

Full cycle stochastic analysis of composite structures under buckling loads. Design & manufacturing scatter

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Abstract: Structures in general are subject to uncertainty due to manufacturing, assembly, environment of work, loads, etc ... This scatter more specifically is associated for example to tolerances of thickness, position, waviness, etc, material mechanical properties distribution, lay-up alignment axes. All these deviations can be taken into account with stochastic analysis to reduce the total cost of the project considering all the phases of product life (manufacturing, assembly, maintainability...) and make a global robust design.

The problem of optimization of composites structures can be addressed in spite of high number of design variables (angles, thickness, lay-up...), failure modes (buckling, strains, cohesive material...), and taking into account the previously mentioned scatter by means of iSight and Abaqus.

Analyses about the tolerances influence (material mechanical properties, thickness, waviness, lay-up alignment axes...) in the critical buckling results of a stiffened curved composite panel have been developed.

The tolerances whose values, in spite of complying with the aeronautical industry current design criteria, generate significant changes in the critical results are identified. On the other hand, the tolerances with very low influences in the critical failure modes are obtained, making possible therefore a cost reduction.

The use of these technologies allows finding improved structures, without an increase of manufacturing non conformities associated with highly optimized structures, and with similar analysis times.

Keywords: composite, buckling, stochastic, robust design, tolerances

1. Introduction

Using stochastic techniques a global analysis is performed in order to take into account some of the main deviations, defects and damages that can appear along the full life cycle of a structure. The objective is to identify the influence of these deviations, defects and damages in the structure behavior and to consider it during the initial design phase. The final number of non conformity manufactured elements, for example, is minimized with these methods and therefore with the associated cost saving.

The analysis carried out is focused on:

- Manufacturing tolerances (material properties, geometric tolerances, thicknesses, lay-up alignment axes ...)
- Damages or defect that can be generated along manufacturing process or the period in service of the structure (delaminations, disbondings, etc).

The damages and defects included in the structure are analyses together with manufacturing tolerances. This is a realistic coupled situation that appears usually along the product life.

A typical stiffened and curved composite panel and two load cases from aeronautical industry have been analyzed to show these effects. Firstly a conventional analysis, with nominal values for all the input data, is presented. In a second step a stochastic analysis considering all the mentioned deviations is performed. This will allow comparing the behavior of the structure with the deviations from the nominal design.

The structural analysis is done through to buckling and static linear perturbations with Abaqus.

Stochastic analysis has been performed with iSight, and the help of TAM in-house software.

2. Nominal analysis

A conventional analysis, with nominal values for all the input data, of a stiffened and curved composite panel, typical in aeronautical industry, has been performed. This analysis is taken like baseline for the subsequent stochastic analysis. The properties of this section have been taken like mean value for the statistical distribution used in following sections.

2.1 Geometry

A general view of the stiffened composite panel is shown in following figure:

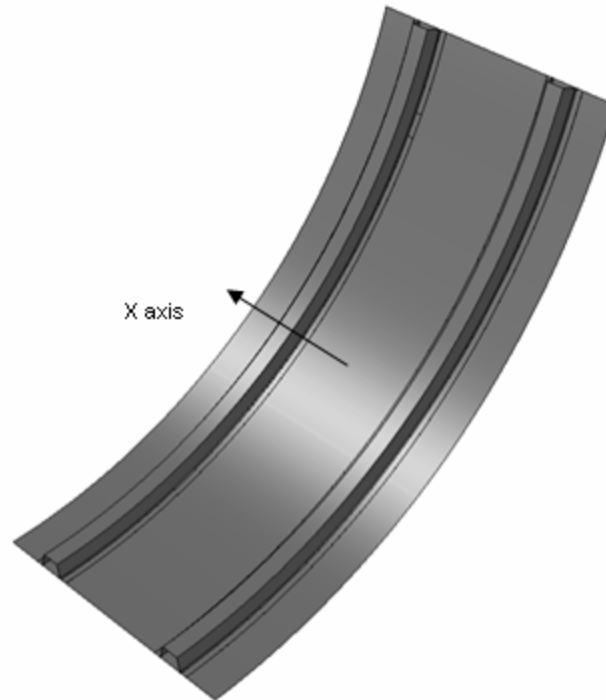


Figure 1 Composite cylindrical stiffened panel.

Curvature radius is 2000 mm approximately. X axis in the Figure 1 is the revolution axis of the structure. The stiffeners have an omega section.

2.2 Materials and lay-ups

Panel and stiffeners are made of uniaxial and biaxial carbon fiber and bonded with adhesive. Mechanical properties of these materials are typical values of aeronautical industry.

2.3 Loads

Three representative load cases have been considered, two with a uniform pressure along the panel one pointing inside of the structure and another one pointing outside of the structure, and another with a shear load model by enforced displacements in axial direction. Both are shown in Figure 2

Uniform pressure tries to model the behavior of a panel under aerodynamic loads.

