Application of Abaqus for Advanced Inelastic Analysis (I : Linear Viscoelastic Materials)

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Abstract: *In the past several years, Abaqus, the leading edge of the implicit method, has added the explicit method to its line-up. Consequently, the ability to solve highly nonlinear problems, especially for two of them, i.e., geometric nonlinearity and boundary condition nonlinearity, has been extensively improved. However, as for material nonlinearity, the issue of defining the constitutive law unresolved. This is due to the fact that this issue may not be resolved solely by using FEM capabilities, but rather may be deeply involved with the management of FEM. Especially for resin materials, the effort made for the exact determination of the mechanical properties might not receive a good return. The circumstance such that many resin-made parts are purchased as completed products and very inexpensively is likely to promote this tendency. We are continuing to carry out research work during these several years aiming to find the approaches to perform practical analysis for resin materials based on the material test results that are given by measurement within the limited test range and the limited number of tests in some cases due to cost. This paper, as the first report of this study, will present herewith our findings on the analysis for some of linear viscoelastic materials.*

Keywords: *Inelastic, Viscoelastic.*

1. Introduction

Resin materials like plastics or rubber inherently demonstrate their unique features known as stress relaxation or creep. As time passes on, this inherent nature makes the material behave rheologically, yielding to relaxation in the internally induced stress. Even if a material like metal or soil foundation seems hard at first glance, it exhibits rheoligical behavior under high temperature or after a long time elapse, and finally its substance flows away. The viscoelastic material model aims to represent the process of relaxation with its elastic modulus decayed over time. Modern research work for viscoelastic materials took place in 1950s followed by its near completion by applying FEM in 1970s. However, the fact is that these days attempts to make practical use of such analysis had gradually shunned in research work. This can be argued such as being blocked by hard tasks with the difficulty in the experimental measurement as well as the difficulty in the analysis taking large deformations and contacts into account. Particularly, it makes the impression that taking the final steps towards the practical analysis for the analysis of linear elastic-viscosity has been set aside, although this analysis can definitely be widely

applicable. This paper covers the results obtained from our research works on the analysis of resin materials based on the experimental measurements of their viscoelastic properties.

2. Generalized Maxwell Model

Currently the generalized Maxwell model as illustrated in Fig.1 is utilized as a typical viscoelastic material model in general purpose FEM codes. This model is composed of multiple Maxwell models in parallel series, and intends to give approximate representations for the time-dependent properties of materials over a wide range. Denoting the modulus of elasticity of each Maxwell model as E_i, and the relaxation time of each dashpot as τ_i, by assigning each τ_i in time sequence, it is possible to create a model that is relaxed one by one beginning with the dashpot with the shortest τ_i followed by the next shortest τ_i. When a constant strain is imposed on this model, the response of stress relaxation yields as follows:

$$\sigma(t) = E_e\varepsilon_0 + \sum_{i=1}^{n} E_i e^{-t/\tau_i}\,\varepsilon_0 \tag{1}$$

Considering the ratio of stress vs. strain at every instance,σ/ε_0=E(t), Eq.(1) can be rewritten in the following expression with a form of pseudo modulus of elasticity E(t).

$$E(t) = E_e + \sum_{i=1}^{n} E_i e^{-t/\tau_i} \tag{2}$$

In other words, the generalized Maxwell model may be regarded as an elastic body of which the modulus of elasticity varies with time, and therefore E(t) is called the 'relaxation modulus'. As this E(t) is the modulus of elasticity defined under the condition of stress relaxation imposed with a strain kept constant, some literature refer to it with the notation of $E_r(t)$, but for simplicity, the notation of E(t) is used here. The response of this relaxation modulus is shown in Fig.1. At first, when a constant strain is given at t=0, E(t) is given as the total sum of all the springs since all the dashpots are frozen not to move. After that, every dashpot begins to be relaxed in the order of the value of τ_i. Consequently, as the spring connected to each dashpot is not able to bear the applied load, E(t) is gradually decreased and finally reaches the modulus of elasticity E_e of the linear spring located at the left end of the model. In the case of E_e =0, the model yields a model in which no residual elasticity is left, i.e., equivalent to fluid.

In general purpose FEM, E_i, τ_i, and E_e of each element composing the generalized Maxwell model are required to be given as the input data. For actual viscoelastic materials, at most 10 to 20 Maxwell models are generally used. It is known that, provided with the order of τ_i in which the next one has approximately an order of time greater by one digit than the former, over all responses can be represented by curves with moderate smoothness. This means that general polymeric materials can be categorized as materials with the elasticity varying during a time domain over 10 to 20 digits.

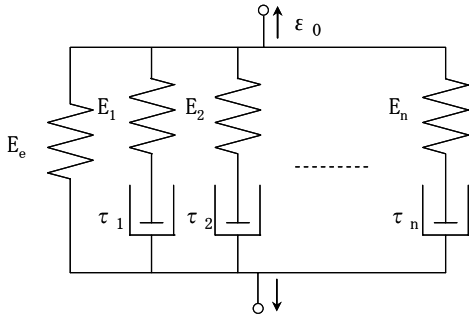


Figure 1 Generalized Maxwell Model.

