

Abaqus-based verification of the Rayleigh damping matrix in dynamic analyses for NPP civil structures

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Abstract: *In dynamic analysis of NPP civil structures the most suitable method proved to be the method of direct integration of equations of motion of the structure-soil system. This method takes account of geometrically nonlinear effects and dashpots with high level of attenuation. In addition, this method allows for receiving a highly effective solution for some types of NPP civil structures. However, the analysis of resultant response spectra has showed a high level of spectral accelerations at elevations of equipment arrangement. One of the main reasons of such result is in a high degree of conservatism, which provides for usage of the Rayleigh damping in material as compared with modal damping. This study describes an approach allowing to receive an improved damping matrix as a Caughey series that brings to diminishing conservatism when defining a dynamic reaction in the system. The study investigates influence of a number of the Caughey series terms to be accounted when defining the improved damping matrix. As a result, there are given also the floor response spectra for a real building considering damping in material as a Caughey series. The dynamic analysis has been implemented within Abaqus software frames.*

Keywords: *Concrete, Dynamics, Response Spectra, Soil-Structure Interaction, Rayleigh matrix, Caughey series*

1. Introduction

When carrying out dynamic analyses of NPP civil structures using Abaqus software the most suitable method proved to be the method of direct integration. This method, in contrast to the modal methods of analysis, allows for considering nonlinear effects, gives a more effective solution for some types of structural members, as well as allows for accounting dashpot elements for modeling energy outflow into soil during vibrations of a structure. Therefore, there has been developed a technique of dynamic analysis of the structure-soil system at seismic impact and aircraft crash based on the method of direct integration of equation of motion. This technique was

repeatedly applied in analyses of floor response spectra for NPP civil structures. However, as the analyses have showed, the technique gives extremely conservative results for the response spectra.

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One of the main reasons for the spectra high peak values is conservative initialization of material attenuation. In case of the direct integration method the material attenuation of the structure-soil system is defined according to the Rayleigh formula as follows:

$$C_I = \alpha M + \beta K, (1)$$

where C_I , K & M - matrices of damping, stiffness and mass of the system, while factors α & β are subject to definition. Having varied values α & β one can approximately choose a desirable damping value within required frequency range. Accurate damping ratios can be obtained only at any two values of circular frequency. The values α & β , in this case, are to be defined according to the formulae:

$$\alpha = \frac{2\omega_1\omega_2(\xi_1\omega_2 - \xi_2\omega_1)}{\omega_2^2 - \omega_1^2},$$

(2)

$$\beta = \frac{2(\xi_2\omega_2 - \xi_1\omega_1)}{\omega_2^2 - \omega_1^2},$$

where ω_1 - is the first tone vibration frequency, and ω_2 - is a maximum tone vibration frequency of important interest (usually $f_2 = \omega_2/2\pi$ is assumed equal to 33Hz at a seismic impact and 50Hz at an aircraft crash and air shock wave effect). The factors of modal damping - ξ_1 & ξ_2 correspond to values of frequencies ω_1 и ω_2 . Any other frequency will have damping as

$$\xi = 0.5(\alpha / \omega + \beta \omega) (3)$$

One can see from the equation (3) that the frequency damping law depends on the factors α & β . Having found an extremum of the function (3) one can show that the damping minimum value is $\xi_{min} = \sqrt{\alpha\beta}$ and obtained at frequency $\omega = \sqrt{\alpha / \beta}$.

Having applied the ratios (2) for NPP arbitrary civil structure made of reinforced concrete at $\xi = 0.04$, $f_1 = 1$ Hz, and $f_2 = 50$ Hz we receive $\alpha = 0.491750$ and $\beta = 0.000249$. The damping minimal value, in this case, is $\xi_{min} = 0.01$ and obtained at frequency $f = 7$ Hz.

It is obvious that such considerable underestimation of damping within actual vibration frequencies in comparison with the rated value $\xi = 0.04$ results in considerable overestimation of dynamic response and, as consequence, in conservatism while calculating spectral accelerations.