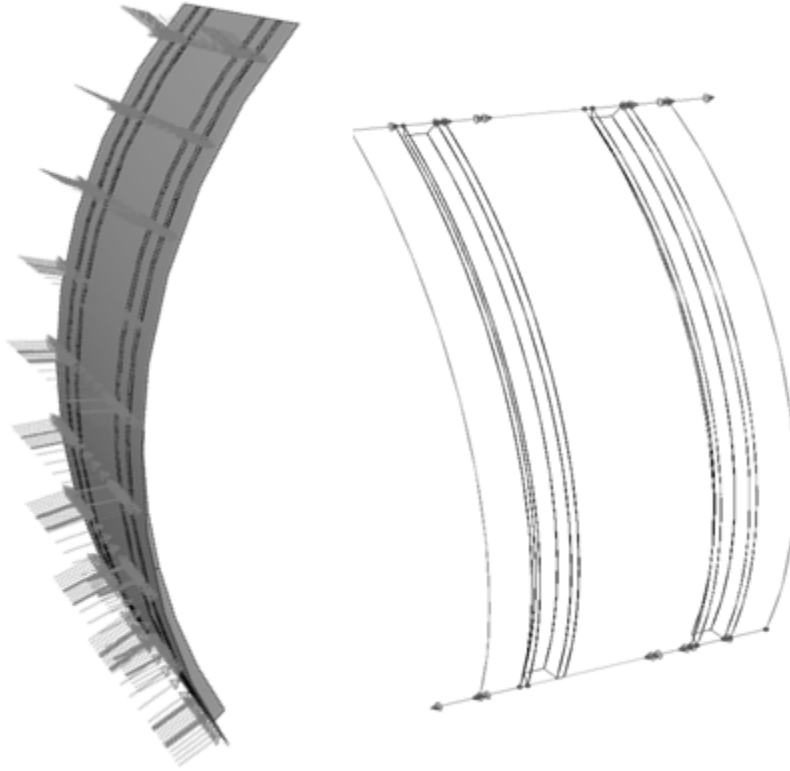


Figure 2 Uniform pressure loads and forced shear displacement.

Displacements in axial direction and opposite side are applied to model a shear load over the panel.

Boundary conditions are simply supported in radial direction in circumferential sides and simply supported in all directions in axial sides.

2.4 FEM Model

Skin and stiffeners have been modeled with planar elements S4R and the adhesive has been modeled with C3D8R element.

Delaminate has been modeled with a shell composite with only a ply.

Disbond has been modeled with a solid homogenous which mechanical properties have been reduced in 6 orders of magnitude.

2.5 Structural analysis

A conventional analysis, with nominal values for all the input data, of the stiffened composite panel is performed. Also two more analyses with a delamination in the middle of the skin and another with two disbondings under the stiffeners are carried out.

In Figure 3, shear and pressure first buckling mode are shown.

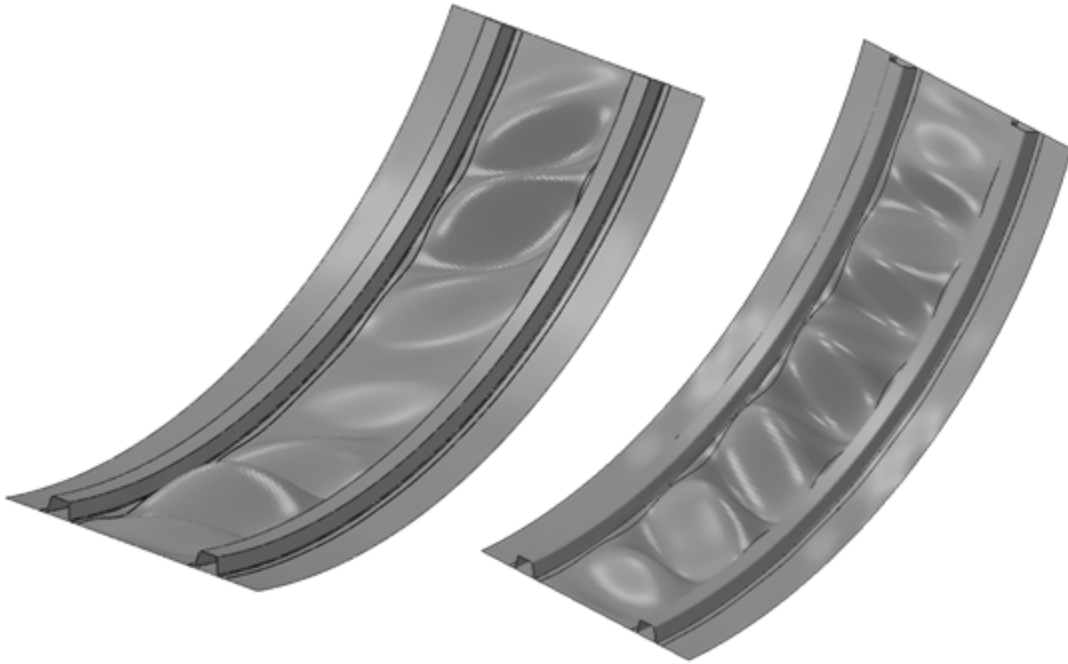


Figure 3 Shear and Pressure buckling respectively.

In following table a resume of the structure reserve factors are shown:

Region	Nominal reserve factor	Damaged minimum reserve factor	Ratios
Buckling shear RF	1.19	1.19	1
Buckling pressure RF	1.14	1.11	0.98
Adhesive	3.24	0.77	0.24
Skin	2.37	0.35	0.146
Foot Stiffener	1.92	0.56	0.294

Table 1 Static analysis reserve factor resume

3. Stochastic Analysis

Beginning with previous nominal sections, in the followings we will do a study taking account of tolerances of material, geometric, lay-up angle, thickness... that take place in manufacturing and typical defects and damage. Then we will see the variation in the strength checking that these cause, identifying the critical tolerances.

In sections 3.1 and 3.2 procedure and statistical distribution of the panel properties that have been used are explained, and how the disbond and delaminate are modeled.

3.1 Procedure

The procedures to do stochastic analysis begin with the selection of input variables statistical distributions. These distributions have been obtained from the bibliography, when no bibliography has been found; uniform distribution has been used, due to it have the maximum possible scatter.

Once statistical distributions have been selected, calculation procedure must be created. In Figure 4 is shown the procedure how it is made in iSight.

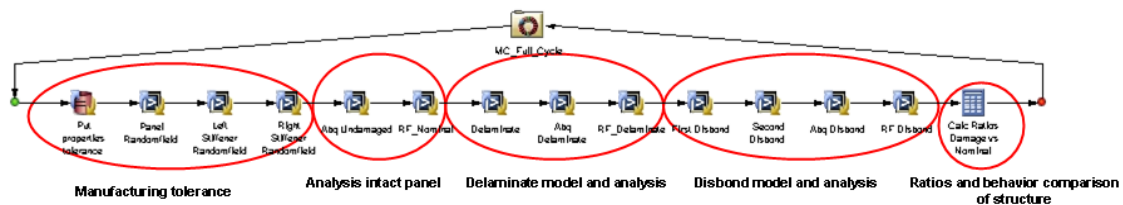


Figure 4 Draft of stochastic analysis procedure.