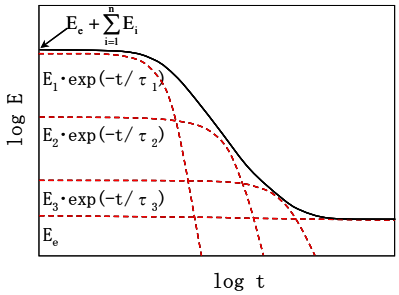


Figure 2 Relaxation modulus for generalized Maxwell model.

3. Time-temperature reductivity

3.1 Temperature-dependency of viscoelastic material

Polymeric materials have not only viscoelastic properties over a wide range of time domain but also very sensitive temperature dependency. Actually, their modulus of elasticity may show variation over a span of many thousands times within a very narrow temperature range called 'glass transition temperature'. It seems realistically impossible to measure these characteristics over a wide range of time and temperature domains by performing only a single testing. Therefore, many studies have been carried out to find methods to estimate the over all characteristics based on the measurement result within the limited range of time and temperature domains utilizing, if successfully derived, the correlation between time and temperature. The characteristic referred to as 'thermo rheoligical simplicity' as shown in Fig.3 is the most typical example of such estimation. This graph shows the relaxation modulus vs. time by logarithmic scaling. In this figure, curves in the left frame are the data measured at each of temperatures T_1 ~ T_7, and the curve in the right frame is generated based on those data and is called the 'master curve' .

First, the data in the left frame is explained. Temperature goes down towards T_1 and rises towards T_7. Upon seeing the scale on the abscissa, it is noted that the respective relaxation curve obtained at each temperature within the time domain of about10~1000sec is plotted. Viscoelastic behavior of polymeric materials is understood as to be dependent upon the size of the space called 'free volume' in which internal molecular chains are allowed to do thermal motion. In the lower side of temperatures domain, they can only elastically behave due to the smaller size of the free volume, hence it is referred to as a 'glass region'. T_1 in Fig.3 corresponds to this state. The value of E(t) in this state appears to be nearly constant at the level of 10^9Pa. As for the modulus of elasticity in the glassy region of polymeric materials, the value at the level of 10^9Pa is said as to be common.

As temperature rises, the free volume is enlarged and molecular chains become able to move against viscous resistance. This means that viscoelastic behavior appears, and this state is referred to as the 'rubbery region'. Each of T_4~T_7 in Fig.3 correspond to this state, and it is noted in this

example that the modulus of elasticity takes the values with an order of 1/1000 as low as that in the glassy region. For distinguishing the temperature at which viscoelasticity occurs, i.e., the temperature at which transition from the glassy region to the rubbery region begins, there is the material intrinsic temperature called 'glass transition temperature'. This temperature is adopted as the typical index to indicate the characteristics of polymeric solid materials. In the example provided in Fig.3, the glass transition temperature exists near the temperatures of T_2 at which the curve begins to descend. The coefficient of viscosity at the glass transition temperature is experimentally found to be about 10^{13} poise, disregarding whatever the material is.

Many attempts to classify these characteristics in terms of the reductivity between time and temperature were made in the early stage of development of viscoelasticity studies (1940s – 1950s). Most remarkable concept is such that the stress relaxation curves measured at different temperatures may be shifted along the abscissa (time axis direction) so that they may be superimposed on the same curve. This hypothesis is referred to as 'thermo rheological simplicity', and a single curve obtained is called a 'master curve'. The generated master curve is plotted as shown in the right frame of Fig.3. This curve is generated by picking the curve at T_3 in curves in the left frame to designate it as the reference curve, followed by shifting other curves towards the right or left properly along the abscissa so that they may be lined to form a single curve. As is clearly noted from this graph, a single and smooth curve is generated.

The abscissa corresponding to the generated curve covers a very wide range of time domain from 0.01sec to 10^9sec (about 30 years). Namely, Fig.3 contains a suggestion that using the test result in the range of 10 ~ 1000sec measured under several temperatures makes it possible for us to predict the behaviors covering from 0.01sec for a short term to a span of 30 years for a long term. Note that logarithmic indication of time is taken as the abscissa of Fig.3. Accordingly, the operation of

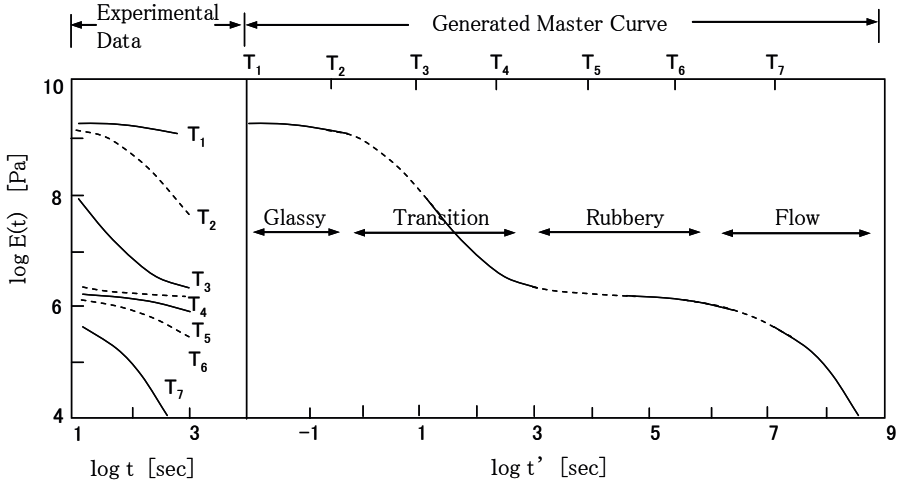


Figure 3 Master curve for temperature-dependant viscoelastic behavior.