In this study we have applied an approach allowing to specify the damping matrix by considering a greater number of terms of Caughey series and thus, to reduce conservatism while defining floor response spectra in NPP civil structures. Similar to the study [1] we shall define the damping matrix as the following series

$$C_l^k = M \sum_{k=1}^p \alpha_k (M^{-1} K)^{k-1} \quad (4)$$

where the factors α_k are defined based on provision of natural mode shapes orthogonality. As applied to the ratio (4) this provision may be mathematically written down as follows:

$$\begin{aligned} \varphi_k^T C_l^k \varphi_k &= 2\xi_k \omega_k = \alpha_1 \varphi_k^T M \varphi_k + \alpha_2 \varphi_k^T \cdot K \cdot \varphi_k + \alpha_3 \cdot \varphi_k^T \cdot M^{-1} \cdot K^3 \cdot \varphi_k + \alpha_4 \cdot \varphi_k^T \cdot M^{-2} \cdot K^3 \cdot \varphi_k + \dots = \\ &= \alpha_1 + \alpha_2 \cdot \omega_k^2 + \alpha_3 \cdot \omega_k^4 + \alpha_4 \cdot \omega_k^6 + \dots \quad (5) \end{aligned}$$

In the equations (4) and (5) the summation is conducted according to a number of modes of special interest ($1 < k < p$, where p – corresponds to a number of considered modes). At any other frequency the damping will be

$$\xi = 0,5(\alpha_1 / \omega + \alpha_2 \cdot \omega + \alpha_3 \cdot \omega^3 + \alpha_4 \cdot \omega^5 + \dots) \quad (6)$$

The equation (4) is a generalization of the Rayleigh equation (1). If in equation (4) only two terms of Caughey series are taken into account, then an obtained ratio shall exactly comply with equation (1).

After calculation of the generalized damping matrix it is necessary to substitute it into the equation of motion (7)

$$Ku + C_l^k \dot{u} + M\ddot{u} = P(t) \quad (7)$$

and make an analysis for definition of dynamic response and appropriate response spectra.

However it is not difficult to see that the generalized damping matrix obtained by equation (4) is denser than the Rayleigh matrix (1). Here, if more components of Caughey series are considered then more densely the matrix (4) will be filled in, and at some number of terms of the series the matrix C may become filled in completely. It will result in essential increase of time for resolution of a dynamic problem. Therefore we shall make use of an approximate method that allows, with account of any number of Caughey series terms, to obtain dynamic problem

resolution time not greater than by consideration as per Rayleigh. For this, we may neglect conservatively the damping matrix terms corresponding to zero elements of the stiffness matrix. It is equivalent to the fact that as for density the damping matrix will be the same as the stiffness matrix, so the dynamic problem resolution time, in this case, will not increase.

The above-described generalized damping matrix account will give unambiguously a positive effect in sense of essential diminishing of conservatism when calculating the floor response spectra. But its realization seems to be impossible for users of the Abaqus software without access to global matrixes of stiffness and mass. Probably, developers of the software will realize soon such an access that will allow to carry out a comprehensive study of efficiency of the proposed approach. And now it is proposed, based on the real test problem, to investigate, in a roundabout way, influence of various number of the Caughey series terms in the generalized damping matrix onto resultant response spectra.