Conceptually the procedure is:

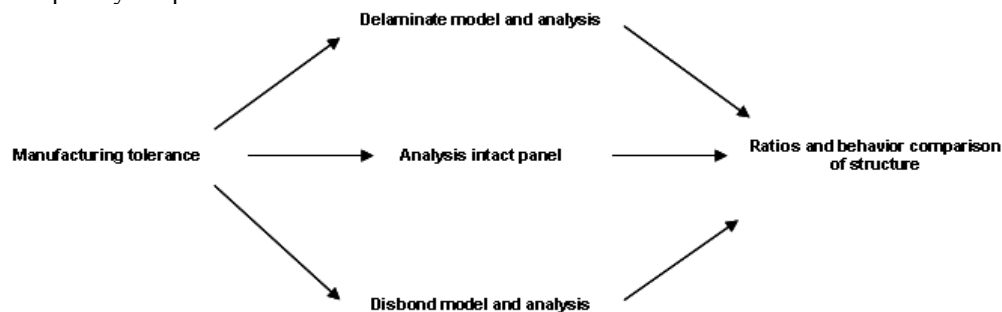


Figure 5 Conceptual flow of the stochastic procedure

Stochastic analysis has been done with iSight. The driver inside iSight that allow us to do stochastic analysis is the named Montecarlo and the sampling method chosen is the descriptive sampling, that has a better convergence (Saliby, 1997) to the output statistical distributions, so less iteration must to be done.

The analyses of the structure have been done through Abaqus/Standard. And random fields applications, disbondings and delaminates positioning, and reserve factors calculation have been done through in-house TAM software. Than have been called with SimCode iSight components, renamed in Figure 4, to clarify its correspondent mission.

Besides the full cycle analysis, partial analyses were done, to check or to deepen in the results.

3.2 Variables distribution

The statistical distributions for manufacturing variables used are explained in following tables.

	Distribution	Unidirectional fiber		Bidirectional fiber		Adhesive
		Coefficient of Variation		Coefficient of Variation		Coefficient of Variation
E_{11}	Weibull	2 %		2 %		16 %
E_{22}	Weibull	2 %		2 %		--
G_{12}	Normal	2 %		2 %		16 %
Coefficient of Poisson	Normal	2 %		10 %		4.5 %
	Distribution	Min Value	Max Value	Min value	Max value	---
Ply thickness tolerance	Uniform	-0.01 mm	+0.01 mm	-0.01 mm	+0.01 mm	---
Angle deviation tolerance	Uniform	-3°	+3°	-5°	+5°	---

Table 2 Materials mechanical properties statistical distributions

Due to than in bidirectional composites and in adhesive the properties in longitudinal and transversal direction are highly related, therefore coefficients of correlation have been applied.

Beside statistical distributions for the variables, to model all manufacturing tolerances geometric deviations have been used. Geometric deviations of the nominal design are not independent in each point. The way these deviations are related along the structure is dependent of the manufacturing process. In our case a bi-sinusoidal deviation have been chose for the skin, one in the axial direction and another one in the circumferential direction.

In Figure 6 is shown a draft of the geometric tolerance wave

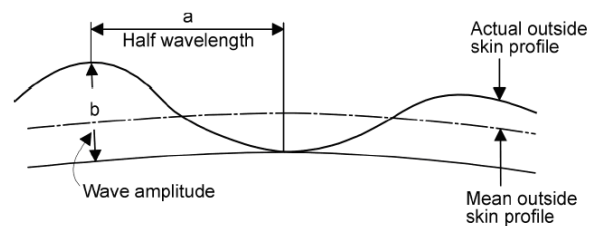


Figure 6 Draft of the geometric tolerances

The parameters of this bi-sinusoidal deviation are explained in following lines and in Equation 1

r_1 : Deviate final radial position.

r_0 : Nominal radial position.

A_1 : Amplitude of circumferential wave.

k_1 : Number of half-wave in circumferential direction.

y_0 : Angle phase for the circumferential perturbation wave.

Θ_{Total} : Total circumferential angle of the structure.

A_2 : Amplitude of axial wave.

k_2 : Number of half-wave in axial direction.

z_0 : Angle phase for the axial perturbation wave.

L : Total length in axial direction of the structure

$$r_1 = r_0 + A_1 \sin\left((k_1 y + y_0) \times \frac{\pi}{\Theta_{Total}}\right) + A_2 \sin\left((k_2 z + z_0) \times \frac{\pi}{L}\right)$$

Equation 1 Bi-sinusoidal equation of the deviation

Previous equation related radial position of each structure point with his axial and circumferential design position. Maximum amplitude of the circumferential and axial wave is 1 mm, and the curvature radius is 2000 mm.

The distribution of theses variables used to cover all the possible geometric tolerances is:

Geometric shape tolerances			
	Distribution	Min value	Max value
Amplitude axial	Uniform	-1 mm	+1 mm
Amplitude circumferential	Uniform	-1 mm	+1 mm
Phase axial	Uniform	0	Axial length
Phase circumferential	Uniform	0	Circumferential length
Axial half wavelength	Uniform	200 mm	Axial length
Circumferential half wavelength	Uniform	200 mm	Circumferential length

Table 3 Geometric shape variables statistical distribution

Minimum value of the half wavelength is 200 mm, because for an aeronautical structure correct aeromothness forbids a relation of half wavelength and amplitude of 0.05, or 0.01 pending of the structure position in the airplane.

It is assumed that defects and damages can be occur in any places of the structure, therefore uniform distribution have been applied. Variables of the position and size of the manufacturing defects and life service damages are exposed in Table 1.

