thin shell and thick shell structures for the strain-concerned applications, this element is currently widely spread for use in industrial application problems. The reduced integration scheme helps in reducing the amount of CPU time. It was fortunate for us that no difficulty in numerical convergence due to possible hourglass behavior in this reduced integration element was met during the entire analysis process. We have confirmed that the fully integrated shell element S4 also gives a comparable result to S4R.

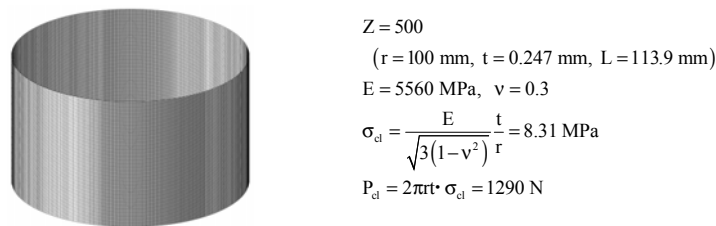


Figure 4. Analysis model for Yamaki's cylindrical shell.

5.2 Eigenvalue analysis

A typical buckling mode obtained from the linear eigenvalue analysis is shown in Fig. 5. In total, more than 100 buckling modes were extracted within a range of 1.01~1.04 of the classical elastic critical stress. The extracted buckling stress is slightly higher than the theoretical stress, because the length of the analysis model is finite. Hunt et al. [18] gave Eq. (3) as a relationship between the

number of axial half-waves m and the circumferential full-waves n , for which they utilized the concept of Koiter Circle [2], [16]. They argue that this equation gives results that match the experimental results derived by Yamaki et al. [6] and Esslinger et al. [17]. The equation identical to this one was also derived from the equilibrium equations for cylindrical shells as found in the literature by Timoshenko [19].

$$n^2 = m\pi\sqrt{12(1-\nu^2)}\frac{r}{L}\sqrt{\frac{r}{t}} - m^2\pi^2\left(\frac{r}{L}\right)^2 \quad (3)$$

As shown in Fig. 6, the curve for the relationship between m and n calculated from this equation becomes upward convex, and accordingly, the maximum number of the circumferential waves is $n=18$ for the condition in this study. The number of waves in every eigenmode obtained from FEM analysis is plotted in Fig. 6; they showed close agreement with the results calculated by Eq. (3). The first mode obtained from FEM analysis was axisymmetric with $n=0$ and $m=13$. In accordance with the classical theory of axisymmetric buckling, the value of m can be represented by Eq. (4) [19], and the FEM solution as $m=13$ is confirmed to be coincident with this theoretical solution. Please note that Eq. (4) yields to the result obtained from Eq. (3) if $n=0$.

$$\frac{L}{m} = \pi\sqrt{\frac{r^2 t^2}{12(1-\nu^2)}} \quad (4)$$

Consequently, extracting eigenmodes by FEM starts from the side with bigger m as shown in Fig. 6, and as the order of mode becomes higher, the value of m tends to reduce (i.e., the direction along the arrow head shown in Fig. 6). On the contrary, the number of axial half-waves m actually observed can be estimated as 3 to 4 at the most for the relatively short cylinder dealt with in this study [7], [8]. For the relationship between the eigenmodes obtained from FEM analysis and the deformation patterns actually observed, the authors believe further review is necessary. Although such circumstances exist, using the results from the linear eigenvalue analysis for the initial imperfection was found to be the most convenient method in practice. Therefore, some appropriate modes were chosen among the obtained linear normal modes (a random choice was made with caution to not omit the necessary number of waves), and they were applied to the incremental analysis.

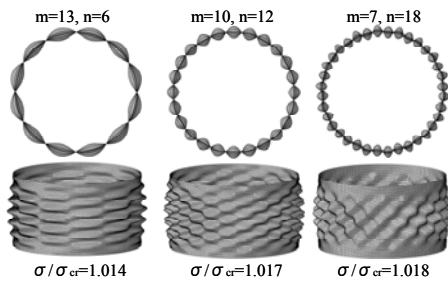


Fig.5. Results from eigenvalue analysis.