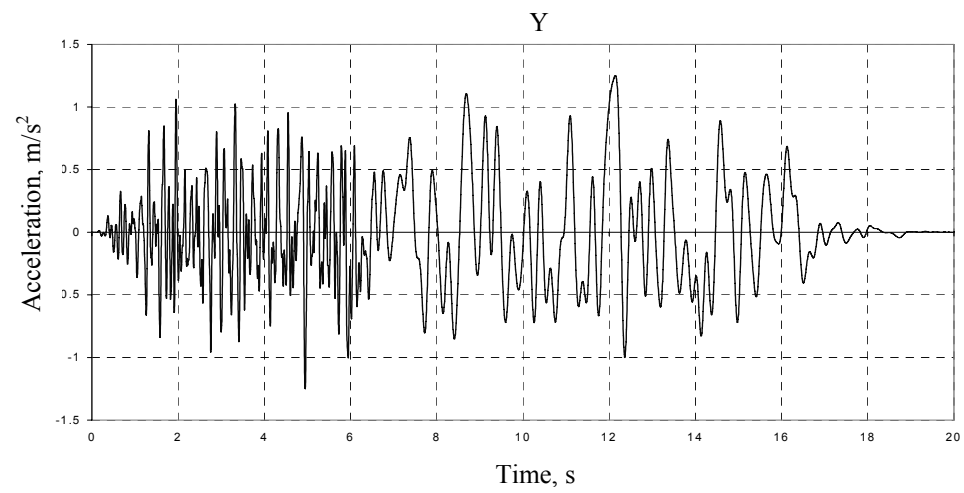
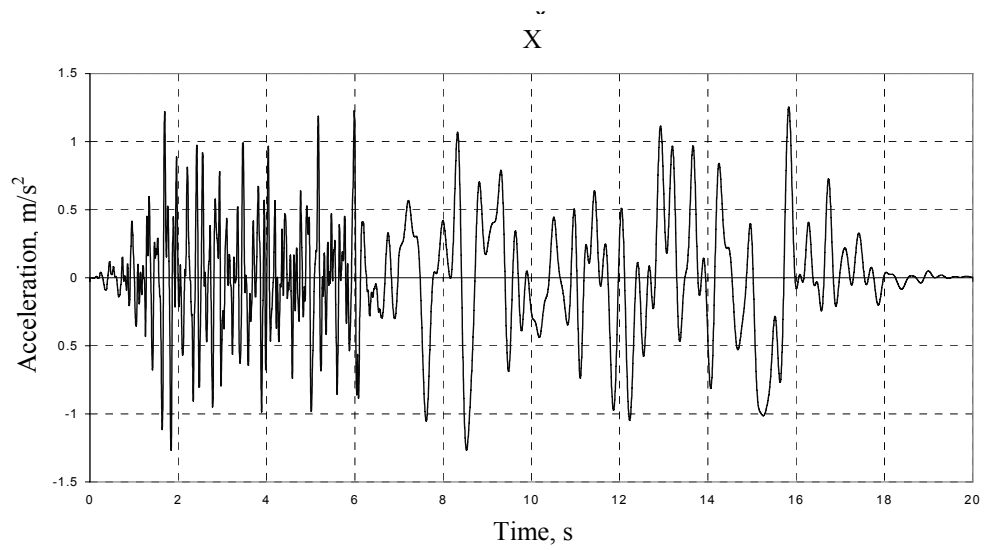
For this purpose in the beginning it is necessary to built according to equation (6) a functional damping dependence on frequency for various numbers of the Caughey series terms. Further it is necessary to take advantage of the method of modal dynamics for definition of dynamic reaction in system. Here, on a basis of the Rayleigh damping, being frequency-dependent, we shall set a modal damping for each frequency of our interest. Such an opportunity is available in the modal method. Similarly we shall act when setting damping for the generalized damping matrix presented as a Caughey series with various number of its terms. As a result, having compared the resultant response spectra corresponding to various number of the Caughey series terms in the damping matrix it is possible to estimate an effect from the account of the generalized damping matrix.

Let us notice that in addition to damping as per Rayleigh the system has got an essential factor of attenuation associated with energy outflow to the soil during vibrations of a building. This, generally speaking, prevailing factor may reduce an effect from the generalized damping matrix account. Therefore, we shall consider the system on solid soils that is equivalent to consideration of a building with rigid closing up in the basis, where is no abovementioned attenuation factor.

On the basis of such an approach of damping definition in the system there has been carried out calculation of response spectra in the building UFA (SFS) of NPP with VVER-1500 at seismic impact.

An initial three-component accelerogram of SSE level is given in Figure 1.

A general view of calculating mathematical model of the building is given in Figure 2.