Delamination			
	Distribution	Min Value	Max Value
Delaminate centroid axial position	Uniform	0 mm	Half of axial length
Delaminate centroid circumferential position	Uniform	0 mm	Half of circumferential length
Delaminate half-width	Uniform	25 mm	400 mm
Disbond			
First disbond centroid circumferential position	Uniform	0 mm	Circumferential length
Second disbond centroid circumferential position	Uniform	0 mm	Circumferential length
First disbond half-width	Uniform	25 mm	400 mm
Second disbond half-width	Uniform	25 mm	400 mm

Table 4 Damage variables statistical distribution

From strength checking point of view the manufacturing defects and the damages during the life cycle are equal, and only changes the size of them. Therefore that is the reason of the wide range of with damages used. Summing up all the variables we have a total of 58 input variables.

3.3 Results

The main results obtained with the stochastic analyses are:

- Statistical distribution of the results associated to the structural analyses. Maximum and minimum reserve factor are identified as consequence of the global set of inputs and their deviations during manufacturing process, defects and damages.
- The percentage of non compliant elements with the initially mentioned structural requirements is determined.
- The specific influence in the results of each input is quantified. With this methodology is possible to identify exactly the inputs to be strongly checked.

Finally these results let resources optimization with a quality and maintenance plan focused on the most influential inputs.

3.3.1 Intact structure

Next table summarizes the reserve factor statistical distribution for intact structure:

	Nominal analysis	Stochastic analysis				
	Nominal	Mean	Most Likely	Coefficient of variation	Min value	Max value
Buckling shear RF	1.19	1.19	1.18	4.5%	1.04	1.45
Buckling pressure RF	1.14	1.14	1.09	6.8%	0.84	1.56
Adhesive RF	3.22	3.08	3.08	3.6%	2.77	3.45
Skin RF	2.37	2.15	2.39	10.2%	1.29	2.5
Stiffener foot RF	1.92	1.81	1.81	2.7%	1.65	1.95

Table 5 Statistic table of the reserve factors of the intact structure

The minimum reserve factor obtained with the nominal analysis is associated to pressure buckling failure and is $RF=1.14$. Including all the manufacturing tolerances this critical reserve factor lowers to $RF=0.84$. That means a 26 % reduction.

Most of aeronautical external aerodynamic surfaces are designed to not to buckle to limit load or to 10% over limit load. No other safety factor is applied on this requirement. In this case that involves that there will be real structures, with manufacturing tolerances, non-compliant with the design requirements.

Below statistical distribution shows that in this case a 5 % of the manufacturing structures are invalid.

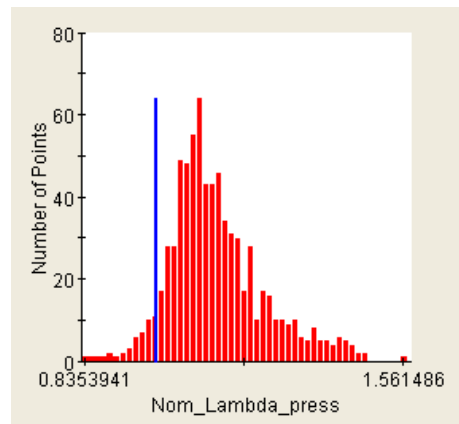


Figure 7 Histogram of the buckling pressures eigenvalues

The specific influence in the main results of each input is quantified, through to Pareto diagrams in Figure 8 and Figure 9.

It can be noticed that the most influential variables for the pressure buckling are:

- Number of half-waves in axial direction, which have a variation between 1 and 5 half-waves, and have an influence of 10 %.
- Uniaxial composite elastic modulus, which have a variation coefficient of 2% and has an influence of 8 %.

It can be noticed that the most influential variables for the shear buckling are:

- Tolerance of the first ply of the panel skin, which have a variation of $\pm 3^\circ$.
- Number of half-waves in axial direction, which have a variation between 1 and 5 half-waves.

It can be noticed that the most influential variables for the skin reserve factor are:

- Amplitude of the waves in the circumferential direction. Being the maximum deviation of the curvature radius of 1 mm in 2000 mm.
- Number of half-waves in circumferential direction, which have a variation between 1 and 12 half-waves.

Other tolerances as shear modulus, thicknesses and lay-up alignment axes have an impact lower than 5 % in the results, as it can be seen in following figures.

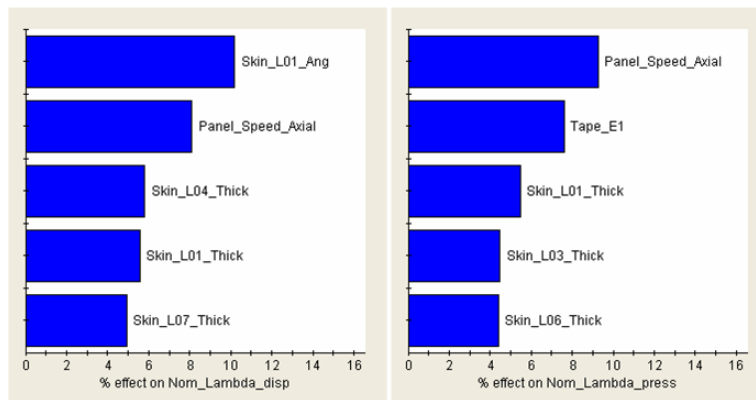


Figure 8 Pareto diagrams for buckling eigenvalues

In above figures blue color means that increasing the values of the variables increase the buckling reserve factors.

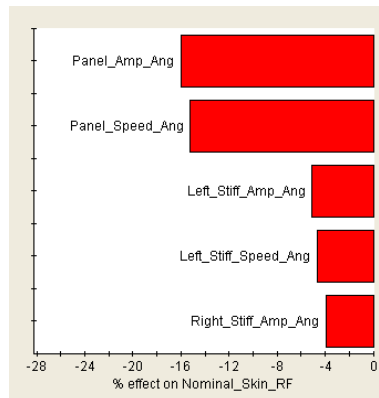


Figure 9 Pareto diagram for skin reserve factor

In above figure red color means that increasing the values of the variables reduce the reserve factor of the skin.

To understand why there are a big reduction in pressure buckling reserve factor, it has been analyzed the samples with reserve factor lower than one, for intact and disbonded structures. It has been found that if we represent the values of phase and number of half-waves in the axial direction of these structures, we see two clusters of points. One of these clusters has a clear lining.