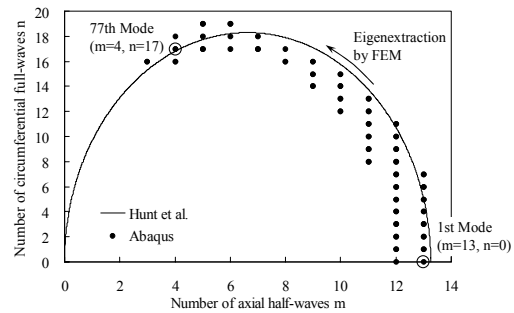


Fig. 6. Wave number space for primary critical load.

5.3 Artificial damping analysis and explicit dynamic analysis

Abaqus enables us to achieve stabilization of the analysis with local instability using the artificial damping method. The load-displacement curves obtained from this analysis are shown in Fig. 7. As shown in Fig. 6, the eigenmodes of the cylindrical shell used in this study have the number of the circumferential full-waves n up to 18 as maximum. For simplicity in practical application, 18 linear eigenmodes were selected corresponding from $n=1$ to $n=18$ sequentially from the lowest order of the eigenmode, and a combined wave shape formed by superimposing those wave patterns was assigned as the initial imperfection. The amplitude of the initial imperfection assigned to the analysis was 0.01 of the shell thickness. As previously mentioned, the magnitude of this initial imperfection is adequately small, and thus such an attempt to apply the initial imperfection intends to perform analysis for a geometrically perfect cylindrical shell. Since the shape of each linear eigenmode appears to be very complicated, the initial state of the model can be regarded as being very close to a state with substantially small and random imperfection in the geometry. Actually, multiple cases with changing the assigned eigenmode from one to the other were analyzed, but all the results confirmed that no significant differences were found among them. Fig. 7 shows the analysis results, including the deep postbuckling region where the deformation largely grows. Viewing the overall buckling paths, local instability appears within the primary buckling region simultaneously with a big drop-down of the load (from Point A to Point D), and in the region after the secondary buckling (from Point E to Point H), unstable progress of the deformation covering a wide range accompanied with mode jumping is observed. The measured values in the experiments carried out by Yamaki et al. are also shown in Fig. 7 by superimposing them onto the results from our FEM analysis. The results from the FEM analysis are confirmed to be in full agreement with the experimental measurement. In addition, as shown in Fig. 7, use of the explicit method gave comparable solutions.

Now let us review the process of buckling following the order of events. At first, within the prebuckling region, the cylindrical shell has slightly out-of-plane deformation due to the prescribed initial imperfection. This deformed shape is noted as being very close to the axisymmetric buckling mode with $m=13$ and $n=0$ as derived from the classical theory. At Point B where primary buckling takes place, as evidently indicated in the deformed shape, local buckling is initiated to occur at the bottom portion of the cylindrical shell. This local buckling is of a dimple shape, i.e., small rounded squares, and it becomes the source of the successive growth of additional unstable buckles spreading over the cylinder's surface. Along with the drop-down of load, the number of local buckles sharply increases, and these buckles are nearly evenly distributed over the cylinder's surface (Point C and Point D). Having been estimated from the deformed shape at Point D, this buckling pattern is considered to be equivalent to the pattern with $m=4$ and $n=17$. As noted in Fig. 6 shown previously, a combination of this m with n satisfies the relation expressed by Eq. (3). A supplemental note is given at this point: the half-wave length of the buckling given by this m or n is about 20mm, which is almost identical to, for example, $3.5(rt)^{0.5} - 4(rt)^{0.5}$, the gauge length of imperfections given by ENV 1993-1-6 [1], [14]. Within the region after passing Point D, the number of axial and circumferential waves simultaneously decreases to eventually reach Point E on a stable deformation path.

The number of the waves at this point corresponds to $m=2$ and $n=12$, which also satisfies the relation expressed by Eq. (3). The behavior as explained thus far – namely the behavior starting from the unstable shape with a larger number of waves to reaching the stable shape with a lesser

number of waves – is found to be identical to the results observed by Esslinger [7], [8]. After passing this state, successive mode jumping can be observed, in which the path follows sequentially Point F, G, and H, with $m=2$ remaining unchanged but with n decreasing one by one. To overcome such a complicated process in the deformation, the whole range of the postbuckling paths could be analyzed employing the full auto-increment seamless calculation.