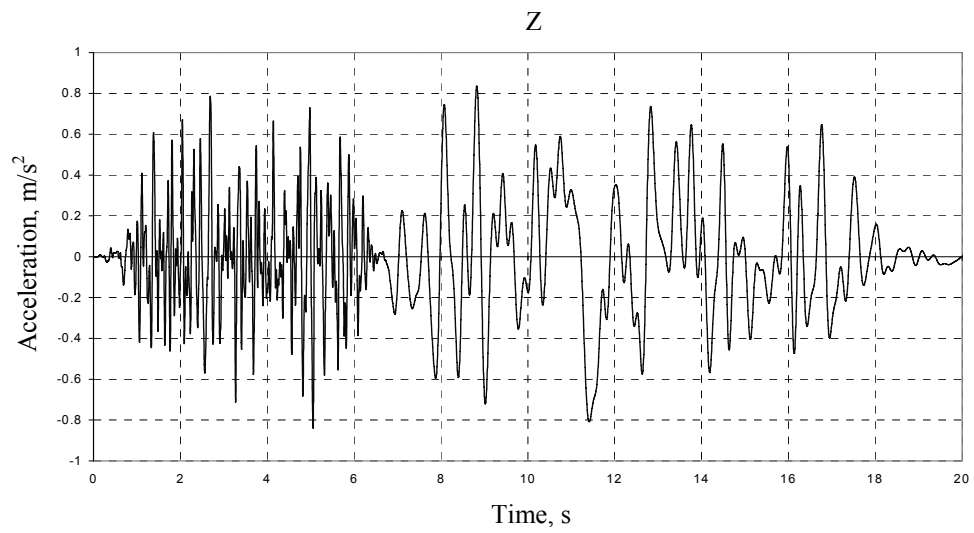


Figure 1. Initial three-component seismic impact

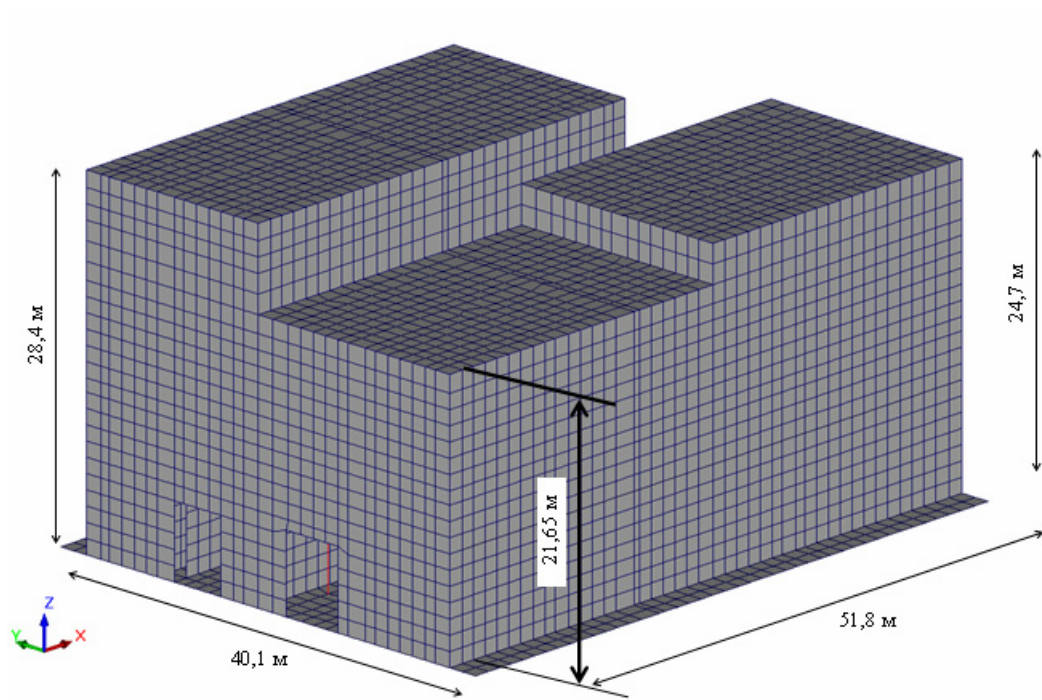


Figure 2. General view of the model of the buildings UFA/UFC

We shall define values of spectral accelerations for the floor at elevation +7.800m.