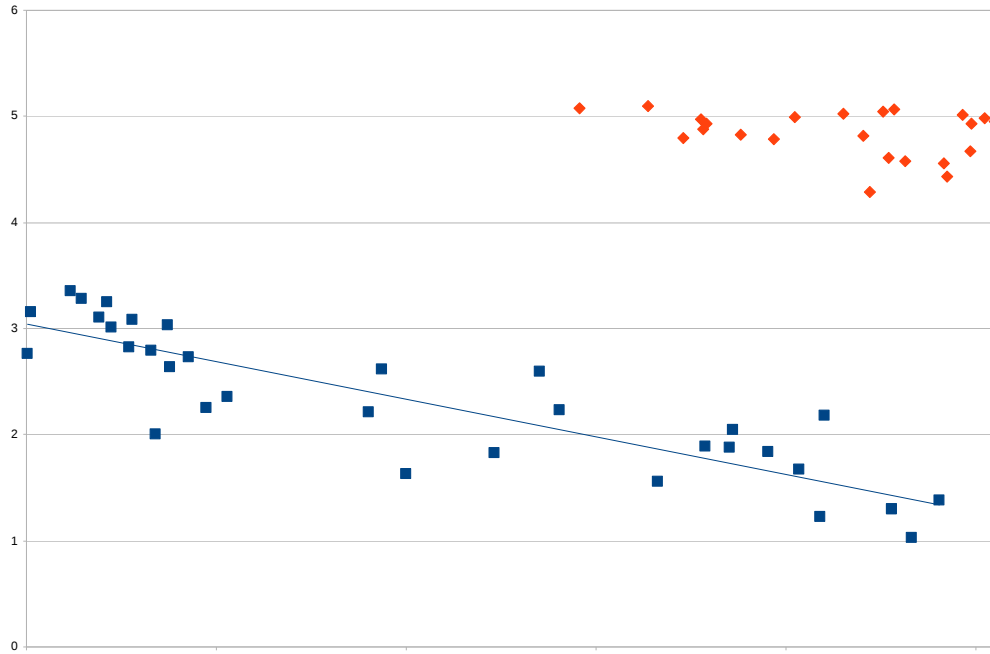


Figure 10 Points with buckling pressure eigenvalue lower than one

Doing minimum less square fitting to the lower cluster, we obtain the following equation:

$$k_2 = 3.04 - 0.0018 \cdot z_0$$

Equation 2 Relation between phase and speed of axial wave

That it has a correlation coefficient of **0.76**.

To know from where this numerical relation come it is used Equation 1 and equaling this to the minimum deviation of the half-wave:

$$(k_2 z + z_0) \times \frac{\pi}{L} = -\frac{\pi}{2} + 2 \cdot n \cdot \pi \Rightarrow \frac{L}{2} (4n - 1) = (k_2 z + z_0) \xrightarrow{z=L/2} k_2 = (4n - 1) - \frac{z_0}{L/2}$$

$$\xrightarrow{n=0} k_2 = -1 - \frac{z_0}{L/2} \Rightarrow \text{Invalid}$$

$$\xrightarrow{n=1} k_2 = 3 - \frac{z_0}{L/2} \approx 3.04 - 0.0018 \cdot z_0 \Rightarrow \text{Cluster1}$$

$$\xrightarrow{n=1} k_2 = 7 - \frac{z_0}{L/2} \Rightarrow \text{Cluster2}$$

Equation 3 Theoretical relation between phase and speed of axial wave

In Equation 3, it is obtained the relation between zero deviation phase (z_0) and number of half-waves (k_2) of the axial random field wave, when the amplitude is maxima in the middle of the panel. In following images are shown the geometric tolerance waves from the random field that corresponds to the first cluster and second cluster of the Figure 10.

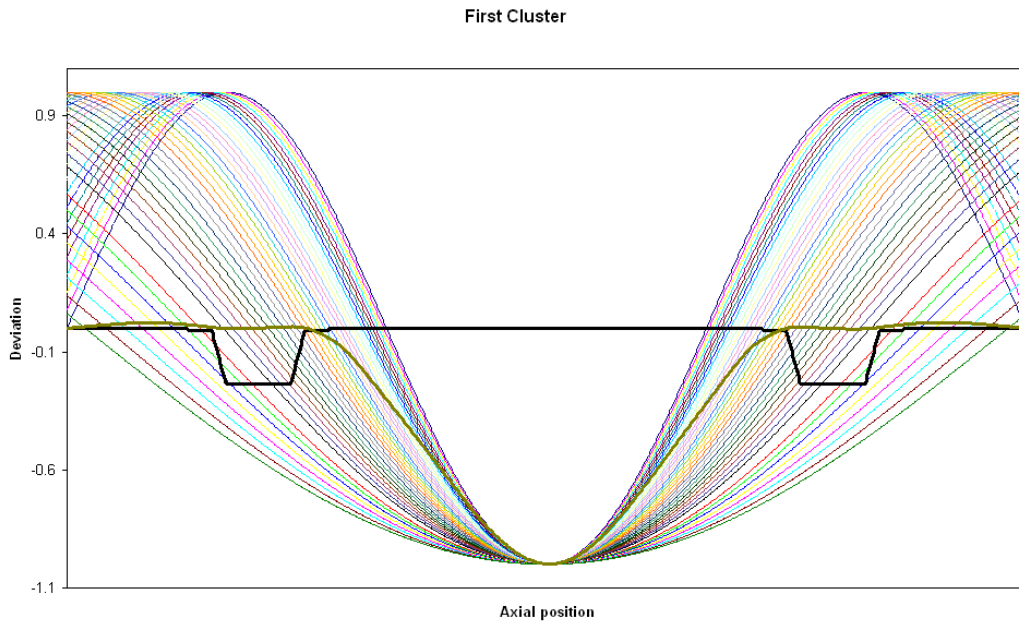


Figure 11 Geometric tolerance waves that correspond to the first cluster.