parallel shift of the curves along the abscissa is equivalent to multiplying the time ratio α_T as shown below to each curve. α_T is defined as 'shift factor'.

$$\alpha_T = t / t' \tag{3}$$

In the above, t is the time at any arbitrary temperature T[K], and t' is the time at the reference temperature T_0[K]. It means that thermo-rheologically simple material exhibits the same degree of the temperature-dependency over all time domains, and when temperature varies from the reference temperature T_0 to any arbitrary temperature T, its relaxation time evenly becomes α_T times as high as the relaxation time at T_0. For the typical form of the time-temperature reductivity, both of the WLF form and the Arrhenius form as shown below are well-known.

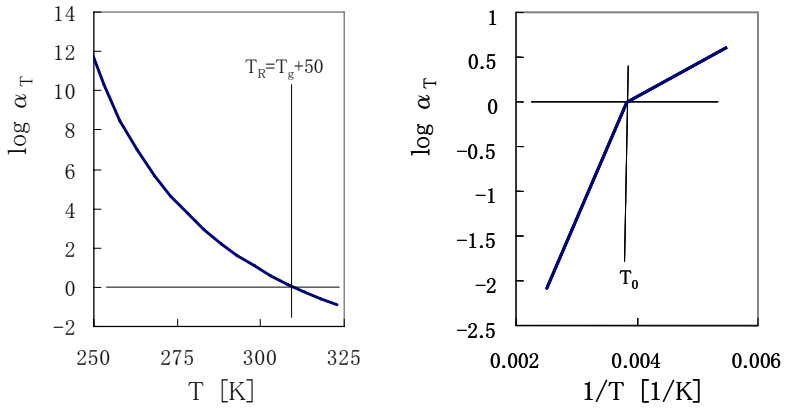
■ The WLF form

The WLF form is proposed by William, Landel and Ferry. Soft and largely deformable polymeric materials, e.g., rubber, can be well-approximated by this form. Employing the temperature by 50K as high as the glass transition temperature T_g as T_R, C_1 and C_2 for non-crystalline polymeric materials are known to take approximately the values as shown in Eq.(4). The relation between α_T and temperature at T_g=260[K] of Eq.(4) is shown in Fig.4.

$$\log \alpha_T = \frac{C_1 (T - T_R)}{C_2 + (T - T_R)} \tag{4}$$
$$T_R = Tg + 50, \quad C_1 = 8.86, \quad C_2 = 101.6$$

■ The Arrhenius form

This reductivity is established by introducing the concept of the chemical kinetics into the rheological process. ΔH_a is the activation energy 〔J/（mol K）〕, T[K] is the test temperature,



(a) WLF (T_g=260K) **(b) Arrhenius (T_0=260K)**

Figure 4 Shift factor for time-temperature reductivity.

and T_0 [K] is the reference temperature. Fig.4 shows the relation between the shift factor α_T and temperature when T_0 in Eq.(5) is taken as 260[K]. As is obviously noted from the form of the expression, if the reciprocal of temperature is given to the abscissa, Eq.(5) gives a linear relation. However, it is common practice to represent the relation with two-line segments approximation applying different ΔH_a before and after of T_0 as shown in Fig.4 The Arrhenius form is evaluated as to be appropriate to relatively hard or somehow crystalline materials.

$$\log \alpha_T = \beta \frac{\Delta H_a}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)$$
$$\beta = 1/2.303 = 0.434 \qquad (5)$$
$$\text{Gas constant} \quad R = 8.314\,\text{J/mol K} = 1.986 \times 10 - 3\,\text{kcal/mol K}$$

3.2 Result from dynamic viscoelasticity test

Regarding the generation of the master curve on the basis of time-temperature reductivity, further explanation is given here using a case of soft epoxy resin material measurement in the test. This resin material exhibits thermo-hardening behavior, and is known as thermo-rheologically simple material. The dynamic viscoelasticity of epoxy resin material was measured using a dynamic viscoelastisity testing device RSA Ⅲ (TA Instruments). The viscoelastic material response subjected to dynamic input data is detailed in the next section. Measurement was carried out in the tensile mode for a rectangular test piece of thin plate with the size of 30x5x1mm. The temperature range in this test spans from -40℃ to 60℃ （T =233～333K）.The response to the strain input at four different angular frequencies ω= 3.16, 10, 31.6, 100rad/sec （about 0.5～16Hz）respectively was measured during the process with continuous temperature rising. Instead of using this type of input procedure, any appropriate method may be employed except methods which might cause longer excitation time are not desirable because of possible temperature-variation of the test piece itself due to viscous heat generation.

Rising rate of ambient temperature is set to 1~2℃ /min according to general industrial standards. Since RSAⅢis equipped with the temperature adjusting system by blowing nitrogen gas with a large flow rate into a constant temperature bath of tough structure, it gives advantage with not simply a wide range of measured temperature, but also excellent stability in temperature control. Generally speaking, viscoelastic materials exhibit very sensitive temperature dependency. Therefore, if it is difficult to control temperature, it should be kept in mind that, irrespective of how the shift factor is adjusted, it is so hard to make the master curve ridden on a single curve.