Calculations were carried out for various number of the Caughey series terms retained in equation (4). Here are inputs for the analysis:

- module of elasticity of concrete $E = 3 \cdot 10^6 \text{ t/m}^2$,
- Poisson's ratio of concrete $\nu = 0.2$
- mass density of concrete $\rho = 2.5 \text{ t/m}^3$,
- velocity of cross waves in soil $V_s = 2500 \text{ m/s}$,
- soil density $\rho_f = 2.5 \text{ t/m}^3$,

- Poisson's ratio in soil $\nu_F = 0.35$,
- damping ratio in concrete $\xi = 0.04$.

When developing a mathematical model of the system the following types of finite elements were used:

- element of beam B31- for modeling crane columns,
- element of plate S4R for modeling walls and floors,
- element of MASS for modeling weight of the crane and other equipment,
- element of spring SPRING2 for modeling stiffness of the soil base.

On the basis of the developed model, first a calculation of modal characteristics of the system was made. Then, damping as per Rayleigh was defined according to the first $f_1 = 1.7428$ Hz and, the nearest to 33Hz, $f_2 = 33.119$ Hz frequencies of natural vibrations of the system using the equation (3).

Further, in view of three and four Caughey series terms the appropriate functions of attenuation were determined according to equation (6). A comparative analysis of these functions is given in Figure 3.

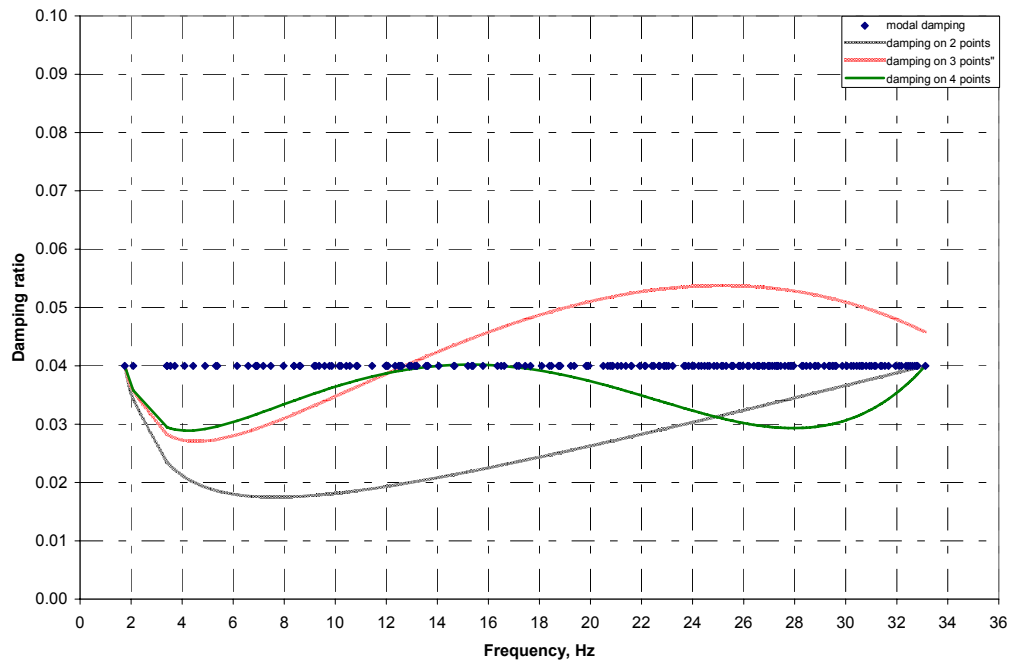


Figure 3.