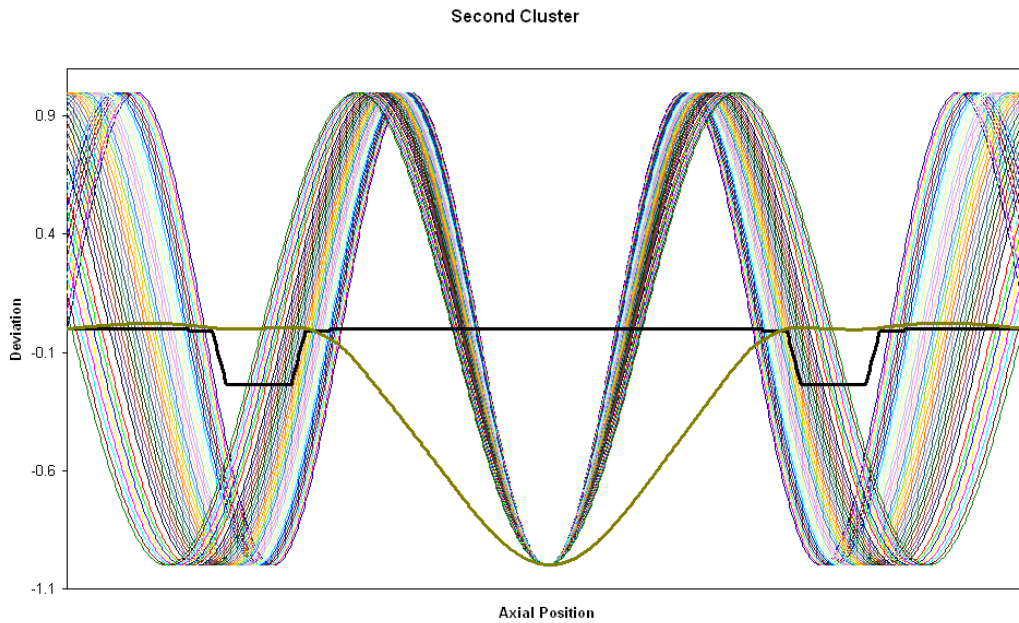


Figure 12 Geometric tolerance waves that correspond to second cluster

In previous images are shown a section of the panel, the geometric tolerances wave than have the relations of Equation 3, and that have eigenvalues reduced at least a 14 % with a maximum deviation of 1 mm.

Taking account the previous geometric tolerances analysis to increase the number of valid structures, it is needed to reduce the maximum deviation to 0.5 mm, or if that is too expensive or impossible warrant than maximum negative deviation is not near to center of the panel.

3.3.2 Delaminated structure

Next table summarizes the reserve factor statistical distribution for damaged structure with a delamination:

	Nominal analysis	Stochastic analysis				
	Nominal	Mean	Most Likely	Coefficient of variation	Min value	Max value
Adhesive RF	0.968	0.857	0.696	21.3 %	0.6	1.63
Delaminate Skin RF	0.224	0.231	0.227	4.4 %	0.196	0.276
Skin RF	0.347	0.363	0.36	4.7%	0.315	0.442
Stiffener foot RF	0.56	0.454	0.46	8.2%	0.345	0.576

Table 6 Statistic table of the reserve factors of the delaminated structure

The delamination generates a hard drop in the reserve factors making that the structure will not be valid for all the fail modes but the adhesive that fails in a 77 % of the cases.

A study about the critical location of the delamination has been performed. Being the critical position when a part of the the delamination is under the stiffener foot.

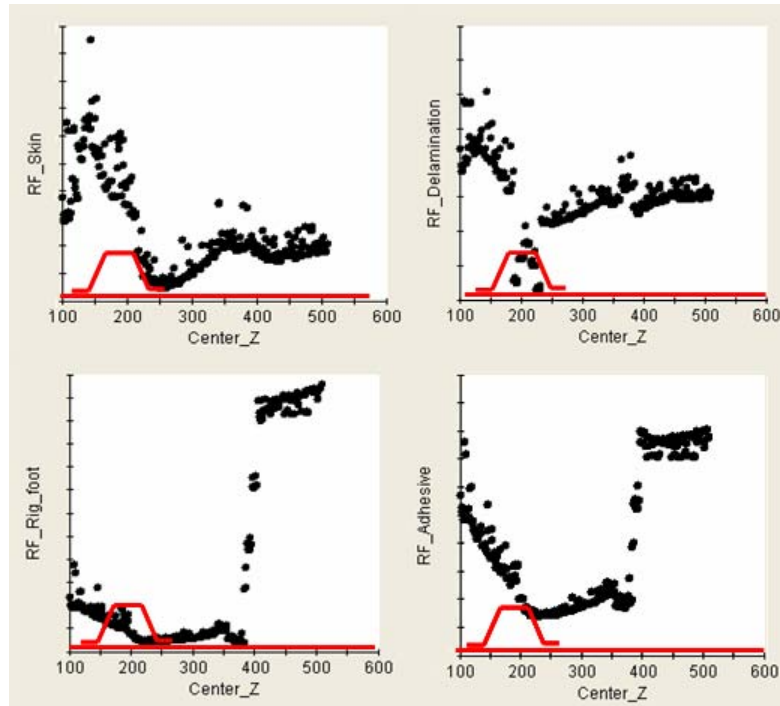


Figure 13 Variation of the reserve factors with the axial axis

3.3.3 Disbonded structure

Next table summarizes the reserve factor statistical distribution for damaged structure with two disbondings under the stiffeners feet:

	Nominal analysis	Stochastic analysis				
	Nominal	Mean	Most Likely	Coefficient of variation	Min value	Max value
Buckling shear eigenvalue	1.19	1.19	1.19	5.50%	1.04	1.43
Buckling pressure eigenvalue	1.11	1.12	1.09	9.50%	0.84	1.49
Adhesive RF	1.53	1.38	1.28	22.9%	0.63	2.7
Skin RF	1.6	1.63	1.66	7.6%	0.97	2.16
Stiffener foot RF	0.72	0.71	0.65	6.1%	0.57	1.03

Table 7 Statistic table of the reserve factors of the disbonded structure

The minimum reserve factor obtained with the nominal analysis is associated to stiffener foot strain reserve factor, and is $RF=0.72$. However minimum reserve factor in the stochastic analysis is lower than one for all the fail modes but shear buckling. The structure has a probability of fail in the stiffener foot of 99.9 %, in the skin of 0.1 %, and in the adhesive of the 13 %.