Also, as it is evidently noted from logarithmic indication of the modulus of elasticity, the values of the load response measured during the test widely vary. Therefore, it is necessary to control the amplitude of the input strain so that appropriate magnitude of load may be generated corresponding to the sensitivity of the load cell. Usually, the amplitude of the strain in dynamic

viscoelasticity tests is about 0.1~1%, and accordingly it is pre-requisite condition that the response under the small strain condition is to be measured.

Fig.5 shows the measurement result of the storage modulus E' and the loss modulus E" against temperatures on the abscissa. In this graph, four curves are obtained for the respective frequencies of ω=3.16, 10, 31.6, and 100rad/sec. In general, as E' indicates the contribution by the energy elastically stored in the material, it may be monotonously decreased against temperature rising or reduction in frequencies. And also, E" indicates the contribution by the energy dissipated by viscosity, and it reaches the peak in the vicinity of the glassy transition temperature. More details are given in the next section.

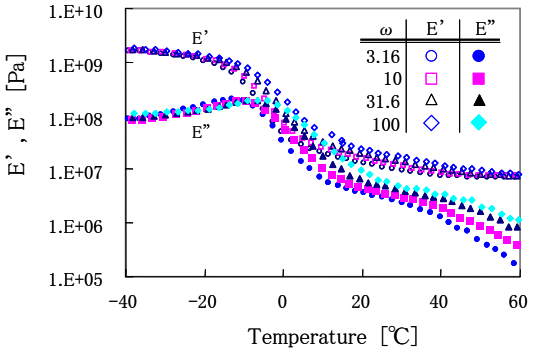


Figure 5 Storage modulus E' and loss modulus E" for epoxy.

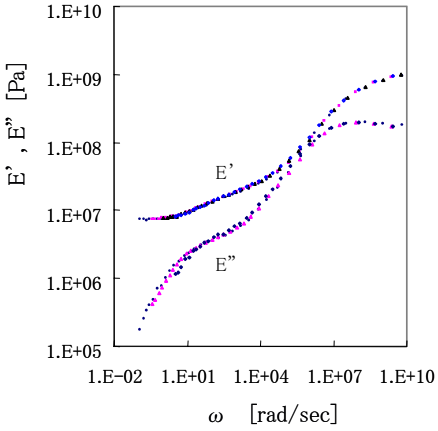


Figure 6 Generated master curve for epoxy (T_g =-14℃).

3.3 Generation of a master curve

Based on the measurement result corresponding to temperature as shown in Fig.5, an attempt is made to generate a master curve. This master curve gives the modulus of elasticity corresponding to time or frequency under the reference temperature. Since Fig.5 shows the result obtained from the measurement with the parameter as temperature and angular frequency, at first, the temperature on the abscissa is to be reduced to angular frequency. In the preceding section, α_T is defined as a coefficient to time as seen in Eq.(3), but it can be given as a coefficient to frequency. In the viscoelasticity theory, the angular frequency can generally be approximated as the function of the reciprocal of time, and so, Eq.(3) can be re-written as Eq.(6), where ω is the angular frequency at any arbitrary temperature T〔K,〕, and ω' is the angular frequency at the reference temperature T_0 [K].

$$\alpha_T = \omega'/\omega \tag{6}$$

For the epoxy resin material used in this test, it is recognized from the previous research works that the WLF form can be applied as the time-temperature reductivity. Then, for the values of temperatures of each measurement data shown in the plots in Fig.5, α_T at each point is calculated using Eq.(4). As input data of the angular frequency ω is known, the angular frequency ω' for the reference temperature can be obtained from Eq.(6). By drawing the relation between resultant ω' and measured E' or E", a new master curve based on the abscissa of angular frequency reduced from time in Fig.5 can be obtained. An example of the master curve generated following the above-mentioned procedure is shown in Fig.6. Since the generated master curve is the result corresponding to the reference temperature, if the shift factor by Eq.(4) is appropriate, it should be noted that the result must be ridden on a single curve. If T_g is taken as -14℃, both E' and E" are found to be in good superimposition and a single smooth curve is generated.

As previously mentioned, in order to generate a master curve by making all curves ridden onto a single curve with no notable dispersion, at first, it is definitely important to improve the accuracy of the measurement. Since viscoelastic material exhibits very sharp temperature dependency, it is the point that the temperature of the test piece is so even and kept stable, and also the temperature should exactly be measured. For the testing device to use, prior to performing the test, it is necessary to capture the time duration required for the initial temperature equilibrium, and appropriate temperature rising rate.

With assurance of the accuracy in the test, the next point is to set up the appropriate time-temperature reductivity. If the WLF form is applied, as the simplest way, C_1 and C_2 in Eq.(4) are fixed and the shift factor may be adjusted manipulating the glassy transition temperature T_g. In general, T_g is known to become the temperature at the peak of $\tan\delta = E''/E'$, the ratio of the storage modulus to the loss modulus. For example, it is noted from Fig.5 that T_g is set in the range of -10～-20℃. Modifying the setting of the value of T_g affects not only whether or not the curves yield a single curve, and also produces big differences between obtained curves. Because of that, the numerical range exhibited by the viscoelastic properties is represented by logarithmic scaling that can cover a very wide range, and the occurrence of the case in which the quality of the identification may cause critical defect in the results from analysis is incidental. We are developing the system to automatically determine the shift factor by employing the advanced optimization technique.