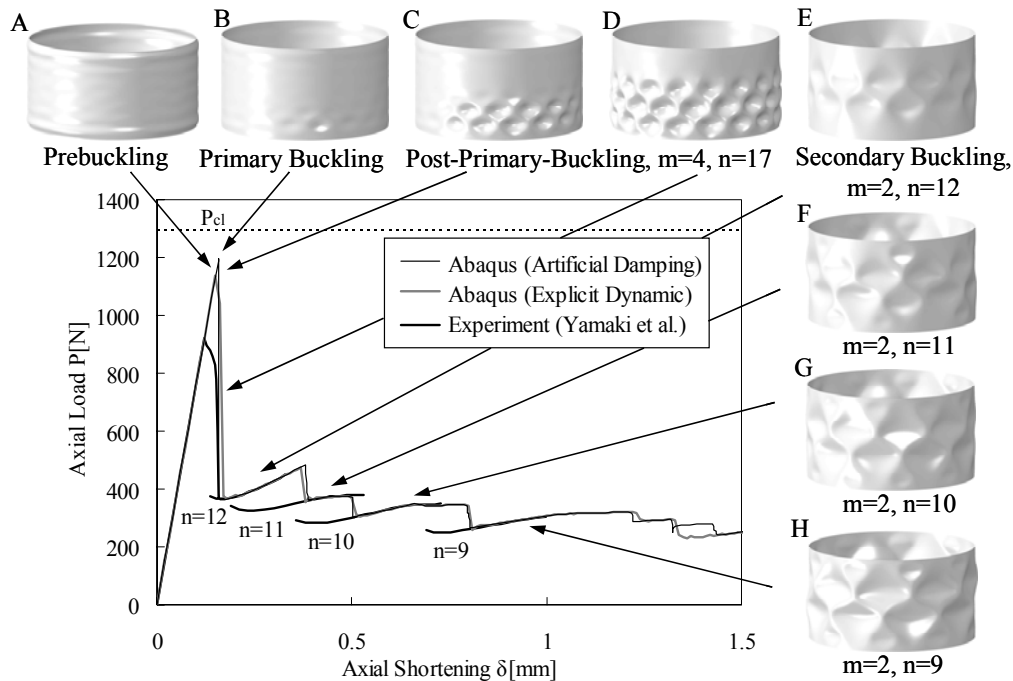


Figure 7. Results from artificial damping analysis and explicit dynamic analysis.

6. Applications

6.1 Comparison with High-Speed Photography

The initial buckle pattern with a larger number of waves (as seen at points B-D in Fig.7) represents the highly unstable primary buckling stage for which any direct confirmation could not be achieved in the ordinary experiment. However, the use of high-speed photography in very carefully performed axial compression tests made it possible to get direct view of the transition from an initial buckling pattern to another, completely different postbuckling pattern. Esslinger studied the buckling process on a number of axially loaded Mylar cylindrical shells using a special high-speed camera with a maximum speed of 5200 frames per second [7]. The thin shells had a radius of 100 mm and wall thickness of 0.254 mm. The Mylar foil had a Young's modulus of 5400MPa with a high elastic limit. The shells had a simple longitudinal lap joint and were fully

fixed at both ends into a thick aluminum plate. The high speed photography was taken with the slowest rate of axial shortening of 0.039mm/sec. For taking high quality photography, the surface of the shell was finished with matted silver tone by spraying silver paint.

Fig.8 shows the experimental results for their longest Mylar shell with $L/r=3.30$. As seen in the figure, the selected high-speed photography well captured clear views of buckles through the transition of buckling path from the beginning to the final stable postbuckling. Number on each picture of the test shell indicates elapsed time (milliseconds) from initiation of buckling. The buckling occurs in the middle of the cylinder, as clearly seen in frame A at 0.33ms. The initial buckle becomes the source to successive growth of unstable additional buckles spreading over the cylinder surface. Such a buckle takes a shape of dimple, i.e., a small rounded square (see frame B at 0.83ms). Many similar buckles are added around the initial buckle and spread over circumferentially (see frame C at 1.3ms).