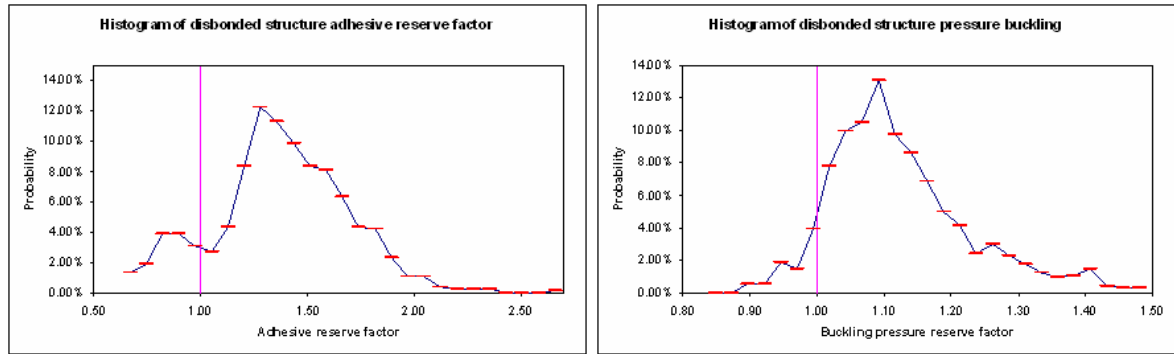


Figure 14 Histogram of adhesive and pressure buckling reserve

In reference to the pressure buckling behavior of the disbonded structure, is similar than for intact structure but with more scatter, and same conclusions were achieved, including the referents to geometric tolerances. Due to we have more scatter the percentage of invalid structures increase, in this case, to an 8 % of the manufacturing structures are invalid.

3.3.4 Comparative and ratios between damaged and intact structures

Next table summarizes the statistical distribution of the ratios between damaged and intact structures:

	Nominal analysis	Stochastic analysis				
	Nominal	Mean	Most Likely	Coefficient of variation	Min value	Max value
Buckling shear eigenvalue ratio	1	1.01	1.00	0.50%	0.98	1.02
Buckling pressure eigenvalue	0.98	0.99	1.00	1.60%	0.90	1.00
Adhesive RF	0.24	0.27	0.22	20.40%	0.19	0.52
Skin RF	0.146	0.17	0.16	16.20%	0.13	0.34
Stiffener foot RF	0.29	0.25	0.25	10.40%	0.19	0.32

Table 8 Statistic table of the reserve factors

In next figure is shown the big skew of the statistical distribution of the ratios. In this case the skew of the distribution is to high ratios, then the number of save structures is greater than expected for a symmetrical distribution.

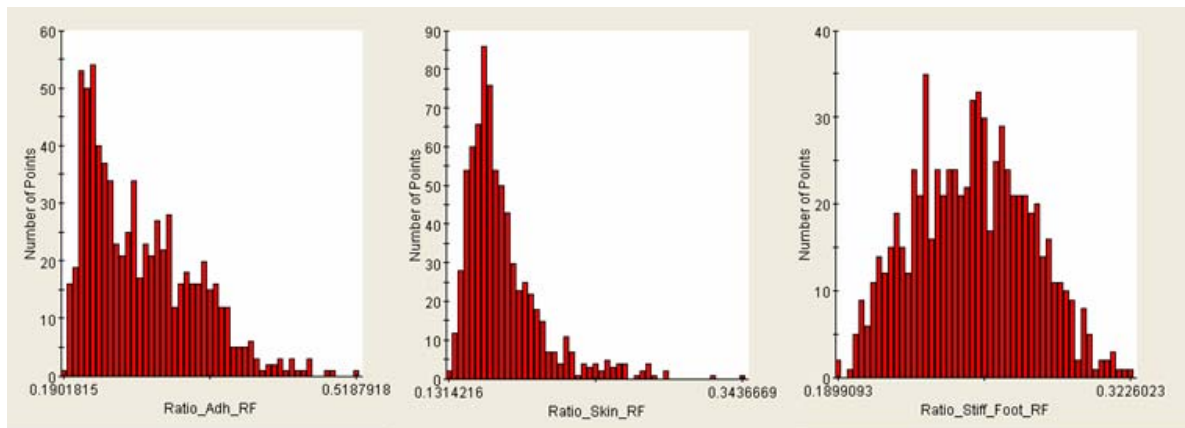


Figure 15 Histogram of the ratios of the reserve factors.

To obtain a structure with a robust design, in the sense that it will have greater mean ratios. So the structure will have more capability to support damages. A lay-up search it has been done, taking account of the all tolerances but the geometric ones.

It has been found that stiffener foot first ply must be changed, due to a changing in the lay-up increase the reserve factor, and reduction ratio with disbondings.

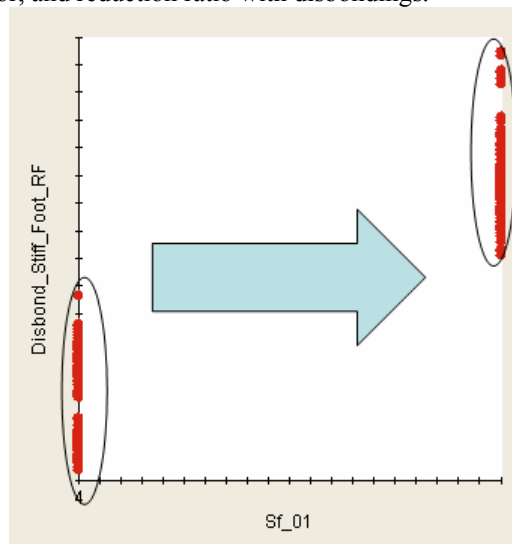


Figure 16 Increase of stiffener foot reserve factor changing lay-up

However first ply on the skin has a correct lay-up angle, as it can be seen in following image.