4. Dynamic characteristics of viscoelastic material

As already shown, the phenomenon of stress relaxation is expressed with the modulus of elasticity that decreases over time. On the other hand, if a harmonically varying strain is imposed, the amplitude of stress is given a different value depending on the applied frequency. That is, viscoelastic materials feature the modulus of elasticity varying not only in a time domain but also in a frequency domain. It is also known that phase lag appears between the input strain and the stress response, which indicates the development of the effect of viscosity. In 1950s when the modern research work for viscoelastic materials began, the approach in which the time domain behavior (as described with hereditary integral) is replaced with the frequency domain behavior problem applying the Laplace transform or the Fourier transform was already the known concept in the field of electric circuits and vibration phenomena. It is recognized that applying this

approach largely helped the rapid development of the viscoelasticity research work in the initial stage.

Nowadays, the dynamic test with frequency domain input is ranked as one of the main measurement methods for viscoelastic materials. One of the big reasons for such ranking comes from its advantage of giving an overall picture of viscoelastic characteristics as well as the fact that the frequency domain test requires a very short measurement time. The general testing device for dynamic viscoelasticity is so designed to be able to test within a measurement time of 10^{-6} to 1sec, while the measurement duration for stress relaxation or creep generally requires a relatively long time of 1 to 10^{6}sec. It gives very large cost saving for the dynamic testing facilities compared to creep testing. As discussed in the previous section, it is possible to estimate the long-term behavior from the measurement results on short-term behavior by utilizing the temperature dependency of viscoelastic materials, and therefore, it becomes possible to identify a viscoelasticity model over a wide range of time domain by performing the dynamic testing.

Using a dynamic viscoelasticity testing device, the storage modulus, the loss modulus, and the loss tangent are measured by giving frequency domain input data. The storage modulus represents elastic characteristic and the loss modulus represents the loss of viscous energy. These characteristics associated with dynamic viscoelasticity are calculated with the following expression, based on the amplitude of the input strain, the amplitude of the stress response, and the phase lag between the input data and the response. The relation between these input data and the response is shown in Fig.7.

$$\begin{cases} E'(\omega) = \dfrac{\sigma_0}{\varepsilon_0}\cos\delta \\ E''(\omega) = \dfrac{\sigma_0}{\varepsilon_0}\sin\delta \\ \tan\delta = E''(\omega)/E'(\omega) \end{cases} \tag{7}$$

The dynamic viscoelasticity characteristics are calculated from the response to the input frequencies. Therefore, in order to make the calculated characteristics yield a model of relaxation form like the generalized Maxwell model, it is necessary for the measurement result from these frequencies to be converted into a function of time. The relation between the characteristics of dynamic viscoelasticity E'(ω) and E''(ω), and the relaxation modulus E(τ) is expressed as follows.

$$\begin{cases} E'(\omega) = \omega\int_0^\infty E(t)\sin\omega t dt \\ E''(\omega) = -\omega\int_0^\infty E(t)\cos\omega t dt \end{cases} \tag{8}$$

The steady-state strain response shows the behavior with a phase lag as seen from Fig.7. Denoting this lag as δ, the loss tangent is given by the following expression.

$$\tan\delta = E''(\omega)/E'(\omega) \tag{9}$$

By substituting Eq.(2) into Eq.(8), the relation between the loss and storage modulus and the generalized Maxwell model can be represented with the following expression as the function of angular frequency ω.

$$E'(\omega) = E_e + \sum_{i=1}^{N} \frac{E_i \tau_i^2 \omega^2}{1 + \tau_i^2 \omega^2} \tag{10}$$

$$E''(\omega) = \sum_{i=1}^{N} \frac{E_i \tau_i \omega}{1 + \tau_i^2 \omega^2} \tag{11}$$

At this point, a master curve associated with E'(ω) and E''(ω) as illustrated in Fig.6 has been obtained from the dynamic viscoelasticity testing. If it is possible to estimate E_e, E_i, τ_i so as to be able to re-create this master curve by applying Eq.(10) and Eq.(11), it can be concluded that all the coefficients to the generalized Maxwell model have been identified. The procedure to be taken is described in the next section.

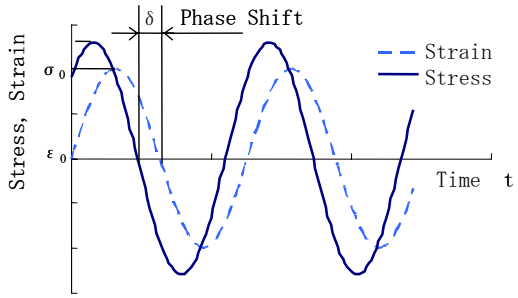


Figure 7 Dynamic stress and strain response for viscoelastic material.

5. Identification of the generalized Maxwell model and development of a curve fit program

When the generalized Maxwell model for time domain is identified based on the master curve in frequency domain using Eq,(10) and Eq.(11), the following three points are should be noted.