Singer [8] suggests that this spread of buckles may be regarded as corresponding to the chessboard pattern with circumferential waves of $n=18$ which is predicted by the linear theory. The shapes of buckles are still roughly square, but their size becomes larger and these buckles may correspond to the waves of $n=13$ in frame D at 2.8ms. From this point the shape of the buckles begins to change. They are further elongated in the axial direction until two-tier diamond pattern is evolved in frame

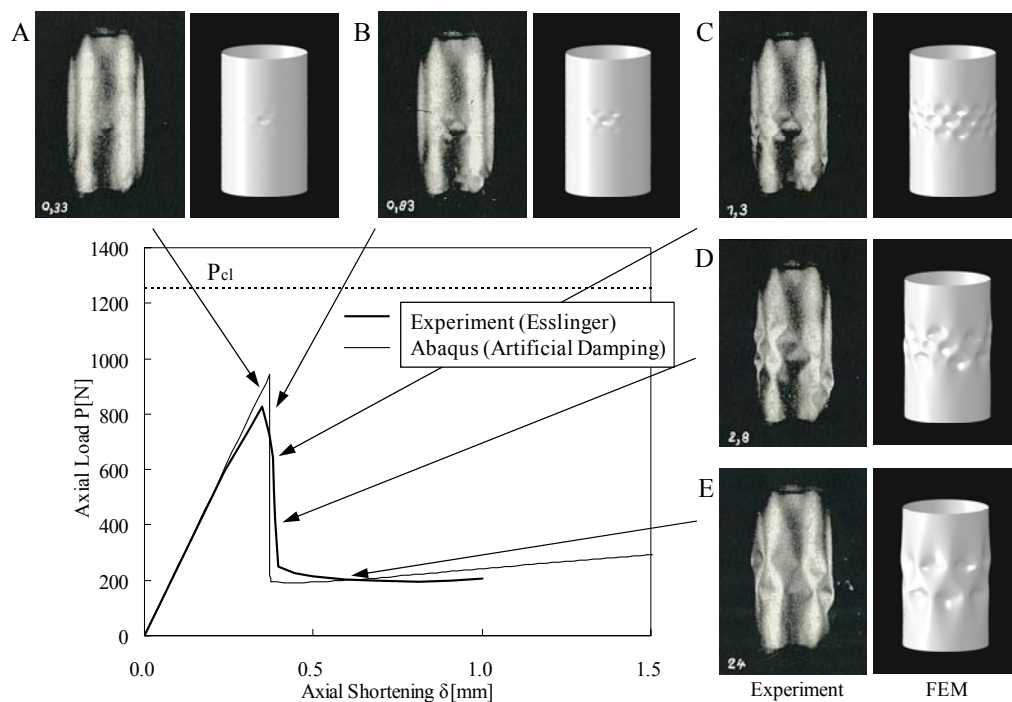


Figure 8. Comparison with High-Speed Photography, Esslinger [7].

E at 24 ms. By comparing frame A at 0.33 ms with frame E at 24 ms, big differences in their shapes and size are seen between the initial unstable buckling pattern and finally stable post-buckling pattern. Our analytical results show good agreement with these experimental results.

6.2 Compression of Temperature Dependent Viscoelastic Shell

Polyethylene terephthalate (PET) is a kind of thermoplastic polymer resin, and nowadays, it is widely used as beverage, food and other liquid containers. Previously mentioned Mylar resin used by Yamaki and Esslinger is the typical brand of PET. PET's property includes a high elastic limit as well as the low glass transition temperature T_g lower than 100°C. Accordingly, the use of PET makes it relatively easy to perform experiments of buckling in the viscoelastic range. In this study, using cylindrical shells made of PET with diameter of 80mm, 100mm in height and wall thickness of 0.3mm, the experiment of buckling under axial compressions and the associated analysis were carried out.