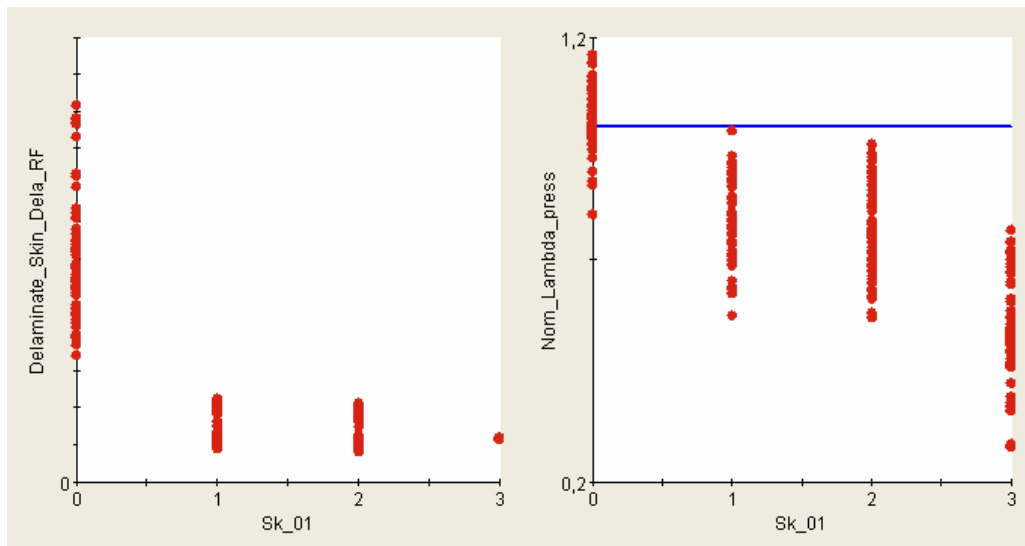


Figure 17 Reserve factor variation with different lay-up angles for the skin first ply

Then changing the rest of the skin and the stiffener foot lay-up it have been obtained following mean ratios:

	New	Old	Increase
Buckling Shear	1.84	1.19	55 %
Buckling Pressure	1.11	1.14	-3 %
Ratio Adhesive RF	0.36	0.24	50 %
Ratio Skin RF	0.30	0.146	105 %
Ratio Stiffener foot RF	0.41	0.29	41 %

Table 9 Increase of reserve factor ratios

How it is shown in previous table with the new lay-up it has been increase the number of valid damaged structures, without weight penalty.

To increase the number of valid structures and take account geometric tolerances, reinforcement is necessary, left for further studies.

4. Conclusion

In this document a composite stiffened panel has been studied taking account manufacturing tolerances, possible defects and damage during airplane life cycle. Allowing us to know how different variable interact, and obtaining design, manufacturing and quality control criteria directly applicable.

Structural analysis and design criteria:

- Statistical distribution of the output variables (reserve factor, displacement...) have high skew. In structural analysis normal distribution is assumed for safety factor calculation purpose, therefore some structures can be invalid unexpectedly.
- Over optimized structure, where each part are sized to the limit, conduct to and a big cost increase in manufacturing and maintenance. Reserve factors are reduced till seven times from nominal structure to damaged one.
- Materials:
 - Tolerances of mechanical properties can be higher, allowing material with “lower quality”.
 - Thickness tolerances can be higher, reducing manufacturing costs.
- Lay-up
 - Uni-axial and bi-axial lay-up ply angle tolerances are $\pm 3^\circ$ and $\pm 5^\circ$. These tolerances have low impact in the results and its spread. Therefore they can be increase allowing a manufacturing cost reduction. Stiffener foot and skin lay-up have been changed increasing the reserve factor and the number of valid damaged structures, taking account the tolerances.
- Shape geometry tolerances
 - The waviness geometric tolerances are the most critical variable of this study. The ± 1 mm usual tolerance can cause a 20 % reduction of buckling reserve factor. In this study we conclude than with a reduction of 50 % in tolerance, the structure behavior is acceptable. In the case we can not warrant a lower deviation it is need to check if the maximum deviation is outside the center of the panel to accept or reject the structure. The previous criteria allow quality control cost reductions, reduction of repair structures and, lower number of stress analysis of not valid structures.
- Damages
 - Delaminate damages are critical and must be avoided, specially under the stiffener foots. The behavior of the structure could not be better without weight increase.
 - The structure can support disbondings, specially with the new lay-up, being critical when both disbond centroid's are coincident.

5. References

1. Engineous Software ,“ Isight-FD v3.1 Component Guide”, 2008
2. Engineous Software, “Isight-FD v3.1 User’s Guide”, 2008
3. Doltsinis, I., ed., Stochastic “Analysis of Multivariate Systems in Computational Mechanics and Engineering”, CIMNE, Barcelona, 1999.
4. Marczyk, J., ed., “Computational Stochastic Mechanics in a Meta-Computing Perspective, International”, Center for Numerical Methods in Engineering (CIMNE), Barcelona, December, 1997.
5. Marczyk, J., “Stochastic Design Improvement: Beyond Optimization, AA/NASA/USAF Conference on Multi Disciplinary Optimization”, Long Beach, USA, September 2000.
6. Niu M. C. “Composite airframe Structures”, Hong Kong Conmilit Press, Ltd, 1992.
7. Saliby. E. , “Descriptive Sampling: An improvement over Latin Hypercube Sampling”. Proceedings of the 1997 Winter Simulation Conference.
8. Peña Sanchez, “Fundamentos de Estadística”, Alianza Editorial, Madrid 2001
9. Simulia, “Abaqus Analysis User’s Guide”, 2008
10. Simulia, “Abaqus/CAE User’s Guide”, 2008

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