In this Figure 3 there are dotted two hundred modes of natural vibrations of the system. We assume these dots as a standard. It is necessary to note that the question concerning which of these modes need to be used for specification of the damping as per Rayleigh is a separate problem. The matter is that when considering the damping function according, let us say, to three modes and with a setting for the additional third mode as – 7Hz, we receive that the maximum of the function in question essentially surpasses 0.04. It is inadmissible because it brings to essential overdamping, in comparison with value 0.04, some modes of our interest.

Therefore the third point was defined by selection based on provision that here, the relevant maximum of damping approximated, in the best way, to the value 0.04. Plotting of the Caughey function as per four points was performed on the basis of the function on three points, with a variation of frequency of the fourth point, provided that the damping value 0.04 is not exceeded.

As it is seen from Figure 3 the damping as per Rayleigh obtained according to two modes, has got an essential dip. With increase of number of the Caughey series terms this dip considerably diminishes, that reduces dynamic reaction in the system and appropriate response spectra.

Below presented are final formulae, which need to be used at definition of damping in the direct integration method in case of various number of the Caughey series terms:

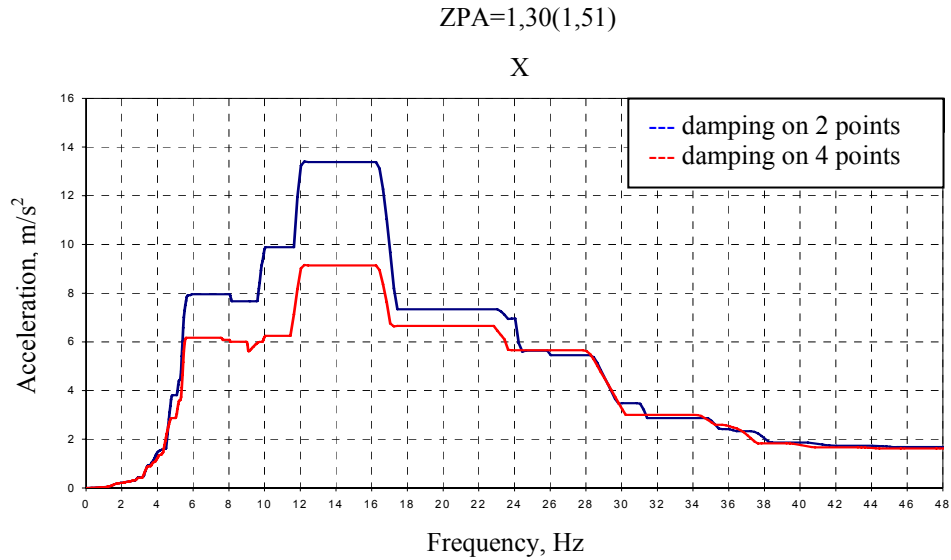
$$C_l^2 = 0.836755 \cdot M + 0.000365 \cdot K$$

$$C_l^3 = 0.771612 \cdot M + 0.000959 \cdot K - 1.2403 \cdot 10^{-0,8} \cdot M^{-1} \cdot K^2 \quad (8)$$

$$C_l^4 = 0.741199 \cdot M + 0.001159 \cdot K - 5.04 \cdot 10^{-0,8} \cdot M^{-1} \cdot K^2 + 7.43 \cdot 10^{-13} \cdot M^{-2} \cdot K^3$$

In the left parts of the presented formulae in equations (8) the top index corresponds to a number of considered Caughey series terms.

As a result of performance of dynamic calculations some response spectra for various Caughey series terms were received. Resultant response spectra are presented in Figure 4.