The measured viscoelastic properties of the PET are shown in Fig.9. The glass transition temperature T_g is found to be about 70°C. The cylindrical shell was placed in a constant temperature bath, and compressed in axial direction at three levels of temperature conditions before and after the transition temperature, i.e., 60°C, 70°C and 80°C. The both ends of the cylindrical shell were fixed with heat-resistant adhesive to an aluminum plate so as to be fully constrained. A downward axial compression with a velocity of 1mm/min was applied at the top of the shell. The total end shortening was 5mm, and the time duration required until finishing compression was 300sec. Fig.9 (b) to (d) indicates the relaxation modulus at each temperature respectively. These figures can be produced from (a) taking the temperature-time reduction factor. In this study, the following expression of WFL shift factor was applied. With display of compression time as 300sec in these figures, three temperature conditions: 60°C, 70°C and 80°C can be regarded as corresponding to the glass region, transition region and rubbery region, respectively.

$$\log(\alpha(T)) = -\frac{C_1(T - T_R)}{C_2 + (T - T_R)} \quad (5)$$

$$T_R = T_g + 50 \quad C_1 = 8.86 \quad C_2 = 101.6 \quad T_g = 70$$

Fig.10 shows the deformed shapes resulted from both of the experiment and analysis. Since the experiment was carried out within the viscoelastic region, permanent deformations remain on the cylindrical shells. Therefore, photography of these deformed shapes can be taken after the experiment. The analysis procedure is similar to the steps described in the above. The initial imperfection was assigned as a combined wave shape formed by superimposing 10 linear eigenmodes selected as those corresponding to $n=1$ to $n=10$.) The implicit analysis method using artificial damping was employed. Abaqus/Standard enables us to apply temperature-dependent viscoelasticity into the buckling analysis using shell elements.

In the temperature condition with 60°C (glass region), the analysis predicted such that, after a mode with circumferential waves of $n=8$ initially occurs, postbuckling mode is subsequently shifted along with decreasing the number of circumferential waves up to $n=4$. Fig.10 shows the final deformed shapes. As shown in Fig.11, the experimental and analytical load-displacement curves are in good agreement. As this temperature condition corresponds to the glass region, the

PET shell behaves as mostly elastic body. Accordingly, the postbuckling behavior is likely to be similar to the tendency derived from elastic study.

In the analysis for the temperature condition with 70°C (transient region), a mode with 8 circumferential waves initially occurs, but, by guess such that it can not jump in snap-through manner to the next buckling mode due to sharp relaxation in the elastic modulus, the end shortening progressed keeping 8 waves. In the experiment, only the mode with 6 waves could be observed, however, it could be confirmed that the number of circumferential waves is relatively increased compared with the results at 60°C. Since the behavior in the transient region appears to be so acute, further improving the accuracy of experiments is needed.