5.1 E_e, E_i, τ_i of positive definite

Since the generalized Maxwell model is regarded as a mechanical model, all the values of these coefficients are preferably always positive. However, as the input rule for them is different among general purpose FEMs. For example, some codes allow negative value input, while others strictly prohibit negative value input. Hence, there is no unified rule among all the codes. Since it is empirically observed that master curves may be oscillated due to the affect of terms with negative values, it is considered reasonable to control the input data so as to be positive-definite (of course, after applying some effective means to assure the obtaining of the converged approximation result).

5.2 Number of terms in the generalized Maxwell model

It is common practice to give the abscissas of a master curve, i.e., frequencies using logarithmic scale covering a range of 10 to 20 digits. To make this master curve approximated with a smooth curve, it is said that the number of terms (number of two-element Maxwell models) should be selected so as to be equal to or above the number of digits of the frequencies. In order to confirm this, a simple calculation was carried out using a single two-element Maxwell model. The calculation was performed under the following conditions.

Elastic model : E=100 Pa

Viscoelastic model : E_1=100 Pa , $\tau_1 = 1\text{sec}$

The relaxation modulus calculated using this model is shown in Fig.8. In the figure, the curve with solid line of the relaxation module is noted to decay over one digit time. It tells that a single Maxwell model is capable to represent relaxation behavior over a time domain of about one digit. Accordingly, when each dash pot is provided with the sequence of τ_i in a manner such that the next one has an order of time greater by one digit than the former, the relaxation behavior over the full range of a time domain can be expressed without a break If the abscissa of a master curve, for example, is represented with a time domain of 10 digits, the number of terms in the generalized Maxwell model may be selected to take 10 or more.

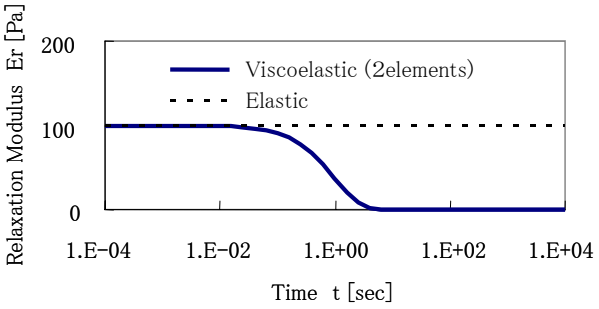


Figure 8 Relaxation behavior of single two-element Maxwell model.

5.3 Smoothness of relaxation spctra

The next task is to organize the model so that the contribution from each term is approximately smoothed. In accordance with the knowledge derived by Emri et al.[(1)], keeping smoothness of discrete relaxation spectra is effective in securing the desirable accuracy of approximation results. δ in the expression denotes the Kronecker's delta.

$$H_i(\tau) = \sum_{i=1}^{N} E_i \tau_i \delta(\tau_i - \tau) \qquad (12)$$

An example of these relaxation spectra is shown in Fig.9. An attempt was made for this example such that the envelope for these discrete spectra is approximated to be piecewise quadratic so that

the smoothness can be maintained subjected to the curvature change along this envelope being not too large. Through the testing of such provisions followed by an approximate calculation, it becomes possible to perform curve fit operation for a master curve even though data is missing. For viscoelastic materials with sharp temperature dependency, it becomes so hard for the temperature control in the measurement device to catch up to the actual response, and resultantly such a critical defect is bound to occur (Fig.10(b)), and therefore, smoothing manipulation for those relaxation spectra is a highly effective measure.

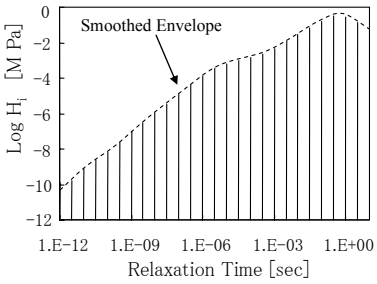
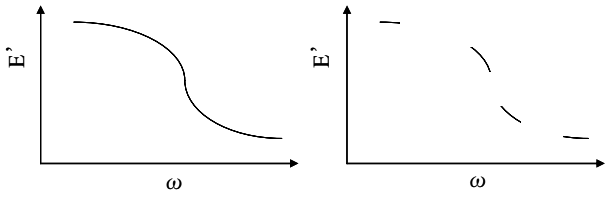


Figure 9 Smoothing manipulation for relaxation spectra.



(a) favorable **(b) defective**

Figure 10 Examples of measured master curve.