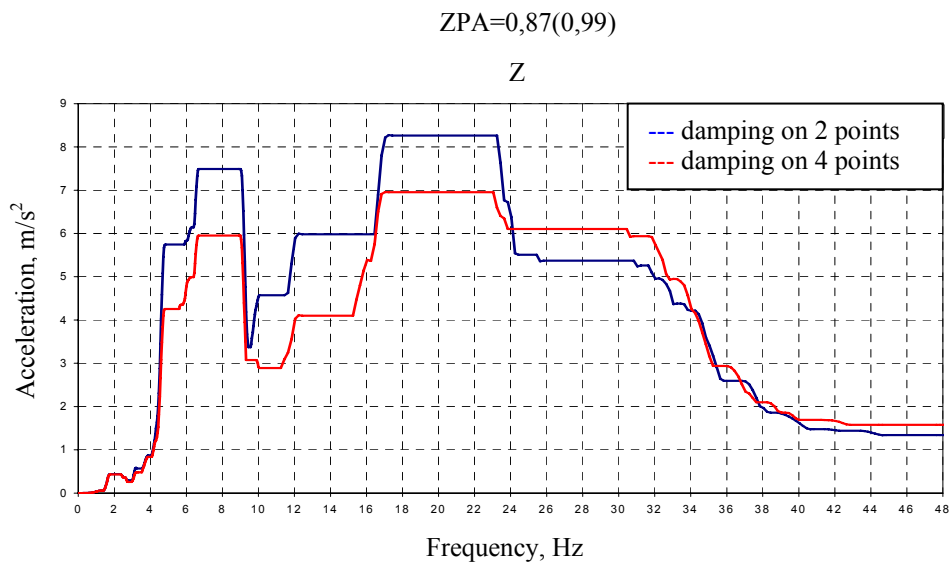
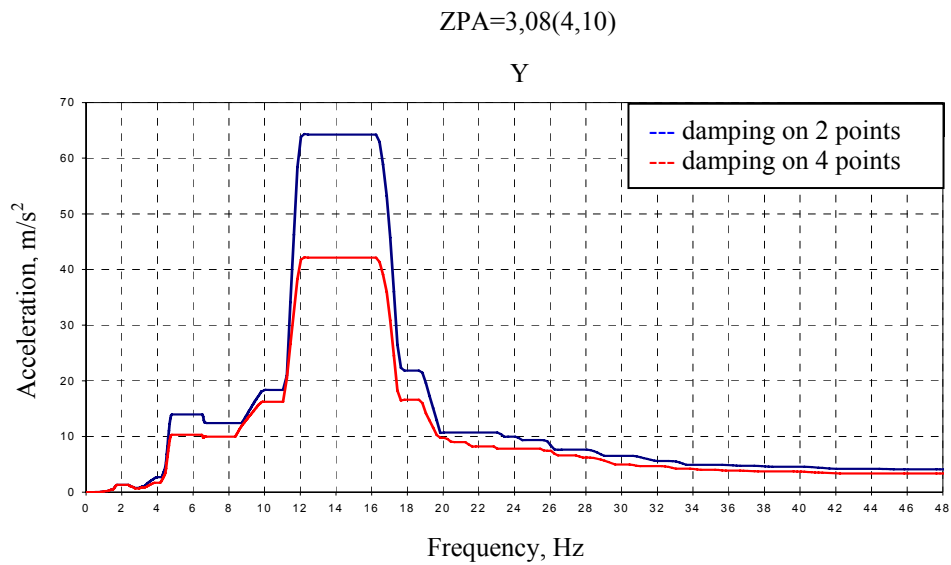


Figure 4. The enveloping response spectra of seismic impact (1500) on stiffen embedment of building UFC-UFA in homogeneous material with the damping factor 0,02 and expansion 0,15. Elevation +7,800

As evident from this Figure 4, with increase of the Caughey series terms up to four an essential decrease (down to 35%) of peak values of spectral accelerations is observed.

2. Conclusions

In the dynamic calculations of civil structures using the direct integration method the floor response spectra obtained are too conservative if the damping as per Rayleigh is applied.

A way of reduction of spectral accelerations is proposed at the expense of damping as a Caughey series.

Verification on the test problem has shown that account of damping as a Caughey series resulted in essential decrease of spectral accelerations (down to 35%)

For the final solution of the problem in question it is necessary to complete the Abaqus program that allows to account damping as a Caughey series in the direct integration method.

3. References

1. K. Bate, E. Wilson, "Numerical methods of analysis and the method of finite elements," Stroyizdat, Moscow, 1982.