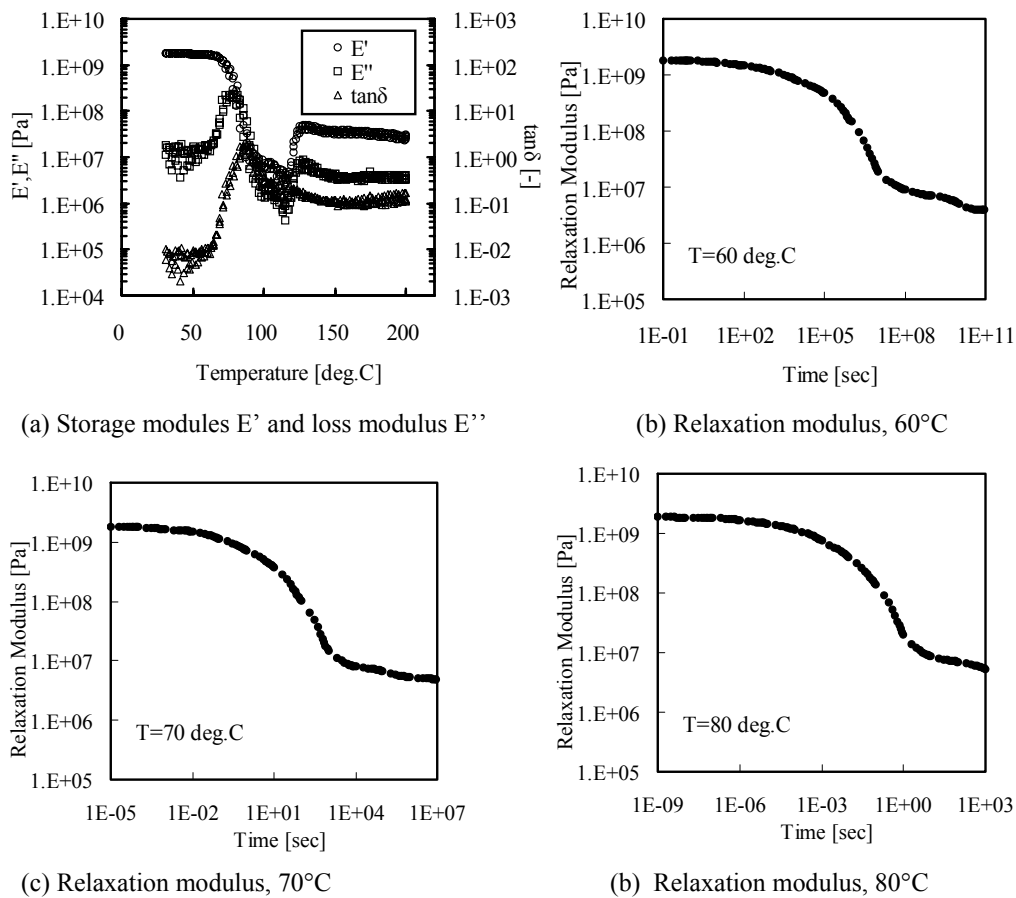


Figure 9. Viscoelastic properties of PET foil.

In the analysis at 80°C (rubbery region), a mode with 8 waves could be initially confirmed, but due to extremely low elastic modulus, its buckling mode is soon collapsed and obscure deformation mode (with no sharp corner edge) is exhibited. Most of such tendency could be captured in the experiment.

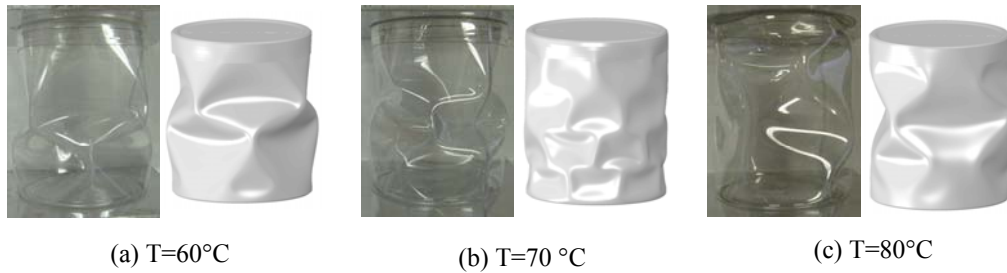


Figure 10. Deformed shapes of PET cylindrical shell. (Left: Experiment, Right: FEM)

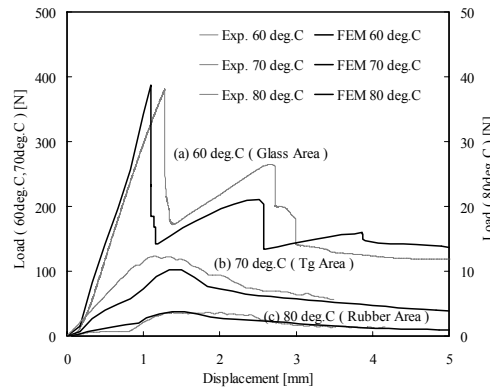


Figure 11. Load-Displacement curves.

7. Conclusions

The analysis capabilities provided with the recent general-purpose FEM codes are regarded as being sufficiently enough to solve the highly nonlinear problems. By such enhancement, many engineers will be released from the burden of developing sophisticated programs, which is a great merit. In this study, the problem of cylindrical shells of perfect geometrical dimensions subject to axial compression is targeted. This is regarded as one of the most difficult tasks involved with the buckling phenomenon. Based on the findings obtained from the previous research works, analysis policies to apply the nonlinear FEM capabilities are formulated and are implemented to be available in practical designs. It is successfully demonstrated that fully automatic seamless analysis using both static analysis and explicit dynamic analysis can be accomplished in a stable process well covering the deep postbuckling region. For the example of practical application of the simulation method described here, a comparison with high-speed photography and applicability to viscoelastic buckling problem were discussed.

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