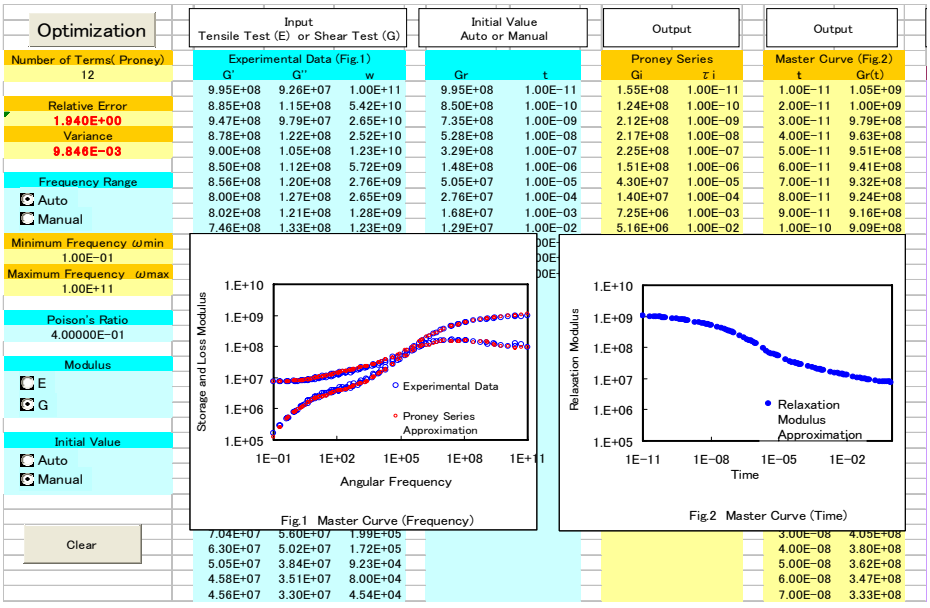


Figure11 Viscoelastic curve fit program using Excel.

With the curve fit program developed by the author' company, the generalized Maxwell model is identified based on the master curve shown in Fig.6(b). This program is designed so as to completely fulfill the constraint condition as discussed in the preceding section.

A sample output from this program is show in Fig.6 The user is only required to enter "Input data", "Number of Prony terms, and "Poison's Ratio" in the specified input field, and then press the "Optimization" button. Then the program automatically performs an approximate calculation. The optimization operation uses the quasi-Newton method. For the quasi-Newton method, it is necessary to set up the initial condition in the vicinity of the optimized value. However, this program incorporates an algorithm that can automatically estimate, from the test results, the initial condition that easily converges.

Now, the master curve for epoxy resin material shown in Fig.6(b) covers the range of about 12 digits in terms of angular frequency, and so, the number of terms in the Maxwell model is set to 12. As mentioned before, the positive values of all of . E_e, E_i, and τ_i are assured to be kept during the calculation. The coefficients of the identified Maxwell model are automatically written into the respective format of the input file for Abaqus, MSC.Marc, or LS-DYNA

6. Application to vibration control with high damping polymer

We now, as an application example, take up the transfer function of a adhesive-backed metal plate with high damping polymer sheet. Fig.12 shows a sample picture of the hammering test to be used. A test piece of a metal sheet with the dimensions 200x150x53 mm is hung by two strings connected at each corner of its upper edge. .By striking multiple points marked on the test piece with a hammer, the transfer function was measured according to the standard measurement method. In the state of the metal sheet with nothing attached, it has the natural frequencies of about 610Hz (1^{st} mode) and 720Hz (2^{nd} mode).

This high damping polymer is a specific resin material that has been developed with the aim of reducing the noise inside the cabin of a car-body and its factory product is processed to form a sheet as shown in Fig.13[(2)]. This sheet is a laminated structure in which its outer surface is an aluminum layer (t=0.22mm), the middle layer is high damping polymer (t=1.44mm), and the bottom layer is an adhesive glue (t=0.12mm), and the user can adhere it onto the sheet panels such as door panels of a car. The composition of this material has not yet been disclosed.

The measurement results for the characteristics of the high damping polymer and the glue are shown in Fig.14. It is noted in the results that the high damping polymer is designed so as to give growth of larger tan δ within the frequency range from 20Hz to 20kHz that corresponds to human's audible range. Using the method discussed in the previous sections, these measurement data were converted into the input data for Abaqus.

Abaqus allows the user to perform the frequency response analysis using the model of viscoelastic material which is defined by the generalized Maxwell model (*VISCOELASTIC). Fig.15 shows the resultant transfer function derived from this frequency response analysis, together with the measurement results for comparison purpose. It is noted from the results that the high damping

polymer is highly effective in reducing the level of the responses, and also the analysis results are found to be in good agreement with the measurement results.

In conventional methods, the effect of the vibration damper material is often expressed in terms of damping coefficient. Now, by incorporating viscoelastic models directly into the analysis, it becomes feasible to perform the simulations exactly for frequency characteristics.

7. Conclusions

We are continuing to carry out research work during these several years aiming to find the approaches to perform practical FEM analysis for resin materials based on the material test results that are given by measurement within the limited test conditions due to cost. This paper, as the first report of this study, presented our findings for some of linear viscoelastic materials. For generating the master curve in terms of the time-temperature reductivity, the procedure taken to process the data derived from the dynamic visco-elasticity tests was discussed. We have successfully developed the curve fit program so that it enables us to identify the generalized Maxwell model based on the created master curve with practical robustness and accuracy. An application of a damping polymer product to reduce the noise inside the cabin of a car-body was demonstrated. By incorporating visco-elastic models directly into the FEM analysis, it becomes feasible to perform the simulations exactly for frequency characteristics of this kind of damping products. We hereafter plan to develop more advanced modeling method for viscoelastic materials.

8. References

1. I.Emri and N.W.Tschoegl, Rheol. Acta, 32, p.311, 1993.
2. REAL SCHILD, Sekisui Chemical Co.,Ltd., http://www.sekisui.co.jp/search/detail-2758.html

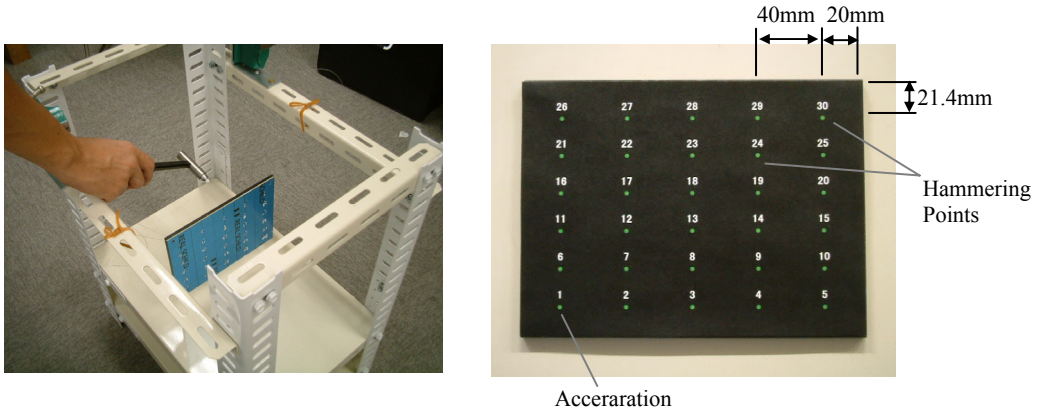


Figure 12 Hammering test for adhesive-backed metal plate with high damping polymer sheet.

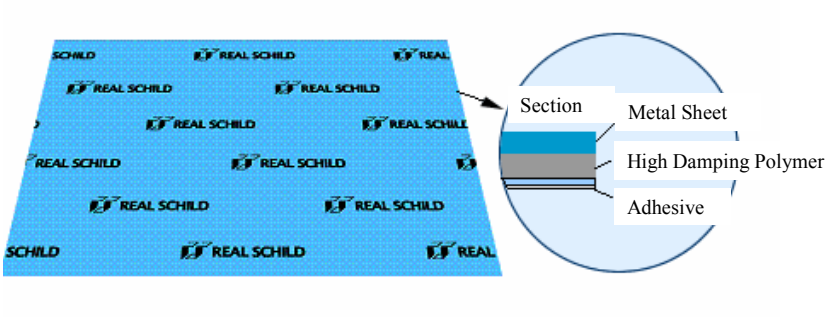
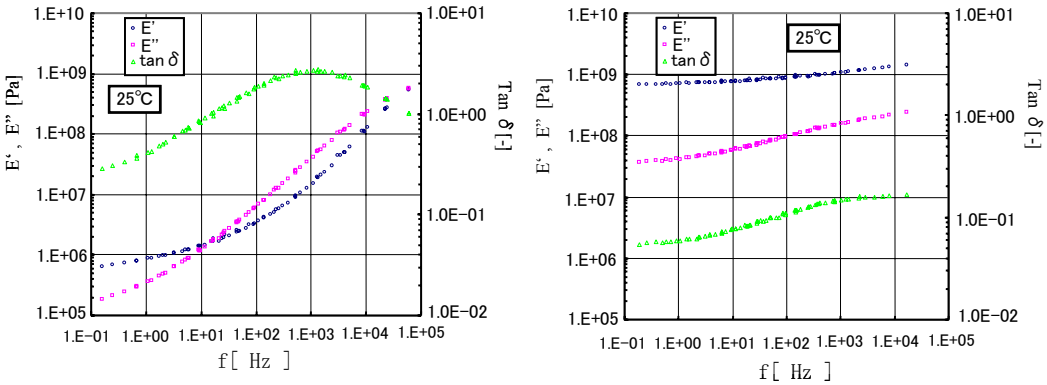
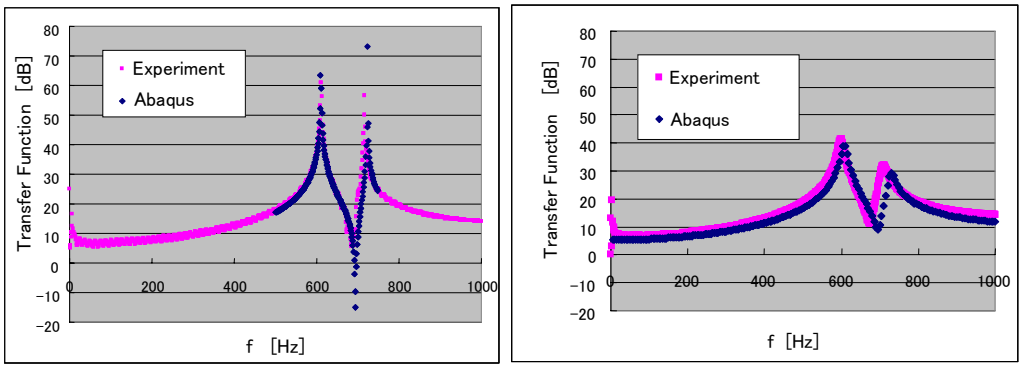


Figure 13 High damping polymer sheet (^{TM} REAL SCHILD).



(a) High damping polymer **(b) adhesive**

Figure 14 Viscoelastic behavior of high damping polymer sheet.



(a) Without high damping polymer sheet **(b) With high damping polymer sheet**

Figure 15 Comparison of transfer functions for adhesive-backed metal plate with high damping polymer sheet.